

Chapter 2

Vibration Testing for the Evaluation of the Effects of Moisture Content on the In-Plane Elastic Constants of Wood Used in Musical Instruments

M.A. Pérez Martínez, P. Poletti, and L. Gil Espert

Abstract The present work provides an experimental-numerical investigation into the effects of moisture content on the in-plane elastic constants of wood for the specific use of the construction of soundboards of musical instruments. The vibrational behavior of a rectangular plate of spruce has been observed using vibration testing under different humidity conditions. The use of a nondestructive test method permits direct examination of a material sample which will eventually become part of a real instrument. The proposed approach is intended to minimize the difference between the numerical and experimental dynamic response through an iterative process, which allows identifying the elastic characteristics of wood specimens. It has been demonstrated that the vibrational behavior of timber varies considerably with variations in humidity. However, not all elastic properties are equally affected by such changes. The most significant variation is found in the transverse elastic modulus, and consequently in resonance modes associated therewith.

2.1 Introduction

Wood is one of the oldest and best-known materials which, given its unique mechanical and acoustical properties, is by far the most widely used material in the making of stringed instruments [46]. Despite the variety of construction materials currently

M.A. Pérez Martínez (✉)

Department of Strength of Materials and Structures, Universitat Politècnica de Catalunya,
C/Colon, 11 TR45, 08225 Terrassa, Barcelona, Spain
e-mail: marco.antonio.perez@upc.edu

P. Poletti

Department of Sonology, Escola Superior de Música de Catalunya, C/Padilla, 155, 08013
Barcelona, Spain
e-mail: paul@polettipiano.com

L. Gil Espert

Laboratori per a la Innovació Tecnològica d'Estructures i Materials, Universitat Politècnica
de Catalunya, C/Colon, 11 TR45, 08225 Terrassa, Barcelona, Spain
e-mail: lluis.gil@upc.edu

available, such as synthetic polymers and carbon fibre composites, luthiers have for the large part continued using wood.

Wood has the capacity to react and adapt to environmental conditions, thus the effects of changing moisture levels are important from a practical point of view and are often a decisive factor for the material's mechanical performance. Generally, the speed of sound increases and the internal damping decrease with a reduction of moisture content and, as a consequence, the musical instrument often sounds noticeably brighter [22]. At the other end of the spectrum, under extremely moist conditions, the expansion experienced by the material due to its hygroscopicity can become large enough that permanent damage results. Other material properties which are critical for the acoustical performance, such as elastic moduli or density, are also affected [18, 44]. Therefore, one of the most important considerations in the process of the construction of a wooden musical instrument is to understand the alterations caused by changes in humidity.

Independent of the basic nature of any material, its elastic properties play a fundamental role in science and technology, permitting a description of the mechanical behavior of a material which is essential to design and experimental stress analysis. There are a wide number of methods to characterize the elasticity of materials. Several static methods have been codified due their simplicity despite the fact that they introduce serious difficulties, primarily their destructive nature, but also including other problems, such as boundary effects, sample size dependencies, difficulties in obtaining homogeneous stress strain fields and localized data, the drawbacks which make these methods less attractive.

In the present work, an initial attempt was made to determine the elastic characteristics of the specimens using extensometric techniques. Aside from the disadvantages of static tests enumerated, this method introduces a series of additional difficulties due to the fact that the reading returned by the sensor represents localized conditions within the material. When examining a material which is not completely homogeneous, the location of the sensor is critical. In the case of wood, different readings are obtained when the sensor is placed over the *earlywood* or *latewood*, a problem which might be resolved by using large strain gauges such that the reading corresponds to the integration of the field deformation of the covered surface. Another problem is that the adhesive can interfere with the reading by producing a local increase in rigidity of the material. Finally, when the specimen is placed under extreme conditions of temperature and humidity, it is difficult to guarantee the perfect adherence of the gauge to the surface of the specimen. Due to all of these drawbacks, the technique of extensometry does not seem to be the best option for the determination of the elastic properties of wooden specimens under different moisture contents.

Given the problem of direct measurement of the elastic properties, an indirect method that allows the measurement of experimental related parameters is useful to derive the unknown properties. A large number of techniques have been developed for the experimental study in the elasticity of wood [46], among them being resonance methods, which are attractive from both the experimental and numerical point of view. These methods are based on a relationship between an analytical

expression describing the vibration behavior of the specimens and experimental results [28]. However, due to the complex nature of the material in combination with problem of boundary conditions, the applicability of this technique is limited. As discussed in the following sections, since no closed form analytical solution exists for the partial differential equation governing the free transverse vibration of a rectangular plate [30], the problem must be addressed by numerical approximations. Finally, the inverse problem of estimating the in-plane elastic constants is reduced to minimizing the difference between the numerical and the experimental response through an iterative process.

The present work provides an experimental-numerical investigation on the effects of moisture content on the in-plane elastic constants of wood for the specific use of the construction of soundboards of musical instruments. This chapter has been organized with the intent of giving the reader a structured and comprehensive knowledge of the problem treated. After a brief overview of the state of the art, Sect. 2.3 discusses wood's orthotropic nature and exposes the constitutive model used in the study. Section 2.4 describes the effects of moisture content in wood and the specimens used, as well as the procedures used to induce humidity changes in the samples and the experimental results of the influence of such changes on the geometry and mass parameters. In Sect. 2.5, the vibrational behavior of a rectangular plate of spruce has been examined using vibration testing under different moisture conditions. Section 2.6 is devoted to the description of the adopted numerical model. In Sect. 2.7, the methodology which allows identifying the elastic characteristics from vibration measurements through an iterative process is presented and discussed. The results of numerical simulations and the comparison with experimental results are then discussed in Sect. 2.8. Finally, Sects. 2.9 and 2.10 contain concluding remarks and suggestions for further lines of inquiry.

2.2 Overview of the State of the Art

In 1680, the experimental philosopher R. Hooke unveiled for the very first time to the eyes of the world the astonishing patterns of movement traced by a vibrating glass plate. In 1787, the scientist E.F.F. Chladni repeated the pioneering experiments of Hooke, providing one of the major 18th century experimental stimuli to 19th century continuum mechanics [6], the results of which are commonly referred to today by his name: *Chladni Patterns* [30]. Several centuries after this discovery, although the methods employed are rather more sophisticated than the bow and flour used by Hooke, the same phenomenon is still used today by both the instrument maker and the scientist to unravel the complex vibration behavior of plates.

Even though 18th century scientists such as J.B. Biot in 1816 [6] were already using dynamic experiments to calculate the elastic moduli of isotropic materials, new approaches continue to be developed today, and in the past decades, a large number of dynamic methodologies for the elastic characterization of rectangular plates have been presented in the literature.

In essence, the modern procedure is based on an optimization process that minimizes the difference between the experimental and the numerical dynamic response. The material's parameters in the numerical model are iteratively updated until the numerical response approximates as closely as possible the experimental response [37]. The values which produce convergence in the response of the system are the unknown elastic characteristics. In other words, the method is basically an optimal curve fitting of vibration test data to equations expressing the dynamics of the test specimen [4].

These methods have in common the requirement of the measurement of a limited number of natural frequencies of the plate, usually with free boundary conditions; the problem is then to relate the mode shapes, natural frequencies and deflection fields to the unknown parameters, since unfortunately there is no closed form analytical solution for the eigenvalue problem in the case of a rectangular plate with free boundary conditions, a difficulty already mentioned in Rayleigh's classical book *Theory of Sound* [39].

In the first approximation, classical analytical methods have been used to determine exact analytical solutions of the governing differential equations for a rectangular plate simply supported or having two opposite edges simply supported with free conditions at the other edges. However, for other combinations of boundary conditions, the solutions are much more complex [23, 30]. In order to provide alternative solutions, many researchers have resorted to approximate analytical methods. Rayleigh's method [3, 12, 33], the Rayleigh-Ritz technique [9, 43] and numerical approaches, e.g. Finite Element Method [1, 2, 15, 16, 25, 35], are the three principal methods¹ usually employed.

At the beginning of the 1980's, Sol [43] developed a method for the identification of the elastic moduli of thin orthotropic plates, based on the comparison of numerically calculated resonant frequencies of a thin plate specimen with corresponding experimental data which allows for the simultaneous identification of the four independent in-plane elastic constants. Deobald and Gibson [9] used a Rayleigh-Ritz technique in order to model the vibrational behavior of an orthotropic rectangular plate with clamped and free boundary conditions. Starting from the natural frequencies of the plates so obtained, they determined the four in-plane elastic properties. Ayorinde and Gibson [3] developed a method using the classical lamination theory and an optimized three-mode Rayleigh's formulation to determine elastic coefficients of orthotropic plates with free boundary conditions. McIntyre and Woodhouse [33] presented a similar approach including the determination of both damping and elastic constants. Similar results were obtained for the determination of stiffness and damping properties of orthotropic composites plates by De Visscher *et al.* [10] using a numerical model of the specimen in combination with the modal strain energy method. Mota Soares *et al.* [35] obtained elastic characteristics by solving an optimization problem of an error functional expressing the difference

¹In [24], the reader can find a comparative overview and a discussion of the advantages and disadvantages of these principal methods for calculating the eigenfrequencies and eigenmodes based on the classical thin plate theory.

between measured higher frequencies of a plate specimen and the corresponding numerical ones. Grédiac *et al.* [15, 16] proposed a method allowing the direct determination of the flexural stiffness from natural frequencies which does not require an initial estimate parameter, based on a set of relevant weighting functions associated with different natural modes of vibration. More recently, Lauwagie [29] has developed a vibration-based identification technique to determine the elastic properties of the constituent layers of layered materials.

The majority of cited references focus on the study of samples of aluminium or composite materials, thereby extending the methodology to anisotropic materials. However, an extensive research regarding wood's elastic characterization has also been carried out.

From a perspective in which wood is treated as a structural material, other authors [28, 36] have offered previous solutions to the problem of the elastic characterization of wood by vibration testing. However they have primarily dealt with samples of large physical dimensions under constant humidity, although moisture content has a significant influence on elastic properties, as shown in [20, 21].

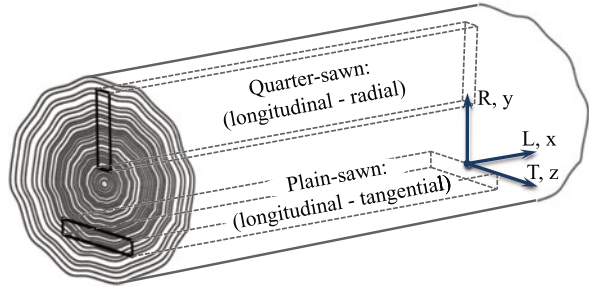
From a musical instrument maker's perspective, in which wood takes precedence over other materials due to its acoustic properties [46, 47], there have also been several discussions of the orthotropic properties of wood, the possible ways to characterize them and their importance in instrument making [22, 34]. For instance, Rodgers [41] concluded, based solely on a finite elements analysis, that only three (longitudinal and radial Young's Moduli and shear modulus) of nine elastic properties were of the first order importance in violin wood selection and analysis. Haines [17] presented an extensive and complete characterization of the musically important properties of wood for instruments, concluding that the ratio between longitudinal and radial Young's Moduli strongly determines the vibrational mode shapes of plates. However, his tests were conducted under closely controlled temperature and constant humidity conditions. In the second part of his work [18], the changes in density and stiffness due to the effects of moisture change were quantified, concluding that both elastic moduli are reduced while the density increases with increasing moisture content. However, the shear modulus was not measured and only two moisture contents were tested. Thompson [44] measured the frequency response of back and front violin plates under six different relative humidity levels, between 15% and 79%, concluding that changes in frequency are significant in a way that decreases as the relative humidity increases; however, in this study, only the second and fifth modes were measured.

