Chapter 2
Planar Mechanisms

Every machine has moving members to perform a designed job. There are Prime Movers or Drives that use energy from some source to provide the necessary power to a Driven Machine or Load. The Prime Movers can be reciprocating steam engines (almost extinct now) which use steam from external combustion of coal in a boiler, reciprocating diesel or petrol engines which contain internal combustion in a cylinder, steam turbines using steam from external combustion of coal, gas turbines that use oil or gas in external combustion to produce hot gases or electrical machines that use electric power produced in power plants and transmitted to the location where it is needed. These prime movers drive a load that could be a propeller of a ship, an automobile for road transportation, a reciprocating or rotating compressor in oil and gas industries, an aircraft for air transportation, a generator to produce electricity or a machine tool that removes metal in a manufacturing process, etc.

As an example, in Figure 2.1a we show a Shaping Machine. The power from an electric motor is transferred through the crank, sliding block, rocker arm and connecting link to the cutting tool. These are rigid bodies to which the cutting tool is connected in a suitable manner to give constrained motion and remove the metal by shaping action. Theory of Machines is concerned with the layout of such members in a given machine, their motion and understanding of how the kinematic quantities, displacement, velocity and acceleration vary with time and kinetics of these members, viz., what forces and moments cause these motions. From these forces and moments, we can determine the stresses and size the members of the machine accordingly. That will be subject of Machine Design, which is not a part of Theory of Machines.

Before we proceed to formally develop the subject in a systematic manner, let us look at Figure 2.1b that depicts schematically the shaping machine of Figure 2.1a. The crank, sliding block, rocker arm, connecting link and the ram here are represented simply by lines with joints connecting them and provide a transfer of motion from the crank to the ram. Comparing Figures 2.1a and 2.1b we find the same motions are achieved by different bodies, e.g., watch the ram and the tool attached to

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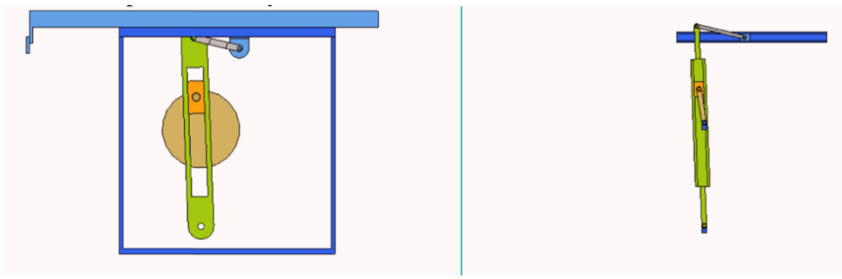


(a) (b)

Fig. 2.1 Shaping machine schematically represented

it.[1] Therefore we recognize that we can schematically represent each machine member by a rigid body. If all the bodies of the machine move in one plane as in this case, the rigid bodies are simply represented by straight line between two joints where the neighboring bodies of the machine are attached. The student should quickly learn to imagine what would be the actual shape of a machine component which might be schematically shown as a straight line (or as a triangle if there are three attachments to this body). Now let us define these machine components in a systematic manner to develop the subject further.

2.1 Basic Kinematic Concepts

The International Federation for the Promotion of Mechanism and Machine Science (IFToMM) is a world body established in 1969 for promotion of Mechanisms and Machines Science. One of its tasks is standardization of Mechanism and Machine Science terminology, the first step of which resulted in a concise publication in 2003 covering various aspects of Mechanisms and Machines. In this book, where necessary, all definitions are taken from these terminology standards.[2]

Machine

Mechanical system that performs a specific task, such as the forming of material, and the transference and transformation of motion and force. It comprises of several rigid bodies connected in such a way that it produces constrained relative motion between them and transmit forces and couples from the source of input power to result in motion, see Figures 2.1a and 2.1b.

[1] The avis in this book can be accessed from extras.springer.com.

[2] IFToMM Commission A, Terminology for the Mechanism and Machine Science, *Mechanism and Machine Theory*, Vol. 38, Nos. 7–10, pp. 598–1111, 2003.

Mechanism

System of bodies designed to convert motions of, and forces on, one or several bodies into constrained motion of, and forces on, other bodies. When we deal with a mechanism, the specific task to which it is assigned is not important, be it a shaping machine or an internal combustion engine, etc. The emphasis is on motion and force of the system and they may be applicable to any machine in which a specific mechanism may be employed. It is also a kinematic chain with one of its components (link or joint) connected to the fixed frame.

Planar Mechanism

Mechanism in which all points of its links describe paths located in parallel planes.

Link

Mechanism element (component) carrying kinematic pairing elements. Thus, the crank, sliding block, rocker arm, connecting link and the ram are the links of the shaping machine.

Bar

Link that carries only revolute joints.

Element (Pairing Element)

Assembly of surfaces, lines or points of a link through which it may contact some other link so constituting a kinematic pair. The connecting rod of an internal combustion engine is shown in Figure 2.2a. The big-end (one element) is connected to the crank-pin and the small end (another element) to the piston-pin. Since it is a rigid body, this link is usually represented as a line (as shown in Figure 2.2b) with its two end elements represented by small circles (joint). In other words, the part of a link that is connected to a neighboring link is called an element.

Joint

The physical realization of a kinematic pair.

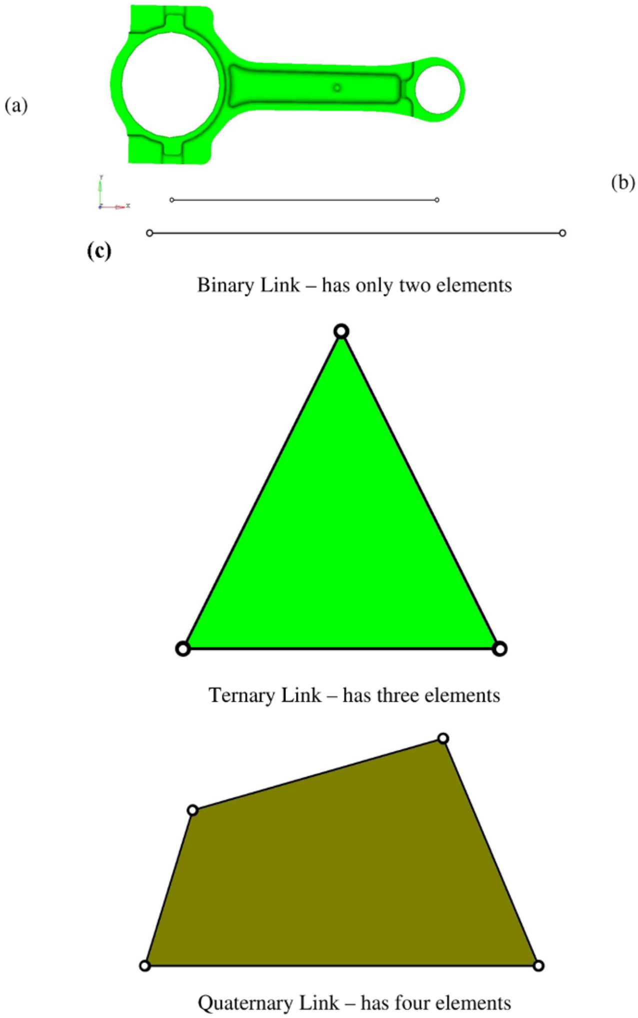


Fig. 2.2 (a) Con rod of an internal combustion engine actual rod represented by (b) a link; (c) different links

Kinematic Pair

Idealization of a physical joint that is concerned only with the type of constraint that the joint offers. There are many types of links, a few of which are shown in Figure 2.2c.