According to the authors' knowledge, this topic has hitherto not been fully studied. Advances in experimental techniques and the versatility and accessibility of numerical methods now makes it possible to perform further study.

2.3 Orthotropic Nature of Wood Properties

Wood is a complicated composite of hard-celled cellulose microfibrils (organic cells known as *tracheids*) embedded in a *lignin* and *hemicellulose* resin matrix. The seasonal variation in the cell wall density of a tree is evident when looking at the end

Fig. 2.1 The principal axes useful for modelling wood as an orthotropic material. The longitudinal axis L is parallel to the cylindrical trunk and the tangential axis T is perpendicular to the long grain and tangential to the annual growth rings



of the cut trunk, where a concentric ring structure formed by the walls of the long slender tracheids can be observed. Commonly referred to as *growth rings*, this architecture composed of alternating layers of *earlywood* (formed in the spring and summer) and *latewood* (formed at the end of the growing season) is responsible for wood's high anisotropic and viscoelastic behavior [46].

Wood may be described as an orthotropic material because its mechanical properties are independent and can be defined in three perpendicular axes (see Fig. 2.1). The longitudinal axis L is parallel to the cylindrical trunk of the tree and therefore to the long axis of the wood fibres as well (parallel to the grain). The tangential axis T is perpendicular to the long grain and tangential to the annual growth rings. Both the tangential and radial directions are referred to as being perpendicular to the grain.

Taking the tree trunk as a series of concentric cylindrical shells and cutting thin radial slices, the growth ring curvature is negligible and occurs in straight parallel lines orthogonal to both the longitudinal and the tangential axis. In the case where the long axis is parallel to the grain fibre orientation and the width is in the radial direction, the piece is said to be *quarter-sawn*. The wood used in soundboards is almost always of quarter-sawn timber, which causes the speed of sound to be higher and the values of damping to be lower than for wood cut at an angle to the grain [22, 46]. In general, the mechanical properties vary the most between the longitudinal grain and the other two radial and tangential directions.

In spite of the fact that wood is a viscoelastic material, viscoelastic phenomena do not appear relevant in the present work because of the small strains and the nature of dynamic problem. Therefore, in the current study, linear elasticity can be assumed as an hypothesis. When linear elastic behavior is assumed, the generalized Hooke's law gives the stress-strain relation, which can be written in matrix form as

$$\{\sigma\} = [D]\{\varepsilon\}, \quad (2.1)$$

or

$$\{\varepsilon\} = [S]\{\sigma\}, \quad (2.2)$$

where $[S] = [D]^{-1}$, $\{\sigma\}$ are the stress components, $[D]$ the elastic matrix, $[S]$ the compliances matrix, and $\{\varepsilon\}$ the strain components [40]. Since $\{\sigma\}$ and $\{\varepsilon\}$ are elements of $\mathbb{R}^6$, there are 36 components in both the stiffness and compliance matrices, but these reduce to 21 because of the symmetry of the stresses and small strains tensor [26]. When three mutually orthogonal planes of material symmetry exist and

the coordinate system employed is aligned with the principal material directions, the number of elastic coefficients is reduced to 9, and such material is called orthotropic. Then, the stress-strain relation takes the form

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}. \quad (2.3)$$

An important feature of orthotropic materials that can be observed in Eq. (2.3) is the uncoupled behavior between the normal stresses and the shear strains, the shear stresses and normal strains, and the shear stresses and shear strains.

For plate-like structures a state of *generalized plane stress* can be considered, which implies that all three transverse stress components are negligible,

$$\sigma_3 = 0, \quad \tau_{23} = 0, \quad \tau_{13} = 0. \quad (2.4)$$

Furthermore the transverse normal strain, ε_3 , is small enough to be ignored. Therefore, for an orthotropic plate material with principal materials axes (x_1, x_2, x_3) coinciding with the plate coordinates (x, y, z) , i.e. the local and global axes coincide, the plane stress-strain reduced constitutive equations can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}, \quad (2.5)$$

where Q_{ij} are the plane strain-reduced stiffnesses. Engineering constants, such as Young's, shear moduli and Poisson's ratio, are used instead of the stiffness coefficients due to its direct and obvious physical meaning. Hence the stiffness coefficients are related to the engineering constants as

$$\begin{aligned} Q_{11} &= \frac{E_x}{1 - \nu_{xy}\nu_{yx}}, & Q_{12} &= \frac{\nu_{xy}E_y}{1 - \nu_{xy}\nu_{yx}} = \frac{\nu_{yx}E_x}{1 - \nu_{xy}\nu_{yx}}, \\ Q_{22} &= \frac{E_y}{1 - \nu_{xy}\nu_{yx}}, & Q_{66} &= G_{xy}. \end{aligned} \quad (2.6)$$

The so-called *thermodynamic constraints* are based on the principle that the sum of the work done by all stresses must be positive in order to avoid the creation of energy. Formally, it can be proven that the matrices relating stresses to strains must be positive-definite [26], i.e., the diagonal elements must be positive. This mathematical condition applied to Eq. (2.5) implies that

$$Q_{11}, \quad Q_{22}, \quad Q_{66} > 0, \quad (2.7)$$

or in terms of the engineering constants,

$$E_x, \quad E_y, \quad G_{xy} > 0 \quad (2.8)$$

whereupon from Eq. (2.6)

$$(1 - \nu_{xy}\nu_{yx}) > 0. \quad (2.9)$$

Using the compliance symmetry condition

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}, \quad (2.10)$$

the inequality in Eq. (2.9) can be rewritten as

$$|\nu_{xy}| < \sqrt{\frac{E_x}{E_y}} \quad \text{or} \quad |\nu_{yx}| < \sqrt{\frac{E_y}{E_x}}. \quad (2.11)$$

In the specific case of wood, the axes are given a special nomenclature, as has been mentioned above, defining a longitudinal and radial direction in reference to the manner in which the specimen has been cut from the original tree trunk. In this manner, the engineering constants are related to the stiffness coefficients in the following way:

$$\begin{Bmatrix} \sigma_L \\ \sigma_R \\ \tau_{LR} \end{Bmatrix} = \begin{bmatrix} \frac{E_L}{1-\nu_{LR}\nu_{RL}} & \frac{\nu_{LR}E_R}{1-\nu_{LR}\nu_{RL}} & 0 \\ \frac{\nu_{RL}E_R}{1-\nu_{LR}\nu_{RL}} & \frac{E_R}{1-\nu_{LR}\nu_{RL}} & 0 \\ 0 & 0 & G_{LR} \end{bmatrix} \begin{Bmatrix} \varepsilon_L \\ \varepsilon_R \\ \gamma_{LR} \end{Bmatrix}. \quad (2.12)$$

As can be seen, the elastic behavior of an orthotropic plate-like structures can be described by only four independent engineering constants, i.e. E_L , E_R , ν_{LR} and G_{LR} .

2.4 Influence of Moisture Changes on Wood

Wood is a hygroscopic material which shows coupling between the moisture transfer phenomena and mechanical behavior at two levels. Moisture changes induce swelling or shrinkage in the material, the so-called hygroexpansive effect; after the initial drying process or *seasoning*, wood continues to interact with the moisture in the ambient atmosphere in order to reach a state of equilibrium, a process which significantly alters the proportion of its total mass which consists of water [46]. Its geometry and elastic characteristics are also altered by these changes in moisture content, and, as shall be demonstrated, its vibrational behavior as well. Furthermore, with a mechanical load acting under moist conditions, an additional deformation is induced [21], known as mechano-sorptive creep.

Most constitutive models consider that the total strain, ε , is assumed to consist of additive strain terms,

$$\varepsilon = \varepsilon_e + \varepsilon_u + \varepsilon_{ve} + \varepsilon_{ms}, \quad (2.13)$$

where ε is the total strain, ε_e the elastic, ε_u the hygroexpansion (swelling and shrinkage), ε_{ve} the viscoelastic and ε_{ms} the mechano-sorptive. The strain terms are treated

by associating a separate differential equation to each one.² Due to the application being studied in this work, the effects of the combined action of simultaneous moisture changes with mechanical loading has not been examined. Furthermore, as has been mentioned above, strain rates are so low that the linear elastic hypothesis can be taken as valid, and therefore the viscoelastic effects that occur in higher strain rates can be neglected.

The moisture content β is defined as the mass fraction of free water in the wood,³

$$\beta = \frac{m_0 - m_{\text{dry}}}{m_{\text{dry}}} \quad (2.14)$$

where m_0 is the initial mass and m_{dry} is the mass of the specimen with all free water removed.

While varnishes and other finishes may retard the movement of water vapour between the atmosphere and the wood, they cannot stop it completely. In any event, the interior surfaces of musical instruments are almost never finished in any way whatsoever, and the ubiquitous presence of acoustic venting holes in the resonating cavity assures that a rapid equalization of internal and external atmospheric humidity is inevitable.

For the constructor of a musical instrument (or for that matter, any large complex wooden object), the more subtle variations of mass and elasticity are far outweighed by the dimensional variation in the radial and tangential directions (wood being essentially stable longitudinally). The larger plate-like objects of many types of musical instruments, including not only the soundboard but also the walls and bottom of resonating cavities, are almost inevitably attached to other structural elements whose longitudinal grain orientation runs in a perpendicular or near-perpendicular orientation relative to the longitudinal grain of the plate itself, elements such as ribbing or barring of soundboards and major structural supports of case walls and bottoms. The fact that the plate-like elements are constrained precisely in the orientation which is dimensionally variable is an inescapable reality which the builder ignores at his own peril. In the best of cases, whenever the ambient humidity is other than that at which the instrument was assembled, the plate will be deformed by the dimensional differences between the restrained and free sides, the free side becoming convex under wet conditions and concave under dry conditions. In the worst scenarios, under extremes of humidity, the tension or pressure applied to the material can become large enough that permanent damage results, either by splitting the wood apart or causing the cell structure to become permanently crushed, a phenomenon known as *compression set*. Wood which has suffered compression set will thereafter be even more susceptible to splitting under dry conditions, as its effective dimension has already been reduced at a cellular level before any excessive drying reduces it further.

²More background information on moisture induced eigen-stresses in timber can be found in, e.g., [20] and an informative and comprehensive review of constitutive models is presented by Hanhijärvi [21].

³Although there is an explicit relation between relative humidity and moisture content [45], these parameters should not be confused.

Fig. 2.2 Distortion caused by changes in humidity when a wooden plate is restrained on one side only. Both pieces of spruce (*Picea Abies*) were cut sequentially from the same plank. The width of the samples is approximately 14 cm

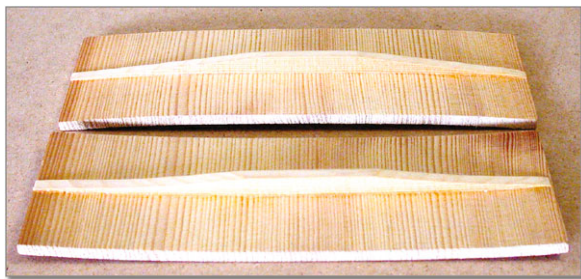


Figure 2.2 illustrates the distortion which can be caused by changes in humidity when a wooden plate is restrained on one side only. Both pieces of spruce (*Picea Abies*) were cut sequentially from the same plank. The single rib was attached to the upper sample after it had reached saturation content using the methods described in Sect. 2.5 below, and to the lower sample after it had been dried to 0% moisture content. The photo was taken after leaving the samples in a stable ambient humidity of 55% for several days.

A soundboard is usually considered to be the structural element which most critically affects the acoustic behavior of the instrument as a whole. In the present work, the behavior of a rectangular plate of spruce (*Picea Abies*), which is the type of wood most commonly used for the soundboards of guitars, violins, pianos, and other string instruments, has been addressed using modal analysis under different conditions of humidity. The wood specimens examined in this work are of quarter-sawn timber, and the dimensions in the growth ring plane are small enough (≤ 2 mm) so that the properties are essentially orthotropic. The wooden specimens had a size of $432 \times 189 \times 5.1$ mm and a density of 470 kg/m^3 under normal conditions.⁴

At this point it is important to justify why only two specimens have been used for this study. No two pieces of wood are exactly identical, and while it is hypothetically possible that two soundboards made from different trees of the same species might demonstrate subtly-different mechanical/acoustical behaviors, the actual limits of any such variations are greatly constrained by the traditional process of material selection, dictated by a set of requirements which include not only the type of wood, the orientation of the cut and the general width of the annular rings, but also a certain raw acoustic characteristic which is empirically determined by tapping the plank and listening to its response. Furthermore, the readily observable dimensional changes caused by moisture variations are quite consistent for a vast sampling of planks of the same species, as is demonstrated by the high degree of correlation between the values which can be found in tables of expansion/contraction rates presented in any

⁴As shown below, uncertainty in the thickness measurement may have major effects on the estimation of elastic properties. To measure the thickness across the whole surface, the authors recommend the use of an accurate caliper thickness with resolution of 0.01 mm. Moreover, the length and width of the specimen should not be the same, in order to avoid the degeneration of the modes (2 0) and (0 2) into ring and x-modes [33], which would complicate the initial estimation of the elastic properties according to the methodology described below.

Table 2.1 Results of the influence of moisture content β on the geometric parameters and mass. Units in mm, g and kg/m³, respectively

β (%)	0.11	1.43	6.01	9.38	15.73	17.22	18.82	20.37	21.80	24.71
Length ^a	431.5	431.5	431.5	431.5	431.5	432.0	432.0	432.0	432.0	432.0
Width ^b	186.5	187.0	188.5	189.0	191.0	191.6	191.5	192.0	192.6	192.5
Thickness ^c	4.90	4.91	4.97	5.02	5.13	5.15	5.18	5.20	5.25	5.32
Mass ^d	175.0	177.3	185.3	191.2	202.3	204.9	207.7	210.4	212.9	218.0
Density	443.8	447.5	458.4	467.0	478.5	481.6	484.7	487.8	488.3	492.8

All test were performed at room temperature $23 \pm 2^\circ\text{C}$ at $38 \pm 2\%$ relative humidity

^aTolerance: ± 0.5 mm; ^b ± 0.1 mm; ^c ± 0.01 mm; ^d ± 0.1 g

number of common technical manuals dealing with wood [45]. Therefore, while the natural inconsistencies of the material might seem to imply a significant factor of uncertainty to those with little first-hand experience with the material and/or little or no knowledge of the craft of instrument making, a level of uncertainty which under other circumstances might require a much larger data set, in regards to the specific application under consideration it is highly unlikely that a study involving a greater number of relevant samples would produce results markedly different from those obtained from the two upon which present work is based.

To measure the dry mass, the plates were subjected to a standard oven-drying process, in which they were kept at a temperature of $100 \pm 2^\circ\text{C}$, the mass of each specimen being checked periodically until no further reduction of mass was detected. The results obtained were 174.7 and 174.9 g. Care was taken to perform the weighing of the sample in ambient conditions as quickly as possible, since reabsorption of moisture can occur very rapidly at low content levels. In order to increase the moisture content, the plates were suspended just above a tray filled with water without allowing them to come into actual contact with the fluid, the tray and plate then being covered with plastic film and left for a period of ten days, after which no further increase of mass was detected.

To simulate the behavior of the specimen it was necessary to estimate which characteristics had changed with respect to the normal conditions of temperature and humidity. Table 2.1 shows the data on the geometry, temperature, relative humidity and mass of the specimen taken during the experiments. There was a significant variation in density with values ranging between 443.8 and 492.8 kg/m³. The length of the plate along the grain direction varied by less than 0.5 mm, which supports the hypothesis that wood is essentially stable longitudinally. However, in thickness and across the grain, differences reached 0.42 and 6 mm respectively under extreme conditions, which corresponds to a variation of $\pm 4\%$ and $\pm 1.6\%$ of the nominal value, respectively.

The reader may be surprised that the tests under the driest conditions were performed with some moisture content. This was caused by the fact that the recommended reversible adhesive to ensure proper coupling between the accelerometer and the specimen is common beeswax. However, the temperature of the specimen remains high enough to melt the wax adhesive for some time after removal from

the drying oven, and therefore it is necessary to allow the specimen to cool for a short period of time, during which some reabsorption of moisture is unavoidable. Alternative solutions might be the use of adhesives whose properties are maintained at higher temperatures such as cyanoacrylate, but such adhesives are not reversible; perhaps the best solution would be to use a laser-based vibration transducer. Unlike traditional contact vibration transducers, laser vibrometers require no physical contact with the test object.

2.5 Experimental Modal Analysis of Wooden Specimens

Experimental modal analysis is the process of determining the modal parameters (frequency, damping and mode shapes) of a linear time-invariant system.⁵ It is generally based upon a theoretical relationship between measured quantities, the so-called Frequency Response Function (FRF), and classical vibration theory represented as matrix differential equations of motion [19].

A system can be described by its equation of motion, which general mathematical representation is expressed by

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\}, \quad (2.15)$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the stiffness matrix, describing the spatial properties of the system, $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are vectors of time-varying acceleration, velocity and displacement response, respectively, and $\{f(t)\}$ is a vector of the time-varying external excitation forces.

By taking the Fourier transform, the differential equation of motion, Eq. (2.15), can be written as

$$[-[M]\omega^2 + j[C]\omega + [K]]\{X(\omega)\} = \{F(\omega)\}, \quad (2.16)$$

where ω is the frequency variable, j the imaginary j-operator, $\{X(\omega)\}$ the output vibration vector, and $\{F(\omega)\}$ the input force vector. Grouping the terms on the left, one obtains

$$[B(\omega)]\{X(\omega)\} = \{F(\omega)\}, \quad (2.17)$$

where $[B(\omega)]$ is the system response matrix. This equivalent representation has the advantage of converting a differential equation to an algebraic equation in the frequency domain (magnitude and phase). The system transfer function $[H(\omega)]$, or FRF, is the inverse of the system response matrix. The FRF relates the Fourier transform of the system input to the Fourier transform of the system response. That is,

$$[B(\omega)]^{-1} = [H(\omega)] = \frac{\{X(\omega)\}}{\{F(\omega)\}}. \quad (2.18)$$

⁵The theory of experimental modal analysis is presented in a shortened, comprehensive form. The interested reader is referred to [11, 19, 32] where more details are given.

The modal parameters such as frequency, damping and mode shapes are finally extracted from measured input-output data $[H(\omega)]$, using numerical techniques, e.g. curve-fitting algorithms, which separate the contributions of individual modes of vibration in measurements [11, 19, 32].

An important feature is that $[B(\omega)]$ and $[H(\omega)]$ are symmetric since $[M]$, $[C]$ and $[K]$ are symmetric. This implies that $H_{pq} = H_{qp}$, which means that a force applied at degree of freedom (DoF) p cause a response at DoF q that is the same as the response at DoF p caused by the same force applied at DoF q . It is said that the structure obeys Maxwell's reciprocity, one of the basic assumptions which underlies experimental modal analysis [19], whose practical results are obvious.

The test environment involves several factors which must be taken into consideration, as well as the appropriate boundary conditions of the structure, especially if the vibration behavior is to be subsequently compared with numerical results. There are many options ranging from completely free to completely clamped edges, simply supported or combinations of them all. From an experimental point of view, arriving at a completely clamped boundary condition is extremely difficult. This detail was also stated in [1, 9], who concluded that it is better to perform the tests with free boundary conditions. Another option is to perform the test with simple supported boundaries. The problem found is that when the specimen is small, as force is applied with the hammer, the specimen itself rebounds and thereby loses contact with the supporting surfaces. To reproduce free boundary conditions in the laboratory several options exist: either the plate can be situated on an undulated foam surface, or it can be suspended either horizontally or vertically. For the present work, the various situations were tested and the results indicated a minimal difference; therefore, a vertical suspension was ultimately employed.

Among different testing configurations, tests were performed with a single-reference testing employing a roving hammer test. To measure a FRF of a structure basically two channels are needed: one channel is used to measure the excitation force and the other one to measure acceleration response of the plate. A stationary monoaxial accelerometer has been attached to a single DoF reference point whereas the hammer roved around exciting the specimen at well distributed measurement DoFs.⁶

The experiments were conducted using a Brüel & Kjaer type 4518-003 uniaxial accelerometer, whose mass is less than 1.5 g, along with a miniature transducer hammer Brüel & Kjaer model 8204 for the excitation of the system. Due to the accelerometer being attached to the test specimen, there is a certain amount of mass which is added to the structure. It is important to assure that the mass of the transducer is negligible when compared with the effective modal mass of a mode of vibration so as not to interfere with the vibrational behavior.⁷ To determine the mass threshold above which the presence of the transducer interferes with the vibration

⁶This is also called Single-Input Single-Output (SISO) testing.

⁷It is generally accepted that a ratio of less than 1:10 is required [11], in this study the ratio was approximately 1:130.



Fig. 2.3 Assembly made for modal testing. Both the applied excitation and the measured response were perpendicular to the plate. To reproduce free boundary conditions a vertical suspension was employed

behavior of the plate, a simple experiment was conducted by gradually adding portions of plasticine modeling clay to a point on the other side of the plate directly opposite the accelerometer, examining the frequencies and vibration modes at each stage. The results showed that there are no variations in the behavior until the added mass reaches 14.3 g, which corresponds with the mass of the plate by a ratio of 1:13. The problem may come when smaller specimens need to be tested and the total added mass becomes significant. In these circumstances, it is advisable to perform an operation known as mass cancellation [32] or, as already discussed above, to use laser vibrometers.

Both the applied excitation and the measured response were perpendicular to the plate. The range of frequencies studied was between 0 and 1000 Hz with a resolution of 0.25 Hz. The data acquisition system was a Brüel & Kjær model 3050-B-6/0. An analysis of the measurements was performed using Brüel & Kjær PULSE v.13, and for the estimation parameters, the modal and structural analysis software ME'Scope VES™ 7754 was employed. Figure 2.3 shows the measurement set-up for modal testing. In order to have sufficient spatial distribution for comparison to a finite element model it is necessary to have a reasonable number of well distributed measurement DoFs. While a larger number of finite matrix measurement points results in a higher description for the mode shape, because of time and cost constraints, the number of measured points was restricted to one hundred FRF's on each plate.