Kinematic Chain

Assemblage of links and joints.

Linkage

Kinematic chain whose joints are equivalent to lower pairs only.

Degree of Freedom [Connectivity]

The number of independent coordinates needed to describe the relative positions of pairing elements. A rigid body has six degrees of freedom as shown in Figure 2.3. Depending on the constraints imposed on the motion, the body may loose one or more of the six degrees of freedom.

Constraint

Any condition that reduces the degree of freedom of a system.

Closure of a Kinematic Pair

Process of constraining two rigid bodies to form a kinematic pair by force (force closure), geometric shape (form closure or self-closed), or flexible materials (material closure).

Force-Closed [Open] Pair

Kinematic pair, the elements of which are held in contact by means of external forces.

Form-Closed Pair [Self-Closed Pair]

Kinematic pair, the elements of which are constrained to contact each other by means of particular geometric shapes.

Figure 2.4 shows a sphere between two parallel plates giving five degrees of freedom for the kinematic pair. A circular cylinder between two parallel plates is shown in Figure 2.5, giving rise to a four degree of freedom kinematic pair.

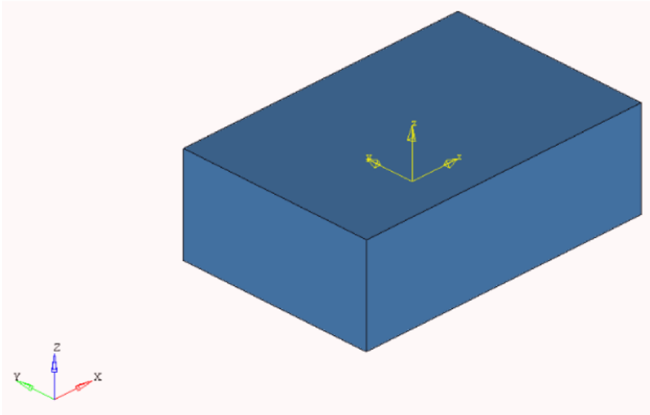


Fig. 2.3 Rigid body with six degrees of freedom

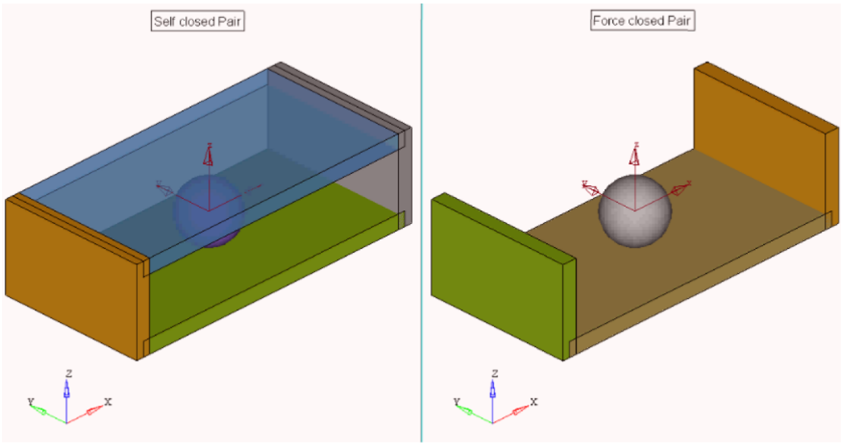


Fig. 2.4 A sphere between two parallel plates giving five degrees of freedom

Planar Contact [Sandwich Pair]

Pair for which the degree of freedom is three and that allows relative motion in parallel planes. A rectangular block between the two parallel plates in Figure 2.6 has only three degrees of freedom.

Spherical Pair

Pair for which the degree of freedom is three and that allows independent relative rotations about three separate concurrent axes.

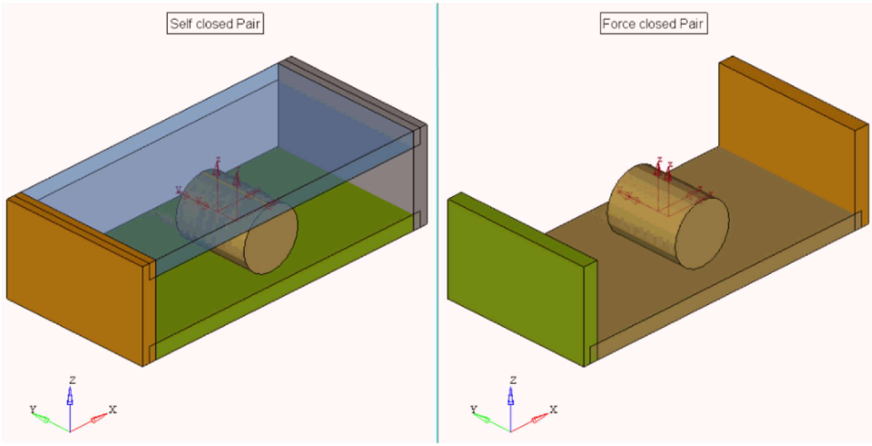


Fig. 2.5 A circular cylinder between two parallel plates with four degrees of freedom

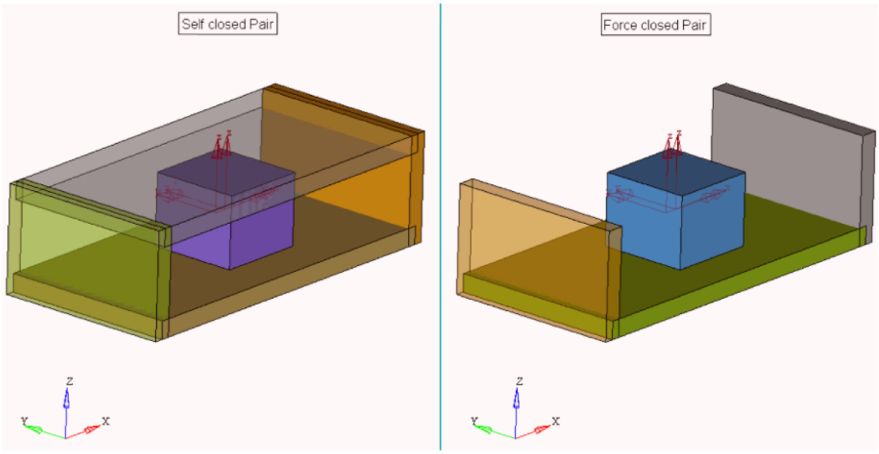


Fig. 2.6 A rectangular block between the two parallel plates has only three degrees of freedom

Cylindrical Pair

Pair for which the degree of freedom is two and that allows a rotation about a particular axis together with an independent translation in the direction of this axis. A round shaft in a coaxial cylinder in Figure 2.7 has only two degrees of freedom.

Turning Pair [Revolute Pair, Hinge]

Pair that allows only a rotary motion between its elements, see Figure 2.8.