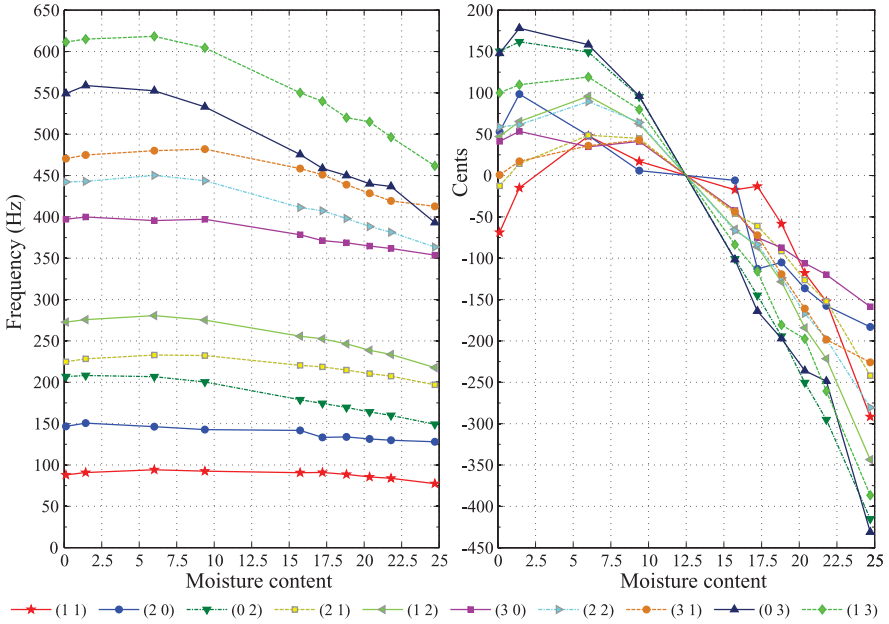


Fig. 2.4 Experimental modes results with variations in moisture content. The maximum variation in moisture content which could be obtained was between 0% a 25%, the latter of which corresponds with saturation. The nomenclature (m, n) identifies the different mode types, where m and n refer to the number of nodal lines parallel to the y direction and x direction, respectively

The selection of the reference location is a critical step of performing an experimental modal test. If the accelerometer location is in a nodal position of a mode, FRF's may not contain strong response for this mode and consequently would be poorly represented or, in the worst case, completely absent. In regular geometries, as is the case of the specimens studied, it is possible to deduce that the nodal lines of the first vibration modes correspond to the symmetry axes or regular subdivision surface. However, with irregular geometry, this task is not trivial. Prior knowledge obtained from pre-testing or a finite element model is very beneficial to accomplish this.⁸

Figure 2.4 shows the evolution of the frequencies of the first ten modes. Both graphics illustrate the same data, though the right graphic uses the proportional unit of cents in order to better illustrate the musical significance of the variations,⁹ since luthiers often tune the resonances of plates to certain notes or musical intervals according to traditional precepts. Clearly the frequencies do not decrease linearly with

⁸If obtaining a large number of modes is desired, the option of taking the test in several stages should be considered, with the position of the accelerometer varied for each test. The final result is obtained by the superposition of all FRF curves.

⁹The cent is a logarithmic unit of measure used for musical intervals which correspond to a one-hundredth of an equal-tempered semitone. The formula $n = 1200 \log_2(f_1 / f_2)$ returns the number of cents measuring the frequency interval between f_1 and f_2 .

increasing moisture content. This fact is empirically supported by luthiers who note that the timbre of a completed instrument undergoes subtle changes with variations of relative humidity, often sounding noticeably brighter with low percentages of moisture.

From a practical point of view, the behavior of wood is moisture-content dependent, and thus the measurements made at the two different times are inevitably inconsistent, an unfortunately unavoidable violation of another of the basic assumptions of time invariance which underlies experimental modal analysis [19]. Considering the rapid reabsorption rate at such low content levels, it is very important to assure that a constant moisture content is maintained during the tests. Given the number of discrete points used to describe the specimen, it is practically impossible to realize the test in only one step without the moisture content changing. One possible solution would be to perform the test in chambers under controlled humidity conditions, which unfortunately would increase the costs prohibitively. As an alternative solution, the test were performed in sequential steps; after drying the specimen in an oven, its mass was determined and the procedure was begun of acquiring the data at the initial points of the mesh, controlling the mass at regular intervals. When variations of mass were detected, which often occurred within less than a minute, the specimen was returned to the oven to reduce the moisture content yet again, a process which was repeated until data from all points of the mesh was acquired. The authors' experience has demonstrated that for the range of moisture content from completely dry to saturation, the order of the modal shapes did not appear to vary¹⁰; therefore, the numerical and experimental modes can be compared sequentially, the number of points can be reduced drastically and, as a consequence, the amount of time required for the test.

Unlike of the finite element approach, where adding additional nodes and elements generally has a direct effect on the results, in a vibration test, a particular frequency response measurement has no relationship to other measurements. However this practice presents a clear drawback: the drastic reduction in the number of DoFs results in poor spatial distribution and consequently the mode shapes would be poorly represented. This fact makes it difficult to make a comparison between the numerical and experimental mode shapes using correlation tools such as the Modal Assurance Criterion (MAC) [32]. Even so, MAC is only an indicator of the vector correlation, i.e. mode shapes, and can indicate only consistency, not validity, but in this application the vector correlation may not be as critical and the frequency correlation is more important.

2.6 Numerical Model of Wooden Plate

Among the wide number of theories that govern the deformation of plates, the well-known *classical plate theory* (CPT), or *Kirchhoff's plate theory*, has been chosen

¹⁰Except modes (3 1) and (0 3) which swap positions at a point close to saturation (see Fig. 2.4 and Table 2.3). However this does not become a critical issue since these modes are not strictly involved in the elastic properties estimation.

since the wood specimens studied satisfy the thin plate condition¹¹ in the sense that the thickness h is small compared to the characteristic lengths and thickness is either uniform or varies slowly so that three-dimensional stress effects are ignored [13]; therefore, CPT is a more suitable, straightforward and useful model.

The partial differential equation governing the free transverse vibration of a rectangular plate is given by

$$D\nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} = 0, \quad (2.19)$$

where w is transverse deflection, $\nabla^4 = \nabla^2 \nabla^2$ is the biharmonic differential operator, D the flexural rigidity which is a function of Young's modulus, Poisson's ratio and plate thickness, ρ is mass density per unit area and t represents time [30].

Although a closed form analytical solution exists for the eigenvalue problem of predicting the natural frequencies of a rectangular plate, this tend to be restricted to regular geometries and simple boundary conditions such a simply supported, clamped edges or any combinations of them. Even the free edges can also be taken into account when the other pair of opposite edges is simply supported or clamped [30]. Unfortunately there is no closed form analytical solution for the case of completely free edges. When the problems do not yield to analytical treatment an alternative practice is needed, e.g. numerical simulation.¹²

2.6.1 The Finite Element Method

The Finite Element Method (FEM) is a procedure for the numerical solution to the mathematical equations governing a problem, and recently has become the dominant discretization technique in structural mechanics.¹³ The basic concept in the physical FEM is the subdivision of the mathematical model into non-overlapping domains of simple geometry termed finite elements. The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements [13].

The matrix equations of motion, Eq. (2.15), describing both individual element and general system models can be derived from the minimum principles, i.e. Hamilton's principle, stated as

$$\delta \int_{t_0}^{t_1} (T + W) dt = 0, \quad (2.20)$$

¹¹The CPT overpredicts frequencies of plates with side-to-thickness ratios of the order of 20 or less, i.e. thick plates [31, 40]. The side-to-thickness ratio for the specimens studied, as can be seen in Sect. 2.4, is about 40.

¹²References [14, 27] may be helpful in order to implement a finite element model to solve the problem of simulation of vibration of a plate. Also, most available FEM analysis packages, e.g. ANSYS, ABAQUS or NASTRAN, among others, are appropriate and useful.

¹³The basic concepts, procedures and whole formulations can also be found in many existing textbooks. The interested reader is referred to, e.g. [5, 13], where more details are given.

where T is the system kinetic energy, W the work done by internal and external forces, δ the variational operator and $[t_0, t_1]$ the interval of time. For a free undamped¹⁴ vibration system, dissipative interior work is neglected and the work performed by externally applied forces is zero; thus Eq. (2.20) becomes

$$\delta \int_{t_0}^{t_1} (T - U) dt = 0, \quad (2.21)$$

where U is the conservative interior elastic potential energy, also called strain energy.

Using an assumed displacement field $u(x, y, z, t)$ written in terms of the nodal DoFs $\{u^e\}$ by means of the so-called element shape functions $[N]$, the kinetic and the strain energy of each element can be expressed in the form of the element mass and stiffness matrices.

The strain field $\{\varepsilon\}$ within the element volume is related to the assumed displacement by

$$\{\varepsilon\} = \partial u(x, y, z, t) = \{\partial\}[N]\{u^e\} = [B]\{u^e\}, \quad (2.22)$$

where $[N]$ is the shape function matrix of the element e , $\{u^e\}$ the vector of the nodal DoFs, $\{\partial\}$ the differential operator matrix and $[B]$ the element strain matrix.

The stress field $\{\sigma\}$ within the element volume is expressed as

$$\{\sigma\} = [D]\{\varepsilon\} = [D][B]\{u^e\}, \quad (2.23)$$

where $[D]$ is the elastic matrix used in Eq. (2.1).

Using the above expressions, the strain and kinetic energies in the element e are given by¹⁵

$$\begin{aligned} U^e &= \frac{1}{2} \int_{V^e} \{\varepsilon\}^T \{\sigma\} dV = \frac{1}{2} \int_{V^e} \{u^e\}^T [B]^T [D] [B] \{u^e\} dV \\ &= \frac{1}{2} \{u^e\}^T [K^e] \{u^e\}, \end{aligned} \quad (2.24)$$

and

$$\begin{aligned} T^e &= \frac{1}{2} \int_{V^e} \rho \{\dot{u}\}^T \{\dot{u}\} dV = \frac{1}{2} \int_{V^e} \{\dot{u}^e\}^T \rho [N]^T [N] \{\dot{u}^e\} dV \\ &= \frac{1}{2} \{\dot{u}^e\}^T [M^e] \{\dot{u}^e\}, \end{aligned} \quad (2.25)$$

respectively, where V denotes volume, $[K^e]$ is the element stiffness matrix, ρ the material density, $\{\dot{u}^e\}$ the velocity, so that $\dot{u}(x, y, z, t) = [N]\{\dot{u}^e\}$, at a point in the element e and $[M^e]$ the element mass matrix.

In order to obtain the complete finite element system model, an assembly process has to be carried out, which consists in the allocation of the individual element's

¹⁴Since the numerical differences between undamped and damped natural frequencies are not significant, the damping is neglected as it causes problems in eigenvalue calculations.

¹⁵The superscript $\{\}^T$ and $[\]^T$ denotes the transpose of a vector and matrix, respectively.

contribution to the system stiffness and mass matrices [5, 13, 14]. Substituting the expressions for the complete system energies into Hamilton's principle, and conducting the routine variation operation [40], results in

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{0\}, \quad (2.26)$$

which is the system differential vibration equation from the finite element discretization, called free undamped equation of motion, where $[K]$ is the global stiffness matrix, $[M]$ the global mass matrix and $\{\ddot{x}(t)\}$ and $\{x(t)\}$ are vectors of time-varying acceleration and displacement response, respectively [19].

Assuming a harmonic motion, the natural vibration frequencies and the associated mode shapes of a homogeneous linear undamped system can be obtained by solving the generalized matrix eigenvalue problem,¹⁶ defined as

$$[\Omega][M][\Phi] = [K][\Phi] \Rightarrow [K] - [\Omega][M][\Phi] = 0, \quad (2.27)$$

where $[K]$ and $[M]$ are the global stiffness and mass matrices, respectively, $[\Omega]$ is a diagonal matrix listing the corresponding eigenvalues (natural vibration frequencies) and $[\Phi]$ is the eigenvector matrix (mode shapes).

2.6.2 Free Vibrations of Kirchhoff Plates

Kirchhoff's plate theory provides a theoretical model of plate behavior by assuming that normals to the undeformed midplane remains straight and perpendicular to the deformed midplane (implying that the transverse shear strain is neglected), that only the transverse displacement w is considered and that transverse normal stress σ_z is negligible.