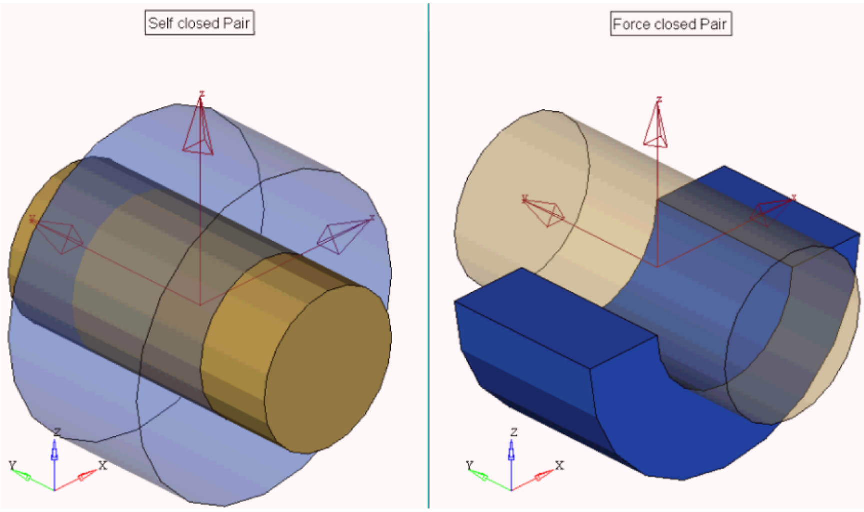


Fig. 2.7 A round shaft in a coaxial cylinder has only two degrees of freedom

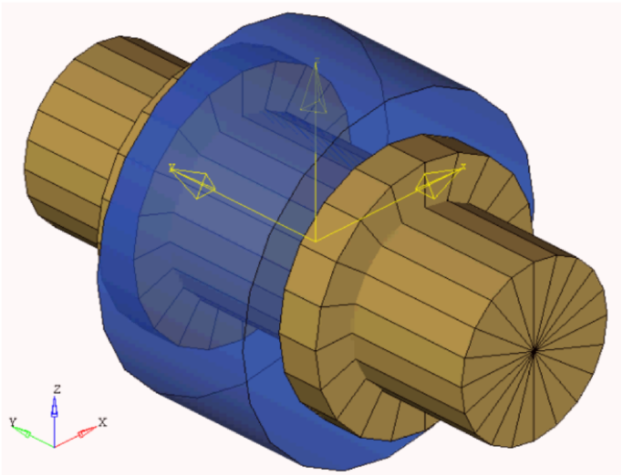


Fig. 2.8 Turning pair/revolute pair/hinge that allows only a rotary motion

Sliding Pair [Prismatic Pair]

Pair that allows only a rectilinear translation between two links, see Figure 2.9.

Screw Pair [Helical Pair]

Pair that allows only a screw motion between two links, see Figure 2.10.

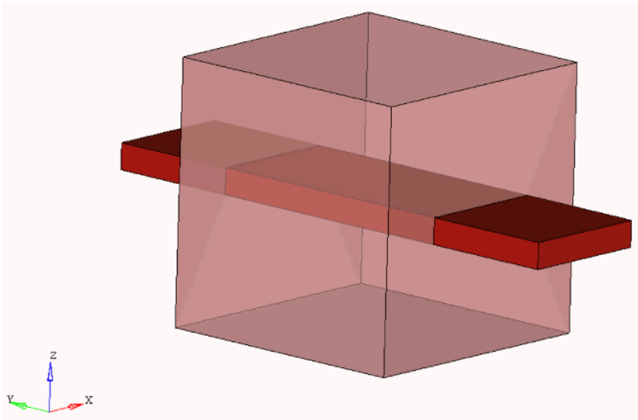


Fig. 2.9 Sliding pair/prismatic pair that allows only a rectilinear translation

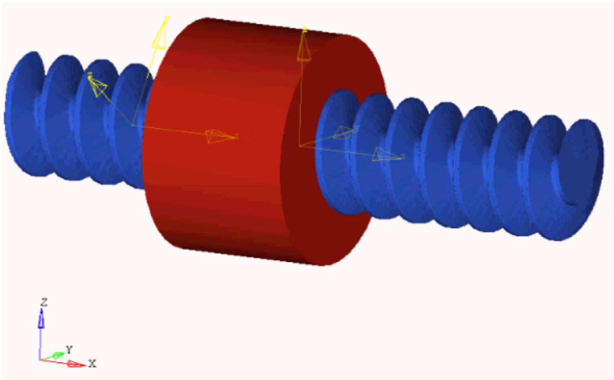


Fig. 2.10 Screw or helical pair

Lower Pair

Kinematic pair that is formed by surface contact between its elements. If A and B form a pair, the path traced by any point on element A relative to element B is identical with the path traced by any point on element B relative to element A.

Higher Pair

Kinematic pair that is formed by point or line contact between its elements.

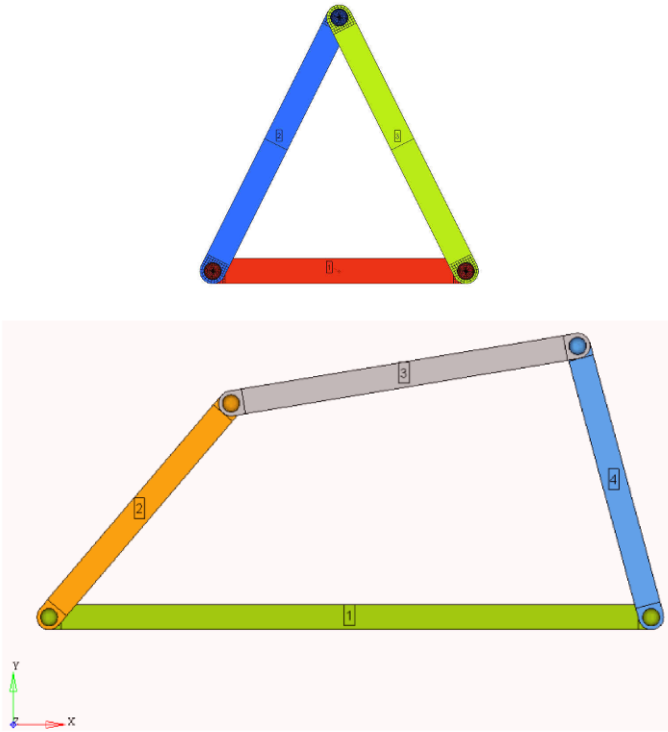


Fig. 2.11 (a) Three links form a structure. (b) Simple kinematic chain with four links

2.2 Elementary Mechanisms

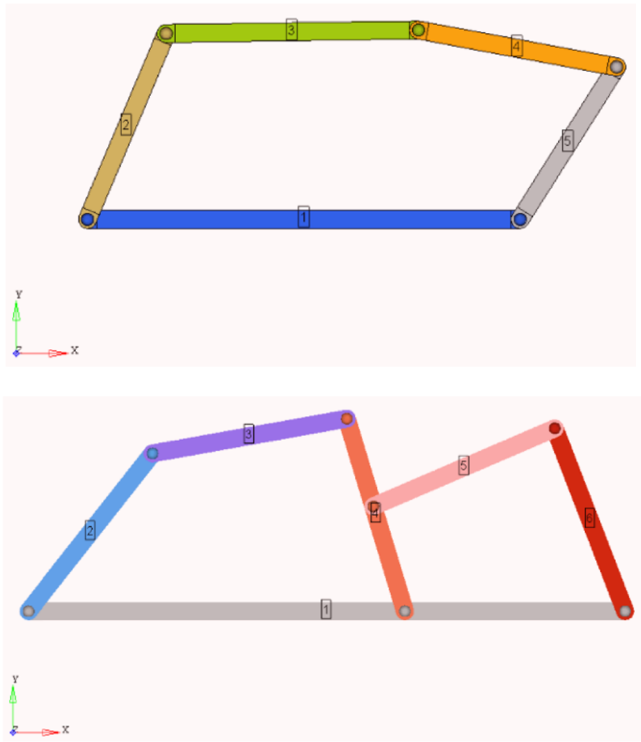
Minimum number of links to form a kinematic chain is three, as shown in Figure 2.11a. Since no relative motion is possible between these links, it forms a structure. The simplest kinematic chain has four links, as shown in Figure 2.11b.

Increasing number of links from four to five as in Figure 2.12a, the constraint of the system is completely lost. If the number of links is increased to six as in Figure 2.12b, the constraint is regained. So we see it is necessary to develop a relation that will tell us whether a given number of links can form a kinematic chain or not.

2.3 Grübler's Criterion for Planar Mechanisms

The number of degrees of freedom, F, of a planar mechanism with n links, j lower kinematic pairs and h higher kinematic pairs is

$$F = 3(n - 1) - 2j - h \tag{2.1}$$



(a) (b)

Fig. 2.12 (a) Five-link chain has no constrained motion. (b) Six-link chain has constrained motion

For constrained motion ($F = 1$)

$$2j - 3n + h + 4 = 0 \tag{2.2}$$

Although Grübler's criterion is applicable in almost all cases, a few exceptions exist, e.g., a fly-press shown in Figure 2.13.

Let us take an example of a six-bar linkage shown in Figure 2.14. Determine the degrees of freedom. There are four binary links and two ternary links. The number of joints are (you can count them directly or use the following formula)

$$\begin{aligned} j &= \frac{1}{2}(2n_2 + 3n_3) \\ &= \frac{1}{2}(2 \times 4 + 3 \times 2) = 7 \end{aligned}$$

The number of degrees of freedom from equation (2.1) is

$$F = 3(6-1) - 2 \times 7 = 1$$

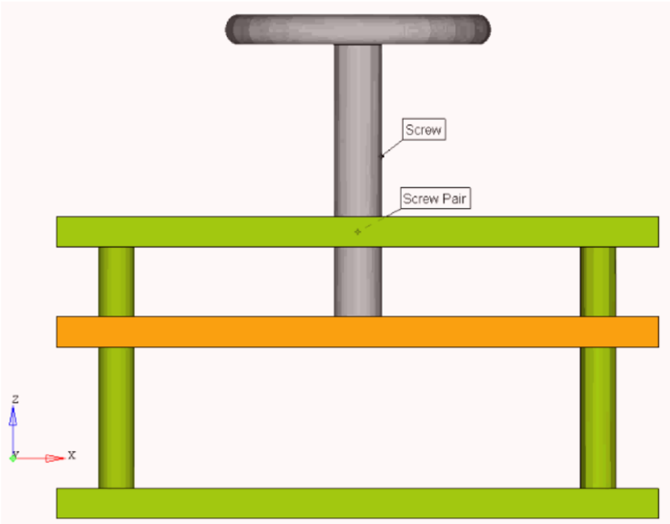


Fig. 2.13 A fly-press – an exception to Grübler's criterion

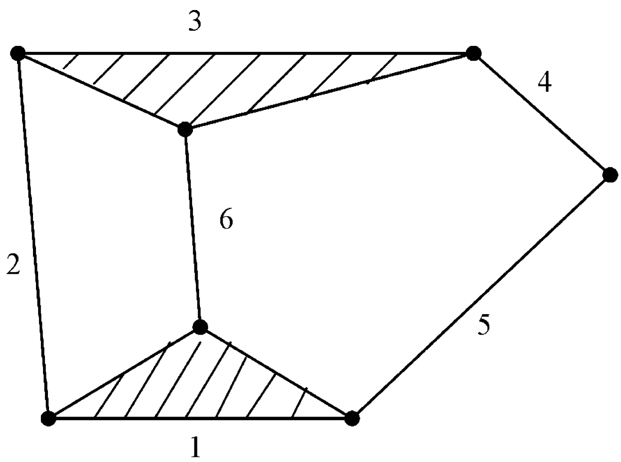


Fig. 2.14 A six-bar linkage

Thus, this linkage has one degree of freedom. If the link 1 is fixed to a frame and the link 2 is driven by a motor, the motions of the rest of the links 3 to 6 will be unique.

Consider another example, an eight-bar linkage shown in Figure 2.15. Determine the degrees of freedom.

There are five binary links ($n_2 = 5$), two ternary links ($n_3 = 2$) and one quaternary link ($n_4 = 1$). The number of joints is

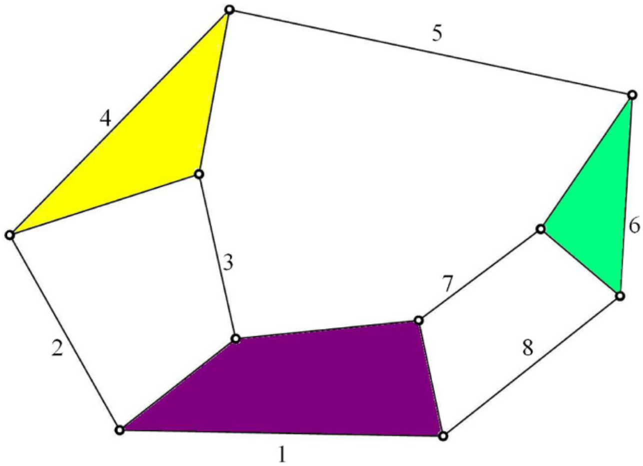


Fig. 2.15 Eight-bar linkage

$$j = \frac{1}{2}(2 \times 5 + 3 \times 2 + 4 \times 1) = 10$$

The number of degrees of freedom from equation (2.1) is

$$F = 3(8 - 1) - 2 \times 10 = 1$$

Thus, this linkage has also one degree of freedom. If the link 1 is fixed to a frame and the link 2 is driven by a motor, the motions of the rest of the links 3 to 8 will be unique.

Now let us consider a seven-bar linkage in Figure 2.16, the number of degrees of freedom can be shown to be two. If the link 1 is fixed to a frame, we need two inputs, e.g., links 2 and 5 to be driven, and then the motions of the rest of the links 3, 4, 6 and 7 will be unique.

Finally consider a six-bar linkage in Figure 2.17 with link 6 having a sliding motion on the fixed frame link 1. There are two binary links, 3 and 6; the remaining four are ternary links. Here, the fixed link 1 has two lower pairs (joints or hinges) and one higher pair (sliding pair) with link 6 which has a lower pair with link 5, therefore it is a ternary link. The number of joints (lower pairs) are $j = 7$ and there is one higher pair $h = 1$. Hence

$$F = 3(6 - 1) - 2 \times 7 - 1 = 0$$

Thus, this linkage has no degrees of freedom. It forms a structure.

Finally, let us show that the five-bar linkage in Figure 2.18 is not capable of producing relative motion. $N = 5$, $j = 6$ and therefore $F = 0$.