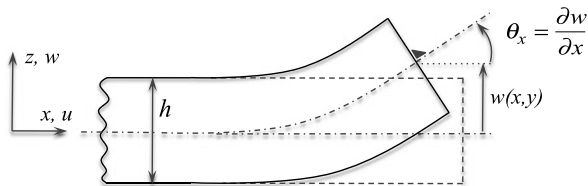
The first and second hypothesis allow the definition of the displacement field through the thickness of the plate and the third hypothesis affects the stress-strain relationship.

From the above assumptions, the displacement field for time-dependent deformations can be derived:

$$\begin{aligned} u(x, y, z, t) &= -z\theta_x(x, y, t) = -z \frac{\partial w(x, y, t)}{\partial x}, \\ v(x, y, z, t) &= -z\theta_y(x, y, t) = -z \frac{\partial w(x, y, t)}{\partial y}, \\ w(x, y, z, t) &= w(x, y, t), \end{aligned} \quad (2.28)$$

¹⁶The eigenvalue problem usually requires a huge computational effort to be solved, especially if the number of elements and consequently the number of DoFs are high. When the number of eigenvalues to be determined is around 30 (as in the present study where the number is limited by the experimental results) there are numerous subroutines readily available based on a vector iteration, i.e. *subspace iteration method* among others, which are well-documented in [5]. It is also highly recommended to use sparse matrices and diagonally lumped mass matrix, given its storage and computational advantages.

Fig. 2.5 Kinematics of thin plate deformation under the Kirchhoff's assumptions



where t is time, u and v are the in xy -plane displacements due to the rotation of the cross section, w denotes the transverse deflection and θ_x and θ_y are the bending rotations of a transverse normal about the xz and yz planes, respectively, as shown in Fig. 2.5. The so-called motion vector $\{u\}$ which contains the transverse displacement and rotations is defined by

$$\{u\} = \begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} = \begin{Bmatrix} w \\ \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix}, \quad (2.29)$$

whose components will correspond to the DoFs of each of the nodes of the discretization.

The strain field of the thin plate associated with the displacement field in Eq. (2.28) can be expressed by

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z \frac{\partial \theta_x}{\partial x} = -z \frac{\partial^2 w}{\partial x^2}, \quad (2.30a)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z \frac{\partial \theta_y}{\partial y} = -z \frac{\partial^2 w}{\partial y^2}, \quad (2.30b)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0, \quad (2.30c)$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = -z \left(\frac{\partial \theta_y}{\partial x} + \frac{\partial \theta_x}{\partial y} \right) = -2z \frac{\partial^2 w}{\partial x \partial y}, \quad (2.30d)$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0, \quad (2.30e)$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0, \quad (2.30f)$$

where the value of the transverse shear strains γ_{xz} and γ_{yz} are due to the first hypothesis. Collecting non-zero components, one obtains

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_y}{\partial x} + \frac{\partial \theta_x}{\partial y} \end{Bmatrix} = -z \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}, \quad (2.31)$$

where $\{\varepsilon\}$ is the strain vector.

The stresses in a linear and orthotropic plate are computed from Hooke's law,¹⁷

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{\nu_{yx}E_y}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [D]\{\varepsilon\}. \quad (2.32)$$

As shown, this relation coincides with Eq. (2.12) for plane stress problems.¹⁸ This coincidence is due to the hypothesis of null normal stress ($\sigma_z = 0$) across the thickness, common to both problems.

The orthogonality hypothesis on which CPT is based only holds for thin plate thickness. As the thickness of the plate increases, normals to the undeformed mid-plane remain straight and unstretched in length but not necessarily normal to the deformed plane.¹⁹ This hypothesis amounts to including transverse shear strains in the classical plate theory. In these cases, the Reissner-Mindlin theory is an improved approximation of the plate's strain [31, 40]. However, since the components of the transverse shear strains γ_{xz} and γ_{yz} are not zero, in the case of an orthotropic material, two more engineering constants appear on the stress-strain relation (G_{xz} and G_{yz} shear moduli). The aim of the work is to analyze the influence of moisture content on the in-plane elastic constants, hence the determination of two transverse shear moduli is beyond the scope of this study, due to the fact that in the thin plate condition the effects of transverse shear do not become significant.

As mentioned above, the problem of determining the natural vibration frequencies and the associated mode shapes of a plate always leads to solving the eigenproblem of the linear undamped system. In order to obtain the complete finite element model of the whole plate, it is first necessary to determine the expressions of strain and kinetic energy of each element and then to place them into Hamilton's principle, resulting in the equation of motion.

The plate strain energy is computed by integrating the strain energy density over the volume,

$$U = \frac{1}{2} \int_V \{\varepsilon\}^T \{\sigma\} dV = \frac{1}{2} \int_V \{\varepsilon\}^T [D] \{\varepsilon\} dV, \quad (2.33)$$

and the kinetic energy due to the three velocities is computed through integration of the kinetic energy of the volume element,

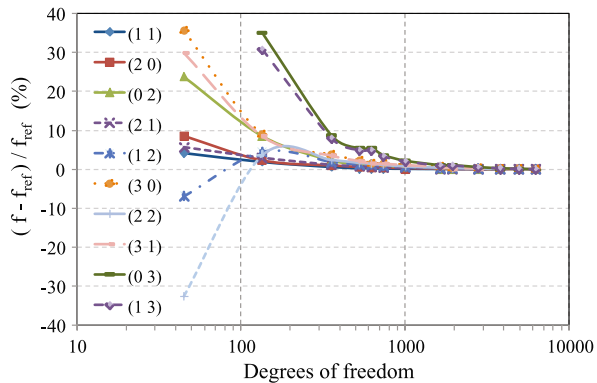
$$T = \frac{1}{2} \int_V \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dV = \frac{1}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_S \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dS dz, \quad (2.34)$$

¹⁷The reader is reminded that the stress-strains relation is exposed in Sect. 2.3.

¹⁸In Eq. (2.12), the axes are given in a wooden nomenclature, in which the subscripts L and R are equivalent to x and y , respectively.

¹⁹In addition, if one tries to analyze thicker specimens, it should be taken into account that the growth ring curvature may not necessarily be negligible, and the effects of such assumptions may become significant.

Fig. 2.6 Evolution of the first ten resonant frequencies as a function of the number of DoFs. Higher order modes converge slower than the lower order modes



where ρ is the mass density per unit volume, h the thickness, S denotes the integration area and the superposed dot on a variable indicates time derivative [40]. By substituting the displacement field in Eq. (2.28) and integrating through the thickness dimension, the kinetic energy can be expressed as

$$T = \frac{\rho h}{2} \int_S \left[\dot{w}^2 + \frac{h^2}{12} (\dot{\theta}_x^2 + \dot{\theta}_y^2) \right] dS, \quad (2.35)$$

where it has been preferred to express T as a function of the nodes DoFs (w, θ_x, θ_y) instead of the transverse deflection w .

Among the numerous readily available finite elements for plate bending, in this work a 4-node quadrilateral element has been implemented in a Matlab code, whose DoFs are the transverse displacement and the bending rotations of a transverse normal about the xz and yz planes. This results in a total of 12 DoFs per element.

Since with a numerical approach the number of nodes generally has a direct effect on the results, before carrying out the simulations it is necessary to conduct a mesh convergence study in order to determine its effect and estimate the computational effort. Due to the fact that there is no closed form analytical solution for the case studied, it is not possible to estimate the accuracy of the numerical approach in absolute terms. In spite of this, a study of the evolution of the first ten resonant frequencies for an increasing mesh density was performed. Figure 2.6 shows the evolution of vibration modes as functions of the number of DoFs. The aspect ratio of an element has been tried to be close to one. As can be seen, higher order modes converge slower than the lower order modes. Accordingly, the mesh density was chosen so that a further mesh refinement results in a changes in frequencies, that is, of an order less than the tolerance defined in the convergence criterion (see Sect. 2.7), which corresponds to a discretization of 70×30 elements, equivalent to 6603 DoFs.

2.6.3 Perturbation of the Equation of Motion

As discussed above, modal parameters (frequency and mode shapes) are characteristics of a structure's mass and stiffness. Considering the effect of wood moisture content as a widespread damage across the whole volume whose influence on the stiffness and density of the material is established, it can be expected that such deterioration affects the natural frequencies and associated mode shapes. Quantitative relations between damage and the resulting changes in modal parameters can be developed from a perturbation of the equation of motion in Eq. (2.26).

The eigenvalue problem for free undamped vibration of a structure is governed by the generalized matrix eigenvalue problem, previously defined as

$$[\Omega][M][\Phi] = [K][\Phi] \Rightarrow [K] - [\Omega][M][\Phi] = 0, \quad (2.36)$$

where $[K]$ and $[M]$ are the global stiffness and mass matrices, respectively, $[\Omega]$ is a diagonal matrix listing the corresponding eigenvalues (natural vibration frequencies) and $[\Phi]$ is the eigenvector matrix (mode shapes).

For the perturbed system, Eq. (2.36) becomes

$$[[K] + [\Delta K]] - [[\Omega] + [\Delta\Omega]][[M] + [\Delta M]] [[\Phi] + [\Delta\Phi]] = 0, \quad (2.37)$$

where $[\Delta\Omega]$ and $[\Delta\Phi]$ are the variation of the natural frequencies and mode shapes, respectively, induced by $[\Delta K]$ and $[\Delta M]$.

Given a non-zero eigenvector $\{\phi\}$, from the generalized Rayleigh quotient, defined as

$$R([K], [M], \{\phi\}) = \frac{\{\phi\}^T [K] \{\phi\}}{\{\phi\}^T [M] \{\phi\}} = \omega^2, \quad (2.38)$$

where ω^2 is the natural vibration frequency, the effect of $[\Delta K]$ and $[\Delta M]$ on a particular mode can be determined [42]. Particularizing Eq. (2.38) for the i th natural frequency and associated i th mode shape, the generalized Rayleigh quotient can be expressed as

$$\omega_i^2 + \Delta\omega_i^2 = \frac{[\{\phi_i\}^T + \{\Delta\phi_i\}^T][[K] + [\Delta K]][\{\phi_i\} + \{\Delta\phi_i\}]}{[\{\phi_i\}^T + \{\Delta\phi_i\}^T][[M] + [\Delta M]][\{\phi_i\} + \{\Delta\phi_i\}]}. \quad (2.39)$$

Because of the fact that $R([K], [M], \{\phi\})$ reaches the stationary value when $\{\phi\}$ is equal to the eigenvector $\{\phi_i\}$ [42], Eq. (2.39) can be simplified as

$$\Delta\omega_i^2 = \{\phi_i\}^T [[\Delta K] - [\Delta M]] \{\phi_i\}, \quad (2.40)$$

which indicates that an increase in mass induced by the increase in moisture content causes a decrease in natural frequencies and *vice versa*, as shown in Fig. 2.4 and represented in Sect. 2.5. On the other hand, a decrease in moisture content leads to a rise in natural frequencies, and therefore intuitively an induced increase in stiffness can be expected, as shown in the following section.

2.7 Elastic Constants from Plate Vibration Measurements

The above experiments have shown that the vibration behavior of a wooden plate is heavily dependent on moisture content (see Fig. 2.4), a fact previously noted by several researchers and commonly observed by musicians and luthiers [18, 22, 44, 46, 47]. In addition to density variations, significant changes in geometry have also been observed (see Table 2.1).

When attempts were made to simulate modal behavior under different moisture contents by employing the dimensional and density variations observed in the specimens, the results did not correspond with observed reality, proving that there must be other variables which intervene in the vibration problem. From Eq. (2.36), it can be derived that the dynamic behavior is governed by the mass and stiffness matrices, hence a perturbation in these matrices will significantly influence the vibration behavior; thus, it is evident that elastic characteristics must also be affected by moisture content.