The study in Section 2.3 helps us in setting up a number of links with an appropriate number of elementary pairs, lower or higher to obtain a desired motion

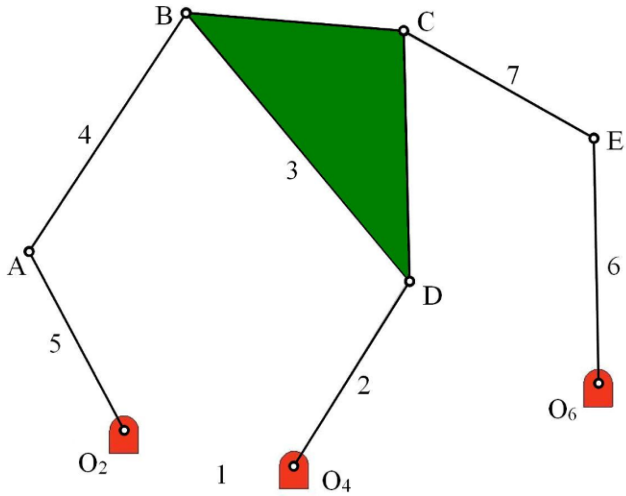


Fig. 2.16 Seven-bar linkage

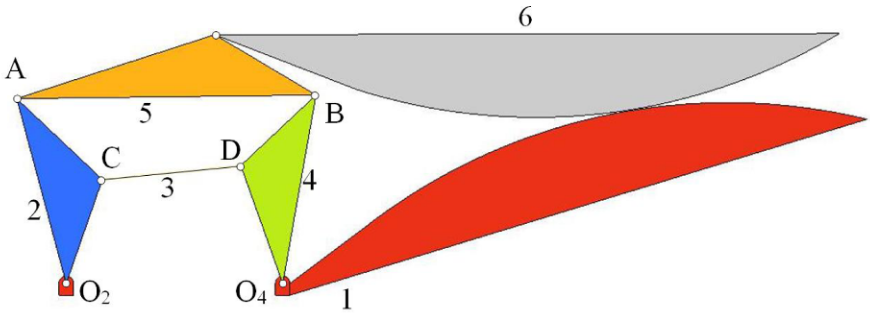


Fig. 2.17 Six-bar linkage

of a new machine or analyze an existing machine. Now let us consider a simple kinematic chain and see what we can do with that.

2.4 Four-Link Chains

Four-link chains with lower pairs are schematically represented in Figures 2.19a, 2.19b, 2.19c and 2.19d.

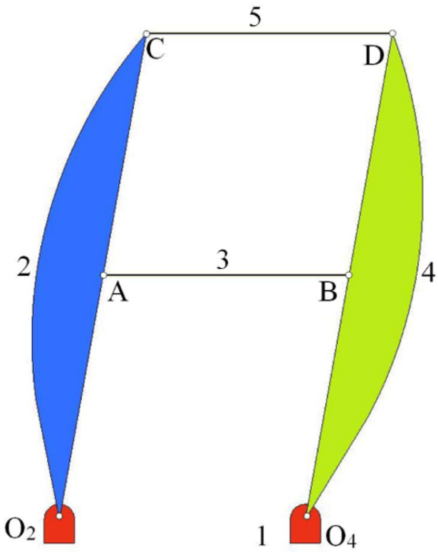


Fig. 2.18 Five-bar linkage

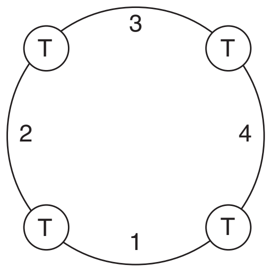


Fig. 2.19 **(a)** Quadric cycle chain with all turning pairs

Quadric Cycle Chain

Four-link chain with all turning pairs (joints or hinges), see Figure 2.19a.

Single Slider Chain

A quadric cycle chain with one of its turning pair replaced by a sliding pair, see Figure 2.19b.

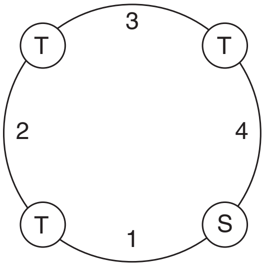


Fig. 2.19 (b) Quadric cycle chain with single slider

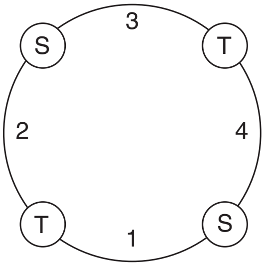


Fig. 2.19 (c) Crossed double slider quadric cycle chain

Crossed Double Slider Chain

A quadric cycle chain with two sliding pairs located opposite to each other, Figure 2.19c.

Double Slider Chain

A quadric cycle chain with two sliding pairs located next to each other, Figure 2.19d.

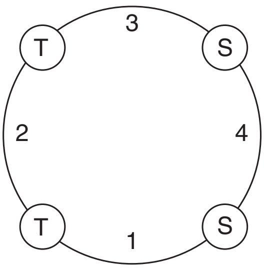


Fig. 2.19 (d) Double slider quadric cycle chain with adjacent sliding pairs

2.5 Kinematic Inversion

Four-Bar Linkage

Linkage with four binary links.

Four-Bar Mechanism

Mechanism with four binary links.

Crank

Link that rotates completely about a fixed axis.

Rocker [Lever]

Link that oscillates within a limited angle of rotation about a fixed axis.

Input [Driving] Link

Link where by motion and force are imparted to a mechanism.

Output [Driven] Link

Link from which required motion and forces are obtained.

Coupler [Floating] Link

Link that is not directly connected to the fixed link or frame.

Slider

Link that forms a prismatic pair (sliding pair) with one link and a revolute (turning) pair with another link.

Sliding Block

Compact element of a prismatic pair that slides along a guiding element.

Guide

Element of a prismatic pair that is fixed to a frame and constrains the motion of a sliding block.

Crosshead

Component between a piston and a connecting rod which, by forming a prismatic joint with the frame, provides a reaction to the component of force in the connecting rod normal to the line of stroke of the piston.

Connecting Rod

Coupler between a piston and or a cross-head and a crank shaft.

Kinematic Inversion

Transformation of one mechanism into another by choosing a different member to be the frame (fixed link).

We defined a mechanism in Section 2.1 as a kinematic chain with one of its components (link or joint) connected to the fixed frame. We can choose any one of the four links in a quadric cycle chain as a ground link that will produce different versions of mechanisms from the same quadric cycle chain. The mechanisms thus obtained are Kinematic Inversions of the original kinematic linkage.

Inversions of Quadric Cycle Chain

Kinematically speaking all inversions of a Quadric Cycle Chain are the same, however, by suitably altering the lengths of links, l_1, l_2, l_3 and l_4, different mechanisms can be obtained. These are described here.

1. *Crank-and-Rocker mechanism* (*Crank-Lever mechanism*): This mechanism is shown in Figure 2.20. Link 2 is the crank and link 4 is the rocker or lever. Link proportions for this case are:

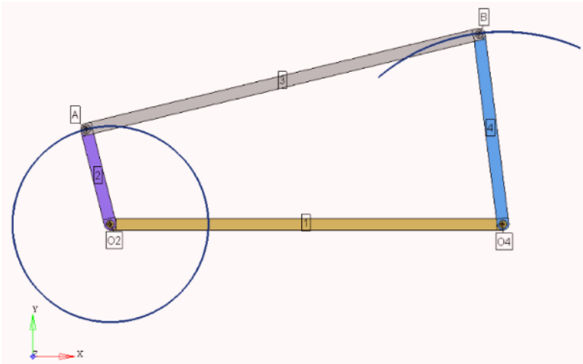


Fig. 2.20 Crank-lever mechanism

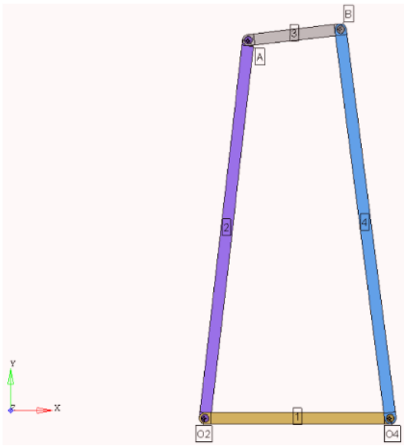


Fig. 2.21 Double-lever mechanism

$$(l_2 + l_3) < (l_1 + l_4)$$
$$(l_3 - l_2) > (l_1 - l_4) \tag{2.3}$$

2. *Double-Rocker mechanism* (*Double-Lever mechanism*): This is a four-bar mechanism with two rockers as shown in Figure 2.21. In this case

$$(l_3 + l_4) < (l_1 + l_2)$$
$$(l_2 + l_3) < (l_1 + l_4) \tag{2.4}$$

3. *Double-Crank mechanism*: Four-bar mechanism with two cranks.

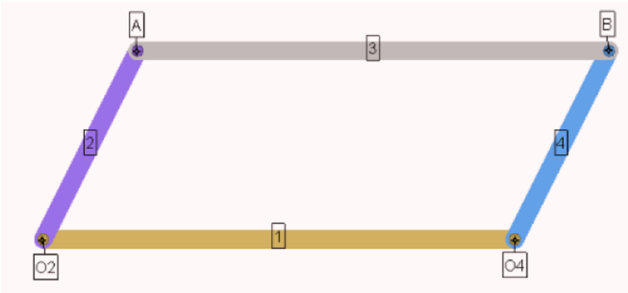


Fig. 2.22 Parallel-crank mechanism

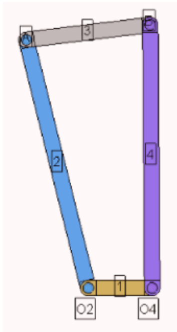


Fig. 2.23 Drag-link mechanism

a. *Parallel-Crank mechanism* Four-bar mechanism having cranks of equal length and a coupler with length equal to that of the fixed link (frame), see Figure 2.22.
b. *Drag-Link mechanism*, see Figure 2.23. In this case, the ground link is the shortest one and the coupler should be longer than that. Both the input and output links perform complete 360° revolutions. The coupler satisfies the following conditions:

$$l_3 > l_1 \quad \text{and} \quad l_4 > l_2$$
$$l_3 > (l_1 + l_4 - l_2)$$
$$l_3 < (l_2 + l_4 - l_1) \tag{2.5}$$

Branching Condition [Change Point Condition]

When

$$(l_1 + l_3) = (l_2 + l_4) \tag{2.6}$$

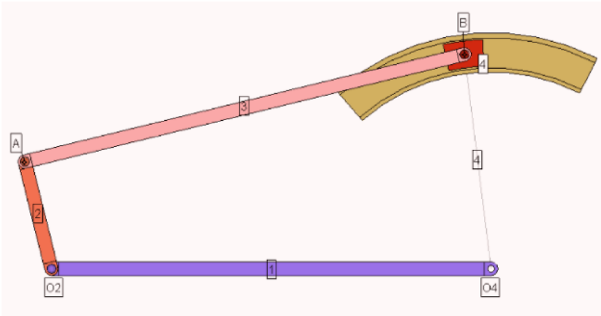


Fig. 2.24 Single slider chain obtained by replacing one revolute pair by a slider

the mechanism suffers from branching or a change point condition. At the change point, the center lines of all links become collinear and the output link may suffer a change in direction unless additional guidance is provided. (Note that a given four-bar mechanism can be drawn in two configurations, one normal-linked and the other cross-linked.)

Now let us find all the inversions of a quadric cycle chain with $l_1 = 10$, $l_2 = 20$, $l_3 = 30$ and $l_4 = 40$ cm.

1. *Crank-and-Rocker mechanism (Crank-Lever mechanism)*
   (a) $l_1 = 20, l_2 = 10, l_3 = 30$ and $l_4 = 40$ cm (Change Point)
   (b) $l_1 = 20, l_2 = 10, l_3 = 40$ and $l_4 = 30$ cm
   (c) $l_1 = 30, l_2 = 10, l_3 = 20$ and $l_4 = 40$ cm (Change Point)
   (d) $l_1 = 30, l_2 = 10, l_3 = 40$ and $l_4 = 20$ cm
   (e) $l_1 = 40, l_2 = 10, l_3 = 20$ and $l_4 = 30$ cm
   (f) $l_1 = 40, l_2 = 10, l_3 = 30$ and $l_4 = 20$ cm

2. *Double-Rocker mechanism (Double-Lever mechanism)*
   (a) $l_1 = 40, l_2 = 20, l_3 = 10$ and $l_4 = 30$ cm
   (b) $l_1 = 30, l_2 = 20, l_3 = 10$ and $l_4 = 40$ cm
   (c) $l_1 = 20, l_2 = 30, l_3 = 10$ and $l_4 = 40$ cm

3. *Double-Crank mechanism*
   (a) $l_1 = 10, l_2 = 20, l_3 = 30$ and $l_4 = 40$ cm
   (b) $l_1 = 10, l_2 = 20, l_3 = 40$ and $l_4 = 30$ cm (Change Point)
   (c) $l_1 = 10, l_2 = 30, l_3 = 20$ and $l_4 = 40$ cm

Inversions of Single Slider Chain

Link 4 of a four-bar mechanism with turning pairs is replaced by a slider; see Figure 2.24 to form a single slider chain.

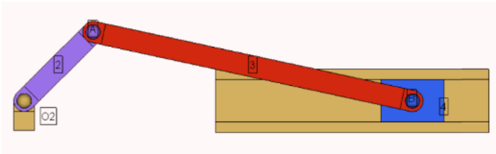


Fig. 2.25 Reciprocating engine mechanism

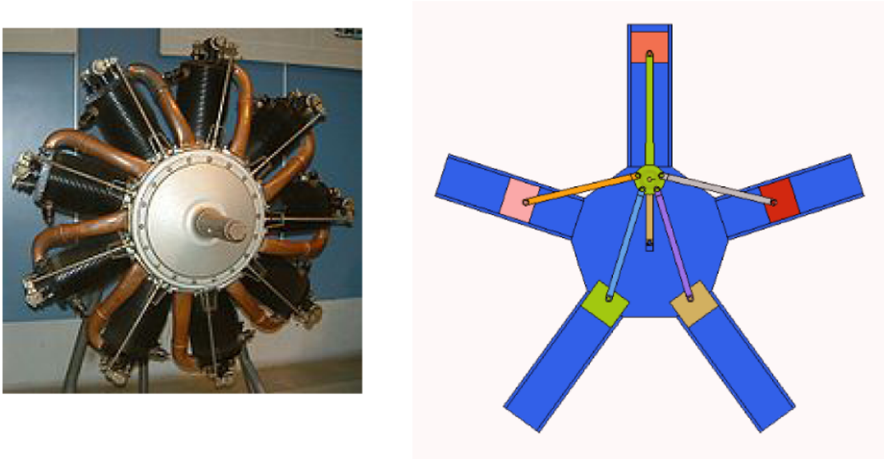


Fig. 2.26 Rotary engine with fixed crank

1. *Inversion with link 1 fixed*: Reciprocating engine mechanism, see Figure 2.25. This is the most common mechanism used to day in all internal combustion engines.
2. *Inversions with link 2 fixed*:

   a. *Rotary Engine*: See Figure 2.26. The engine shown has five cylinders. Out of the five connecting rods, one is a master connecting rod and the other four are slave rods. The crank 2 is common to all five cylinders and is fixed. Link 1 is the engine block which rotates. Le Rhône 9C, a typical rotary engine of WWI is shown on the left. The copper pipes carry the fuel-air mixture from the crankcase to the cylinder heads.
   The design was used mostly in the years shortly before and during World War I to power aircraft, and also saw use in a few early motorcycles and cars. By the early 1920s the rotary aircraft engine was becoming obsolete, mainly because of an upper ceiling to its possible output torque, which was a fundamental consequence of the way the engine worked. It was also limited by its inherent restriction on breathing capacity, due to the need for the fuel/air mixture to be aspirated through the hollow crankshaft and crankcase, which directly affected its volumetric efficiency. However, at the time it was a very efficient solution to the problems of power output, weight, and reliability. The