Given the problem of direct measurement of the elastic properties, an indirect method that allows the measurement of experimental related parameters is useful to derive the unknown properties. Resonance methods²⁰ are based on a relationship between an analytical expression, describing the vibration behavior of the specimens, and experimental results.

The inverse problem of estimating the in-plane elastic constants is reduced to minimizing the difference between the numerical and the experimental response through an iterative process, in which the resonant frequencies obtained numerically for the initially estimated parameters are compared to the experimental resonant frequencies, followed by an update of the parameters in order to minimize the difference in the responses until convergence is achieved for a predefined error criteria. The flowchart of the procedure followed in this work is shown schematically in Fig. 2.7.

Prior to beginning the iterative process, it is necessary to introduce initial estimations of the in-plane elastic moduli to the numerical model. The success of the convergence depends in part upon the correct choice of these initial constants. These variables have been estimated using the simple methodology proposed in [28], which is based principally on the assumption that the resonance modes (2 0), (0 2) and (1 1)²¹ are influenced principally by elastic constants E_L , E_R and G_{LR} , respectively. This assumption may be demonstrated by conducting a sensitivity analysis of

²⁰Nevertheless, the main drawback of the resonant technique applied to samples of wood is related to the type of cut and shape of the specimen. For solid wood, it is easy to obtain plates in quarter-sawn, i.e. the long axis is parallel to the grain fibre orientation and the width is in the radial direction (LR) (see Fig. 2.1), or plain-sawn, i.e. the long axis is parallel to the grain fibre orientation and the width is in the tangential direction (LT). However, it is very difficult in the radial tangential plane (RT), especially when it comes to thin slices, as well as maintain the integrity of samples and ensure orthotropic behavior.

²¹As mentioned above, the nomenclature (m, n) identifies the different mode types, where m and n refer to the number of nodal lines parallel to the y direction and x direction, respectively.

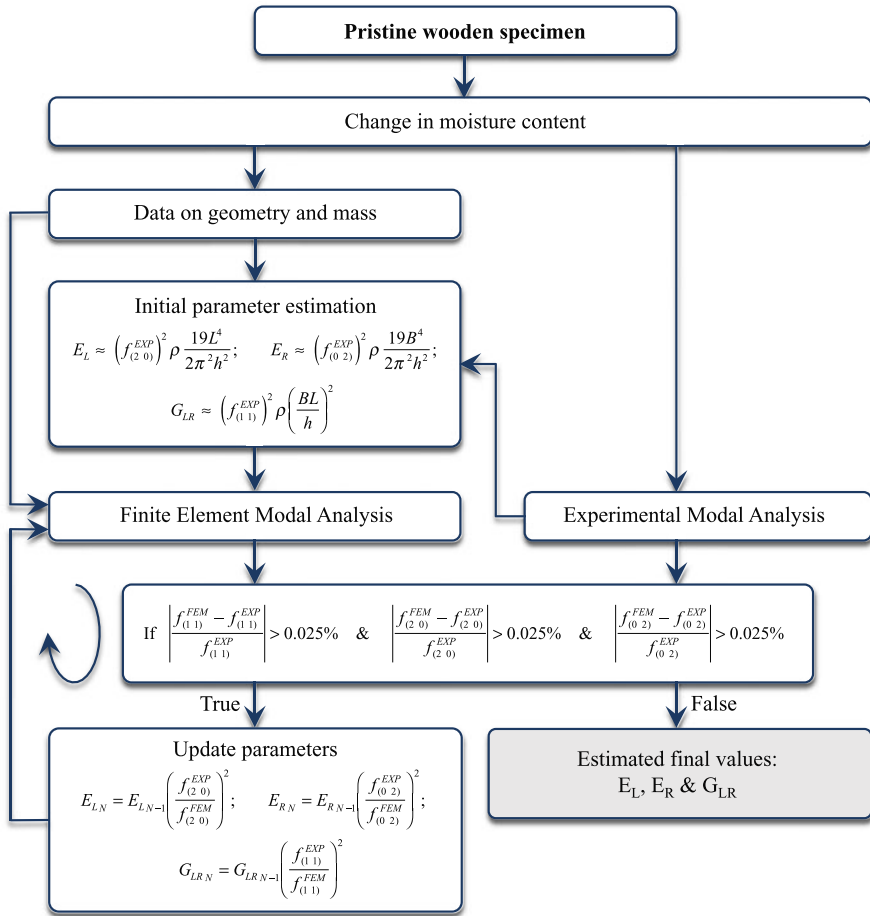


Fig. 2.7 The procedure, based on [28], is essentially an iterative process to minimize the difference between the numerical and the experimental response. An initial estimation of the elastic constants is required. The error function which defines the convergence criterion is based solely on frequency values. The updating of variables ensures a rapid convergence

numerical model response from the variation of input parameters, as explained here below.

In Fig. 2.8, the effect upon the numerical frequencies of the ten first resonance modes caused by an increase of 15% in the reference values for the elastic moduli E_L , E_R , G_{LR} and ν_{LR} has been represented. It is evident that some elastic moduli influence some mode types more predominantly than others. That is, a specific eigenmode can be more sensitive to the variation of some elastic constants than others. For example, the first mode, corresponding to torsional mode (1 1), is clearly dependent on the shear modulus G_{LR} . Similarly the second and sixth mode, mode shapes (2 0) and (3 0) respectively, or the third and ninth mode, mode shapes (0 2)

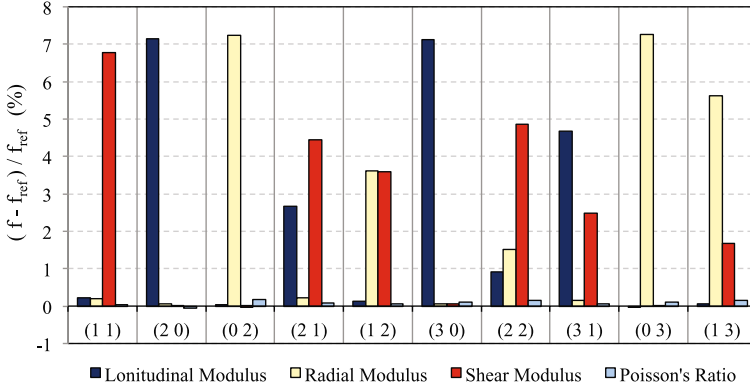


Fig. 2.8 Percentage variations of the numerical frequencies of the ten first resonance modes caused by a 15% increase in the references values for the elastic moduli E_L , E_R , G_{LR} and ν_{LR}

and (0 3) respectively, are essentially sensitive to the elastic moduli E_L and E_R , respectively. In modes in which $m \neq 0$ or $n \neq 0$, the influence is shared.

Another important conclusion derived from sensitivity analysis, also discussed in [28, 41], is that the influence of the Poisson's ratio on the numerical frequencies is minimal. Since it does not seem feasible to use the bending modes for the estimation of Poisson's ratio, consequently this work has been limited to the identification of three of the four in-plane elastic moduli, even though the results show that this does not become a critical issue.²²

The in-plane elastic moduli may be initially estimated by using the closed form eigenvalue solution for free and torsional beam, assuming $B \gg h$, that is

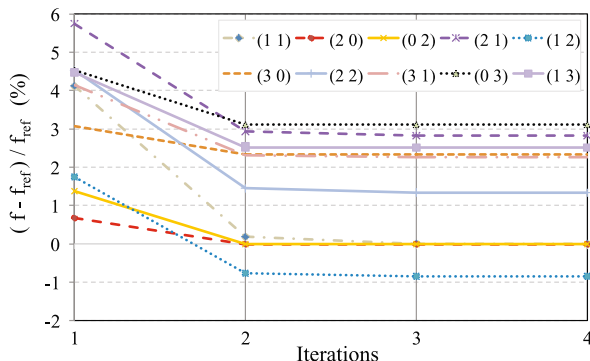
$$E_L \approx f_{(20)}^2 \rho \frac{19L^4}{2\pi^2 h^2}, \quad E_R \approx f_{(02)}^2 \rho \frac{19B^4}{2\pi^2 h^2}, \quad G_{LR} \approx f_{(11)}^2 \rho \left(\frac{BL}{h} \right)^2, \quad (2.41)$$

where ρ is the density, L the plate length, B the plate width, h the thickness and the subscript $(m n)$ denotes the mode shape.

One might think that the numerical approximation, and consequently the iterative process described, is entirely dispensable if a beam model is assumed to define the behavior of the plate. However, as discussed below, a refinement in the solution is required in order to minimize the differences between the numerical and experimental response. Moreover, the model allows the determination of the influence and sensitivity of different parameters involved in the problem of plate behavior, as well as identifying any deficiencies or errors in the experimental data.

²²As described in [28], the Poisson's ratio can be estimated from an in-plane compression mode. To perform a vibration test in which one wishes to acquire bending and compression modes simultaneously, the use of a biaxial accelerometer and a biaxial excitation are required.

Fig. 2.9 Convergence graph of the iterative process for the specimen with moisture content 6.01%. The error is completely minimized for the first three modes, since the update is done only using them



Once the numerical response for the initially estimated parameters is obtained, the eigenfrequencies associated to the mode shapes (1 1), (2 0) and (0 2) are compared with the corresponding experimental frequencies. If the convergence criterion is not achieved, parameters are updated as follows:

$$\text{if } \left| \frac{f_{(m,n)}^{\text{FEM}} - f_{(m,n)}^{\text{Experimental}}}{f_{(m,n)}^{\text{Experimental}}} \right| > 0.025\% \Rightarrow P_N = P_{N-1} \left(\frac{f_{(m,n)}^{\text{Experimental}}}{f_{(m,n)}^{\text{FEM}}} \right)^2, \quad (2.42)$$

where f is the numerical or experimental frequency associated to the mode (m, n) , P the parameter to be update, i.e. E_L , E_R and G_{LR} , N is the iteration number and the percentage corresponds to the maximum error defined to achieve convergence.

It must be observed that in both the case of the error function as well as that of the update parameters function, the calculations are done for each independent mode, which means that each elastic constant depends upon and is updated based upon a single mode. Among the wide variety of error functions, most of which calculated the error by comparing several modes simultaneously, it is preferable in this case perform the modal correlation and updating the elastic constants individually, as proven by the results. This is due to the independent and nearly uncoupled behavior of the modes analyzed, as illustrated in Fig. 2.8.

In Fig. 2.9 the convergence graph of one of the cases studied (moisture content 6.01%) has been represented. As can be seen, convergence is achieved after only a few iterations, as was also true in all the other cases. It should be pointed out that the error is completely minimized for the first three modes, since the update is done using them only. Despite this, as shown in the following section, the errors that occur in other frequencies for any specimen are less than 5%, which prove that the implemented routine is appropriate.