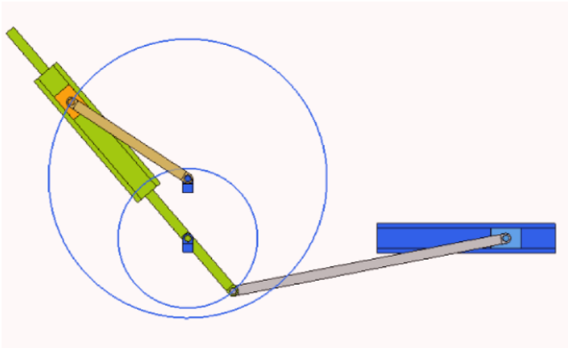


Fig. 2.27 Whitworth quick return motion mechanism

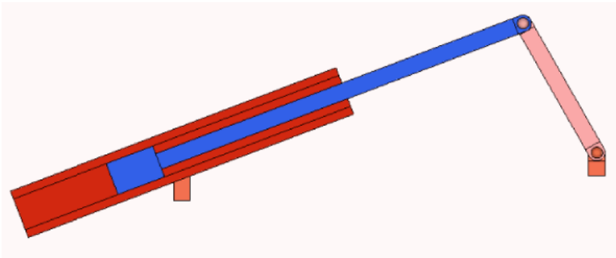


Fig. 2.28 Oscillating cylinder engine

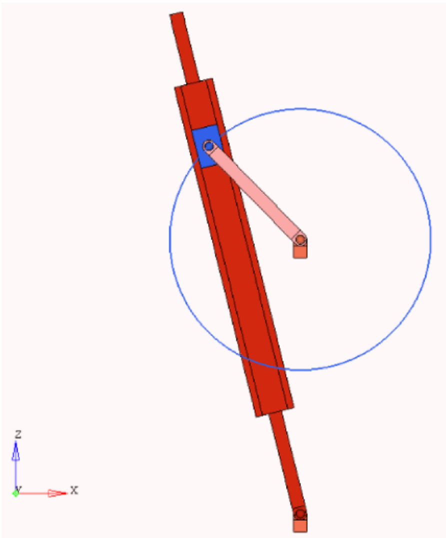


Fig. 2.29 Quick return mechanism

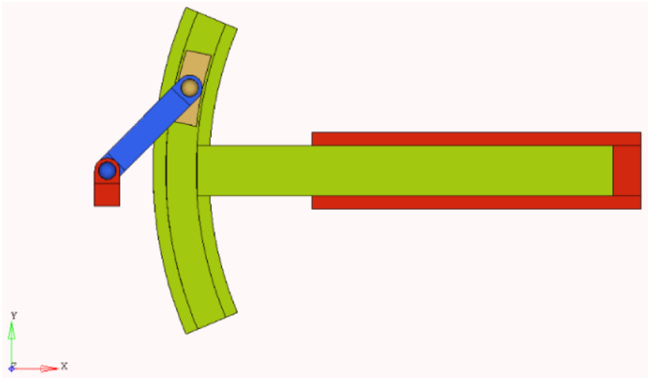


Fig. 2.30 Formation of a double-slider chain

rotation of the bulk of the engine's mass produced a powerful gyroscopic flywheel effect, which smoothed out the power delivery and reduced vibration. Vibration had been such a serious problem on conventional piston engine designs that heavy flywheels had to be added. Rotary and radial engines look strikingly similar when they are not running and can easily be confused, since both have cylinders arranged radially around a central crankshaft.

b. *Whitworth Quick Return mechanism*: See Figure 2.27. Link 3 is the crank here. Slider 4 drives link 1. Link 1 drives a cutting tool through the connecting rod 5. The forward stroke starts with link 3 in position *AQ* and ends at *AP* through *AS*. The return motion is faster from *AP* through *AR* to *AQ*. These linkages are most useful in saving time, since the return stroke, which is an idle stroke, is faster than a useful forward stroke when metal is removed.

3. *Inversions with link 3 fixed*

   a. *Oscillating Cylinder Engine*: See Figure 2.28.
   b. *Quick Return Mechanism*: See Figure 2.29. Link 2 is the crank; link 4 is the rocker arm. The forward stroke starts with link 2 in position *P* and ends at *Q* through *S*. The return motion is faster from *Q* through *R* to *P*.

4. *Inversion with link 4 fixed*: These are same as case 1.

Inversions of Double Slider Chain

Figure 2.30 illustrates how a double slider chain is formed.

1. *Inversion with link 1 fixed*: Scotch–Yoke mechanism, see Figure 2.31. This is a four-bar mechanism in which a crank is connected by a slider with another link which, in turn, forms a prismatic pair with the frame. Kinematically, this is the same as fixing link 3.

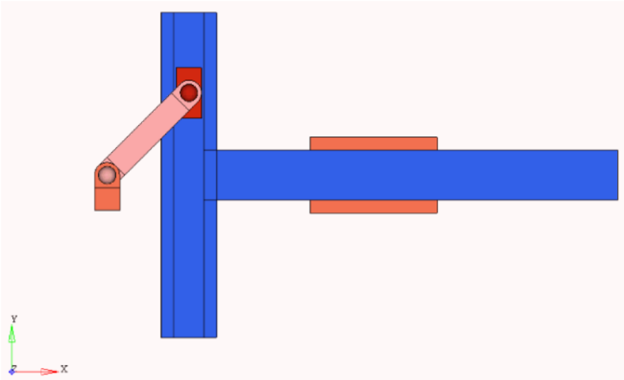


Fig. 2.31 Scotch–Yoke mechanism

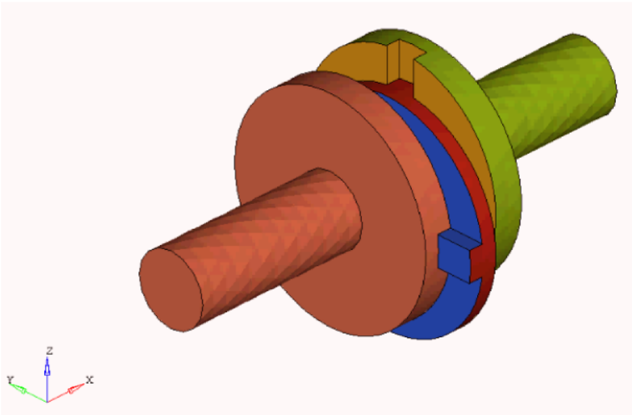


Fig. 2.32 Oldham's coupling

2. *Inversion with link 2 fixed*: This gives rise to Oldhams coupling in Figure 2.32. Links 1 and 3 have slotted grooves to form sliding pairs with corresponding faces of link 4.
3. *Inversion with Link 4 fixed*: This gives an Elliptic Trammel mechanism, see Figure 2.33.

We looked at a quadric cycle chain and various useful mechanisms it can generate and that are exploited in developing machines. In a similar way, one can explore multi-link chains and make a systematic study to identify possible new machines. This is out of the scope of the present book.

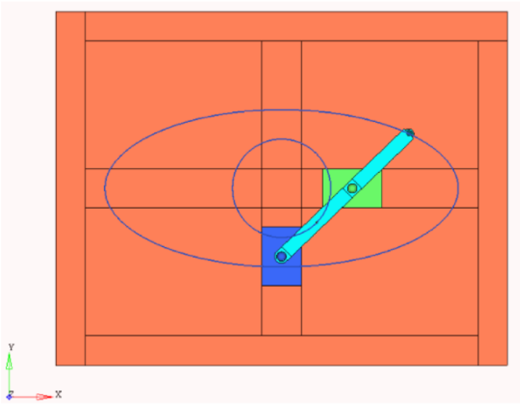


Fig. 2.33 Elliptic trammel mechanism

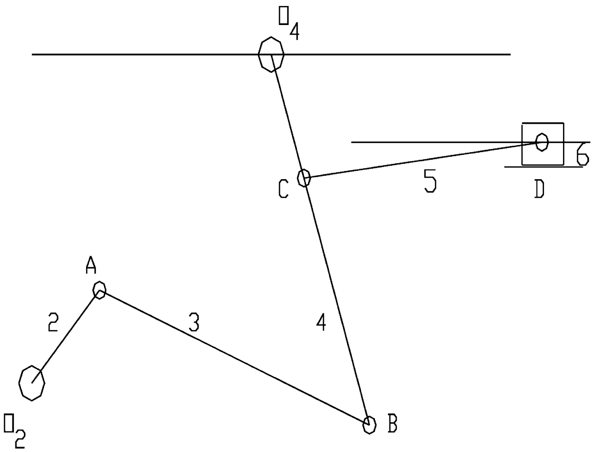


Fig. 2.34

2.6 Additional Problems

1. Refer to Figure 2.34. Identify (a) the number and type of links, (b) the different elements, (c) the kinematic pairs and their type, (d) draw a schematic diagram representing its kinematic chain, (e) is there a basic quadric cycle chain in the linkage, if so what type, and (f) determine the number of degrees of freedom.
2. Refer to Figure 2.35. Identify (a) the number and type of links, (b) the different elements, (c) the kinematic pairs and their type, (d) draw a schematic diagram representing its kinematic chain, (e) is there a basic quadric cycle chain in the linkage, if so what type, and (f) determine the number of degrees of freedom.
3. Refer to Figure 2.36. Identify (a) the number and type of links, (b) the different elements, (c) the kinematic pairs and their type, (d) draw a schematic diagram

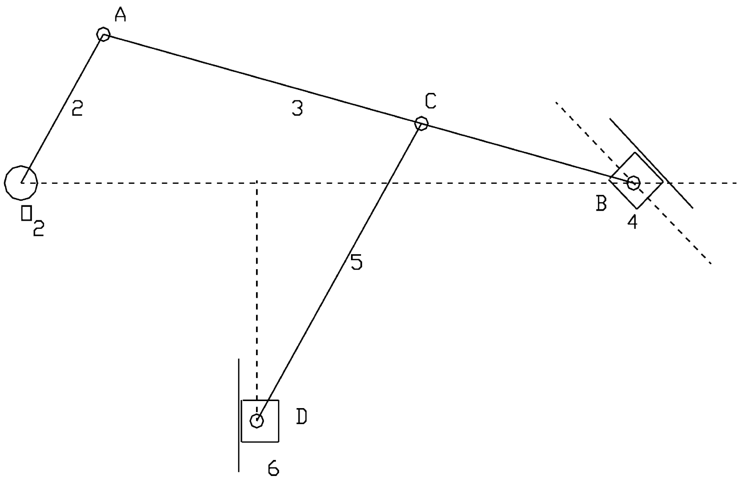


Fig. 2.35

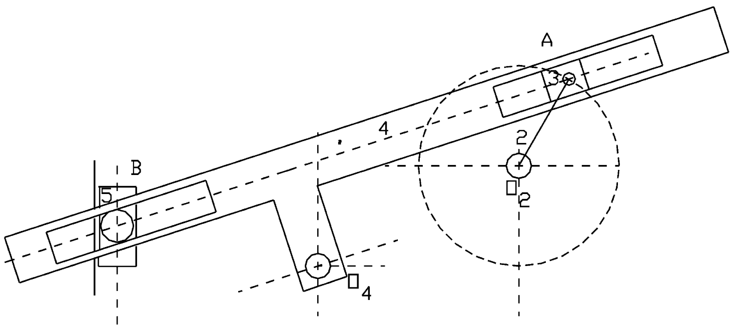


Fig. 2.36

representing its kinematic chain, (e) is there a basic quadric cycle chain in the linkage, if so what type, and (f) comment on the number of degrees of freedom.

4. Refer to Figure 2.37. Compare this linkage with that of Figure 2.34 above and make any comments you have.
5. Refer to Figure 2.38. Identify (a) the number and type of links, (b) different elements, (c) kinematic pairs and their type, (d) draw a schematic diagram representing its kinematic chain, (e) is there a basic quadric cycle chain in the linkage, if so what type, and (f) determine number of degrees of freedom.

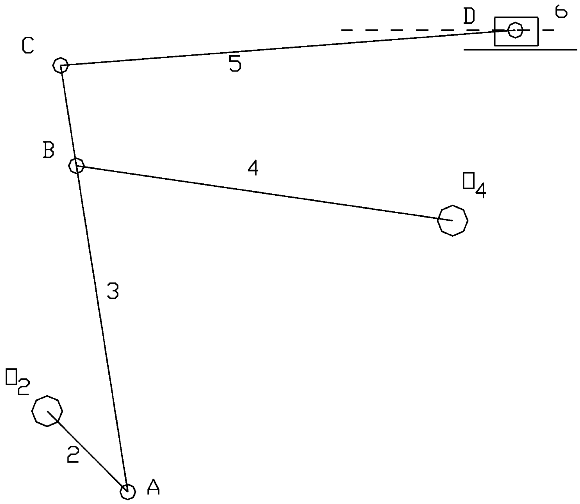


Fig. 2.37

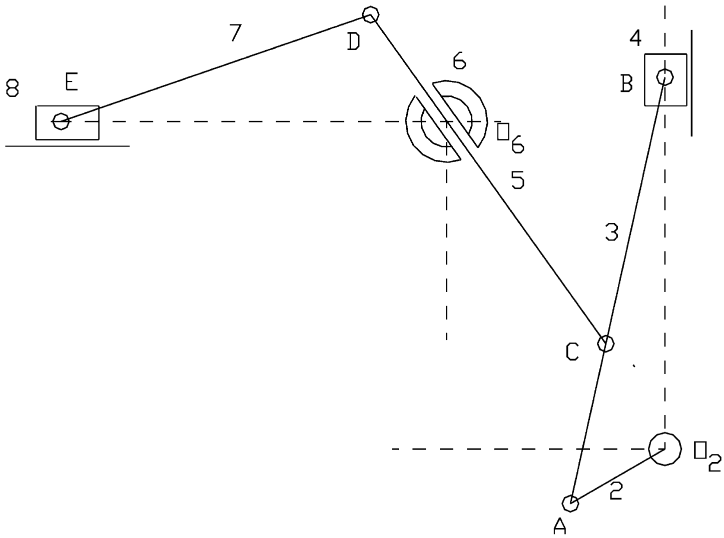


Fig. 2.38

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