2.8 Results

In the previous sections, the influence of moisture content upon wooden plate vibration behavior has been experimentally demonstrated. Moreover, procedures and

Table 2.2 Initial and final estimated Young's and shear moduli (E_L , E_R and G_{LR} , respectively, in MPa) percentage error and elastic ratios under different states of moisture content β

β (%)	0.11	1.43	6.01	9.38	15.73	17.22	18.82	20.37	21.80	24.71
E_{Li}	13274	14049	13254	12604	12199	10816	10857	10458	10021	9563
E_{Lf}	13102	13871	13080	12439	12065	10685	10736	10347	9919	9491
Error	1.31	1.28	1.33	1.32	1.11	1.22	1.12	1.07	1.04	0.76
E_{Ri}	920.5	947.1	963.5	913.8	745.4	711.1	671.8	635.5	595.6	512.3
E_{Rf}	887.8	923.0	938.6	890.1	725.6	692.3	653.9	618.3	579.4	497.6
Error	3.69	2.61	2.65	2.66	2.74	2.72	2.74	2.77	2.79	2.94
G_{LRi}	927.4	996.5	1088.3	1054.5	1015.5	1024.1	969.3	908.6	859.5	721.1
G_{LRf}	845.0	914.0	1004.8	974.9	942.7	956.1	903.7	846.1	800.4	668.1
Error	9.75	9.03	8.31	8.16	7.72	7.11	7.26	7.39	7.39	7.94
E_{Rf}/E_{Lf}	0.068	0.067	0.072	0.072	0.060	0.065	0.061	0.060	0.058	0.052
G_{LRf}/E_{Lf}	0.064	0.066	0.077	0.078	0.078	0.089	0.084	0.082	0.081	0.070
G_{LRf}/E_{Rf}	0.952	0.990	1.070	1.095	1.299	1.381	1.382	1.368	1.381	1.343

Subscripts i and f indicate initial and final estimation, respectively

methodologies to identify which variables are affected and quantitatively assess the magnitude of the variation due to such moisture changes have been discussed. The value of elastic moduli were determined using the exposed iterative method, solving the inverse problem.

It should be noted that the rapid convergence is due in part to the proper estimation of the initial variables, involving not only the validation of assumptions, but also requiring that density variations, changes in the geometry and overall experimental frequencies have been properly determined. In Table 2.2, the initial and final estimated elastic properties, the percentage error between them and the elastic ratios for each of the states of moisture content are presented. In spite of the difference between specimens of the same species (spruce), good agreement between Ref. [45] and obtained elastic ratios have been observed. It is noteworthy that in all cases the initial estimate tends to stiffen the plate. Although errors are relatively small for the elastic moduli, the shear modulus error is almost 10%,²³ demonstrating the need for refinement from the numerical model.

Figure 2.10 shows the variation of the final estimated elastic characteristics E_L , E_R and G_{LR} as a function of moisture content for *Picea Abies* specimens. Although the species studied are different, there is a clear trend that roughly corresponds with that published in [46] and a good correlation has been observed with the recently

²³To determine the consequences that would cause the error in a modal simulation, the influence of shear modulus in the first ten resonance modes represented in the graph in Fig. 2.8 must be analyzed.

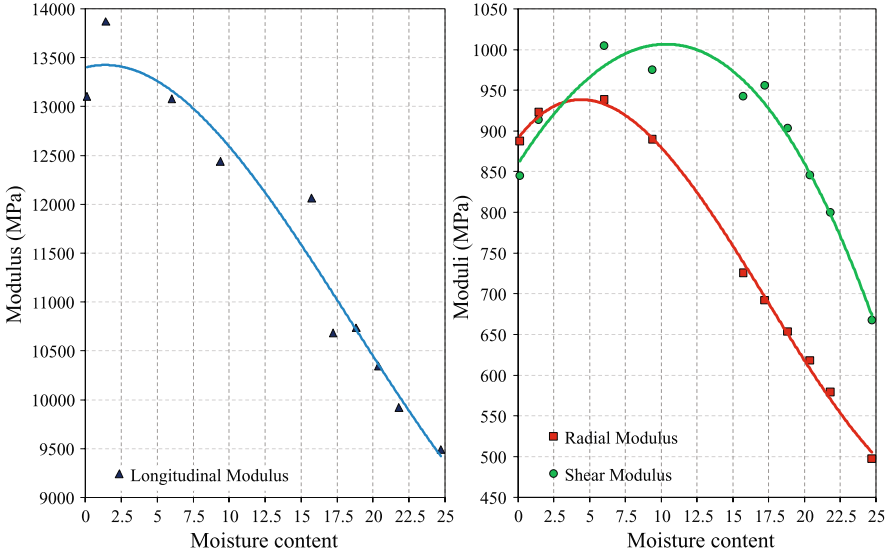


Fig. 2.10 Changes in longitudinal (*left*), radial and shear moduli (*right*) as a function of moisture content. The most significant variation is found in the radial modulus, and consequently in resonance modes associated therewith

published results in [38] for a proposed computational micromechanical model for the influence of moisture on softwood.

As anticipated in Sect. 2.6, Eq. (2.40) indicated that an increase in mass induced by the increase in moisture content caused a decrease in natural frequencies and *vice versa*. On the other hand, a decrease in moisture content led to a rise in natural frequencies, and therefore intuitively an induced increase in stiffness could be expected as a result of the alleged elastic moduli dependence on moisture content. While this hypothesis has been shown to be true, it should be noted that the behavior is clearly not linear.

Fitting a third order polynomial curve is excellent for the radial elastic modulus, but for the longitudinal and shear moduli, major discrepancies occur. Not all elastic properties are equally affected by moisture content changes, as can also be derived from the elastic ratios shown in Table 2.2; the most significant variation is found in the radial modulus, in which the difference between the maximum and minimum modulus reached approximately 88%, while for the shear and longitudinal moduli, variation is about 51% and 47%, respectively.

From the illustrated results, one can conclude that the elastic moduli is reduced with increasing moisture content and *vice versa*; however, while the minimum value corresponds to the saturation point, the maximum value does not correspond to the driest condition. This fact can be analyzed from a micromechanical approach.

The properties of the wood's cell walls are responsible for its macromechanical behavior. As mentioned earlier, its highly anisotropic behavior is due to alternating layers of earlywood and latewood. In terms of the microscopic structure, wood is essentially composed of cellulose, hemicelluloses and lignin. Latewood has a higher

cellulose content, which is the main contributor to the stiffness, and a lower lignin content than earlywood. Unlike cellulose, hemicellulose is strongly influenced by moisture changes, and along with lignin, it is the primary cause of moisture sorption. At lower moisture content, the hard-celled cellulose microfibrils, known as tracheids, are stiffer and conversely they become softer at higher moisture content. The dependence of stiffness constants of hemicellulose and lignin on moisture content can be found in [7, 8, 38], in which it is observed the lignin stiffness has a clear trend that roughly corresponds with the curves in Fig. 2.10.

The methodology results in numerical eigenfrequencies that fit closely with the experimental frequencies. Based on the estimated elastic properties²⁴ displayed in Table 2.2 and taking into account the geometric and mass variations experienced by the specimen displayed in Table 2.1, wooden plate vibration behavior has been simulated under different moisture conditions. Experimental and numerical modes, the corresponding mode shapes and the relative deviation between frequencies under different moisture conditions have been presented in Table 2.3. Figure 2.11 shows a comparison between the numerical and experimental modal analysis under 6.01% moisture content conditions for the first ten mode shapes. As expected, the minimum error corresponds to the first three modes. This fact is due to the independent use of the first three eigenfrequencies for the updating of the elastic characteristics at each iteration; even so, the maximum error for the rest of the modes is below 5%.

The main source of error is due to the inevitable heterogeneity of wood's properties, such as a local dispersion and orientation of the grain, irregularities in the thickness, non-uniformity in the cross-grain direction and densities. Inaccuracies in geometrical or mass measurements are other possible sources of error.

In order to quantify the influence of these inaccuracies in the eigenfrequencies obtained in the numerical model, a sensitivity analysis has been carried out by increasing the nominal values of length, width, thickness and mass by 5%. Variations in relation to a reference value of each of the resonance modes of the wooden plate analyzed are illustrated in Fig. 2.12. A decoupled behavior has been observed between the x -bending and y -bending modes with a strong relation to the length and width, respectively, as well as constant errors of 5% and -2.5% in all resonance modes, corresponding to variations in thickness and mass, respectively. It must be emphasized that these increases would result with an error in the nominal measure of approximately 10 g, 21.6 mm, 9.6 mm and 0.26 mm, respectively, all of which are well above tolerance levels (see Table 2.1).

Errors in the estimation of elastic properties as a result of inaccuracies in the geometrical measurements and mass are also quantifiable. The percentage errors in estimating the elastic properties for a 5% increase in length, width, thickness and mass have been represented in Fig. 2.13. Although errors illustrated are much higher than tolerance, it is demonstrated that geometric variation is not necessarily negligible and therefore should be taken into account in order to obtain results which are as accurate as possible.

²⁴Since the Poisson's ratio does not have a significant effect on bending modes as shown in the graph of sensitivity (see in Fig. 2.8), a reference value of 0.35 has taken to perform simulations [45].

Table 2.3 Experimental (Exp) and numerical (FEM) resonant modes (in Hz), corresponding mode shapes and percentage deviation between frequencies under different moisture conditions β

β	Modes	(1 1)	(2 0)	(0 2)	(2 1)	(1 2)	(3 0)	(2 2)	(3 1)	(0 3)	(1 3)
0.11%	Exp	88.0	146.7	206.8	224.8	272.8	397.1	442.3	470.4	549.2	611.5
	FEM	88.0	146.7	206.8	230.2	270.3	405.7	436.6	481.5	569.3	625.7
	Error	0.02 ^a	0.00	0.00	2.40	−0.92	2.16	−1.29	2.36	3.65	2.32
1.43%	Exp	90.8	150.6	208.2	228.3	275.7	399.9	443.0	474.9	558.9	615.0
	FEM	90.8	150.6	208.2	236.9	275.0	416.5	447.6	495.0	573.1	632.6
	Error	0.02	0.00	0.00	3.78	−0.27	4.15	1.03	4.22	2.54	2.86
6.01%	Exp	94.2	146.3	206.7	232.9	280.6	395.6	450.2	480.1	552.5	618.3
	FEM	94.2	146.3	206.7	239.5	278.2	404.9	456.1	491.0	569.0	633.1
	Error	0.00	0.00	0.00	2.83	−0.86	2.35	1.31	2.26	2.99	2.40
9.38%	Exp	92.5	142.8	200.4	232.4	275.3	397.1	443.7	481.9	533.0	604.4
	FEM	92.5	142.8	200.4	234.7	271.3	395.1	446.4	480.1	551.5	615.3
	Error	0.00	0.00	0.00	0.98	−1.44	−0.51	0.61	−0.38	3.47	1.79
15.73%	Exp	90.7	141.8	178.9	220.5	255.7	378.4	411.4	458.5	475.5	550.0
	FEM	90.7	141.8	178.9	230.9	253.4	392.5	429.9	474.4	492.1	560.1
	Error	0.00	0.00	0.00	4.73	−0.91	3.72	4.50	3.47	3.50	1.83
17.22%	Exp	90.9	133.3	174.4	218.6	252.6	371.3	407.4	451.0	458.7	539.8
	FEM	90.9	133.3	174.4	226.1	250.6	369.0	425.7	456.0	479.9	549.6
	Error	−0.03	0.00	0.00	3.44	−0.81	−0.61	4.49	1.10	4.62	1.82
18.82%	Exp	88.6	133.9	169.5	214.8	246.5	368.7	398.1	438.9	450.0	520.0
	FEM	88.5	133.9	169.5	222.7	243.8	370.7	415.8	453.0	466.3	534.4
	Error	−0.02	0.00	0.00	3.67	−1.10	0.54	4.46	3.22	3.62	2.77
20.37%	Exp	85.6	131.5	164.1	210.5	238.7	364.7	388.3	428.5	440.0	515.0
	FEM	85.5	131.5	164.1	216.5	235.8	364.0	402.7	442.4	451.4	517.1
	Error	−0.02	0.00	0.01	2.83	−1.22	−0.18	3.70	3.25	2.58	0.40
21.80%	Exp	83.9	129.9	159.9	207.4	233.6	361.8	381.4	419.3	436.8	496.5
	FEM	83.9	129.9	159.9	212.8	230.4	359.6	394.6	435.8	439.7	504.5
	Error	−0.02	0.00	0.00	2.58	−1.35	−0.61	3.46	3.94	0.66	1.61
24.71%	Exp	77.4	128.0	149.2	196.9	217.7	353.8	363.6	412.7	393.1	461.7
	FEM	77.4	128.0	149.2	201.4	213.7	354.4	367.7	420.4	409.8	469.3
	Error	0.00	−0.01	0.01	2.30	−1.83	0.17	1.12	1.86	4.25	1.65

^aNon-zero errors when equal frequencies are due to rounding off decimals

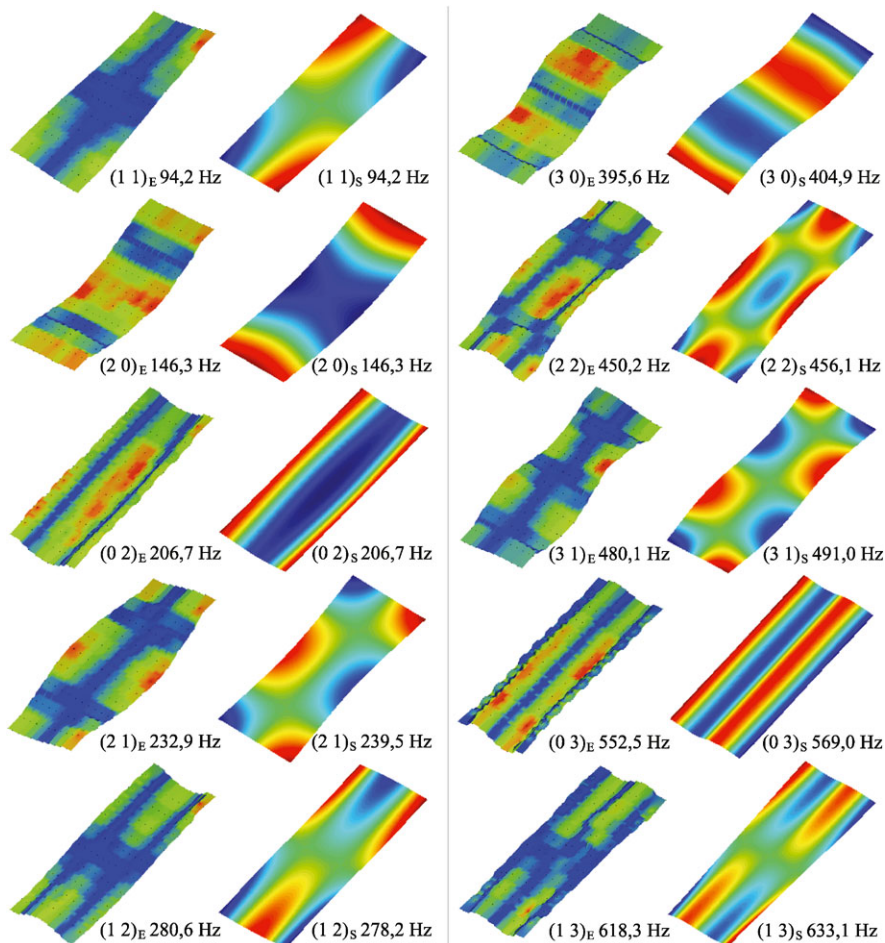


Fig. 2.11 Comparison between the numerical and experimental modal analysis for the first ten mode shapes under 6.01% moisture content conditions. The subscripts E and S denote experimental and simulated results, respectively. Possible sources of error are wood's heterogeneity, thickness irregularities, non-uniformity in the cross-grain direction and densities, as well as inaccuracies in geometrical or mass measurements

Finally, another source of error may be due to the curvature in the transverse direction which the specimen assumes under different moisture conditions. Although none of the sides of the plate are attached to any structural element, as mentioned in Sect. 2.3 (see Fig. 2.2), it has been observed during the tests that the plate becomes slightly curved across the grain under different moisture conditions, with less than 1 mm deflection in the radial-tangential plane, as shown schematically in Fig. 2.14. This distortion is due to the imperfect perpendicularity between the earlywood and latewood. This curved geometry has an influence on the modes in that it tends to soften the bending modes in the y -plane and stiffen them in the x -plane;

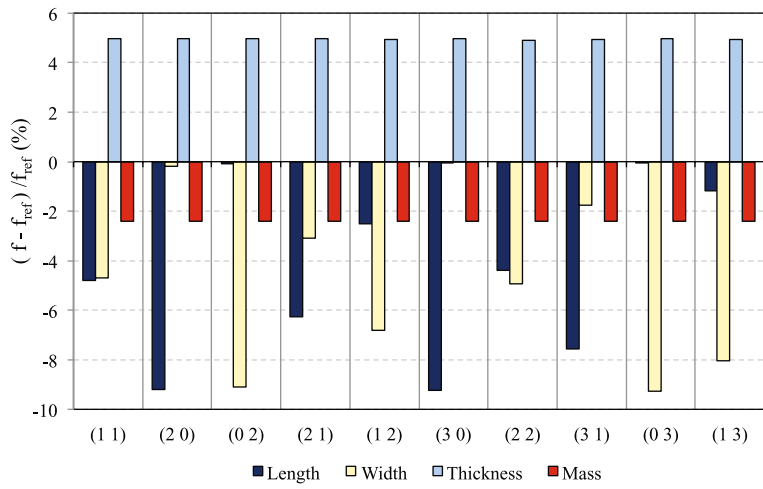
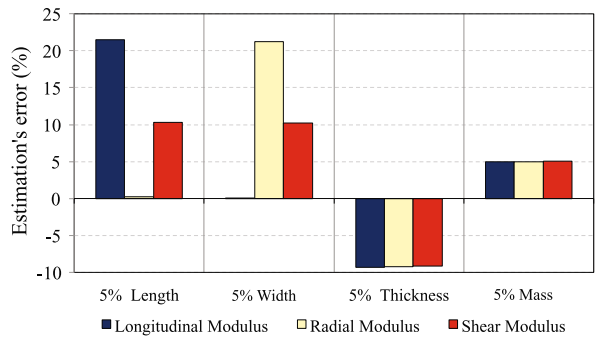


Fig. 2.12 Percentage variations of the numerical frequencies of the ten first resonance modes caused by increasing the nominal values of length, width, thickness and mass by 5%

Fig. 2.13 Percentage errors in estimating the elastic properties E_L , E_R and G_{LR} introduced by increasing the nominal values of length, width, thickness and mass by 5%

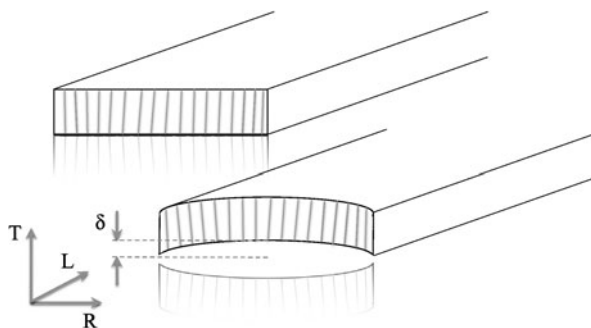


consequently, the experimental resonance would be slightly lower and higher, respectively. Since the deflection is minimal, the curvature of the plate mid-surface in the reference configuration can be considered zero, hence Kirchhoff’s plate theory can be used. However, it may become a critical issue for thicker specimens, or for grain orientations other than quarter-sawn.

2.9 Concluding Remarks

Tradition is very strong in stringed musical instruments making. Despite scientific effort to introduce new materials such as synthetic polymers and carbon fibre composites, wood remains by far the most widely used material, due to its aesthetic appeal and unique mechanical and acoustical properties. However, its susceptibility to moisture changes and the irreversible consequences that may result therefrom

Fig. 2.14 Curvature adopted by the plate under moisture conditions due to the imperfect perpendicularity between the earlywood and latewood. δ denotes deflection, and L, R and T are the longitudinal, radial and tangential directions, respectively



forces the builder to focus on this aspect, becoming one of the most important considerations in the process of construction.

The aim of this contribution is to illustrate the dynamic effects caused by moisture in the wood, to provide a practical tool for quantifying them, and finally to reaffirm the benefits of applying the techniques of simulation and modal analysis to the problem of furthering the understanding of the complex behavior of musical instrument.

It has been shown that wood's dynamic behavior is heavily dependent on moisture content. Wood changes dimension differently in the longitudinal, radial and tangential directions; in the latter two orientations, the increments have a near-linear relationship to moisture content until saturation point. The most significant effects were observed in the radial direction due to the major contribution of earlywood, and the strong influence of humidity on cell wall components. One can conclude that the lower the percentage of earlywood, that is, the denser or more fine-grained the wood, the less sensitive to moisture changes.

The combination of numerical with experimental modal analysis is a potential alternative approach for the elastic characterization of materials. Its nondestructive nature permits direct examination of a material sample which will eventually become part of a real instrument.

The use of numerical approximation to solve the plate's vibration problem has allowed sensitivity studies of the influence on the dynamic response of the parameters involved in the problem, showing, for example, the influence of a isolated parameter in global behavior, the errors in the estimation due to geometric inaccuracies, or which of the elastic properties are of first order importance in bending modes. In addition, numerical approximation allows one to work with irregular shapes such as a real instrument's soundboard. In this case, vector correlation tools, e.g. MAC, are useful, since the comparison between the numerical and experimental mode shapes may not be trivial.

In general, the results of the study agree well with the empirical knowledge of many luthiers regarding changes in timbre associated with variations in ambient humidity. A high degree of correlation between the numerical and experimental results and cited references shows that the global dynamic behavior of the wooden specimen is accurately described assuming a homogeneous, orthotropic and linear elastic material. Furthermore, given the geometric characteristics of the specimen

and in accordance with the results, plate theory can properly be applied. It must nonetheless be pointed that if one wishes to analyze specimens other than quarter-sawn, samples which are thicker or curved, or samples with varying thicknesses, these assumptions must be reconsidered.

2.10 Prospects for the Future

While the general lines to follow in the evaluation of the effects of moisture content on the in-plane elastic constants of wooden specimens have been roughly sketched out in the present work, it is certainly necessary to refine the numerical model by including other parameters such as Poisson's ratio, the consideration of more modes for the error functions and examining aspects other than just modal frequencies, incorporating a theory that provides a solution for thicker plates by using a shear deformation theory, as well as dealing with plates which vary in thickness, in which case three-dimensional stress effects could not be neglected.

Future lines of enquiry might be to compare the results with the estimation of elastic properties using ultrasonic inspection techniques, and the effects upon dynamical behavior due to different types of glue or varnish used in the construction of instruments.

Finally, since the most important aspect in the making and preservation of a musical instrument is its sound quality, future lines of enquiry might include the preservation of samples of the wood used in the construction of soundboards of specific instruments in order to allow the correlation of observed variations in the elastic characteristics or states of stress of the material with any acoustic alterations which might be detected in the instrument's sound, a study which could be carried out without having to subject the instrument itself to the sort of testing needed to establish mechanical characteristics of its component parts. In addition, further work is required on the influence of wood's moisture content on the internal friction, i.e. damping, given its significance for the final musical result.

2.11 Summary

The present work illustrates the effects on the vibration behavior caused by moisture content in wood and provides a mixed experimental-numerical tool to quantify these effects on the in-plane elastic constants. The vibrational behavior of a specimen of quarter-sawn spruce has been observed using vibration testing, from dry conditions to the saturation point. It has been shown that wood's dynamic behavior is heavily dependent on moisture content, and a significant variations in geometry and mass measurements have been observed. The methodology results in numerical eigenfrequencies that fit closely with the experimental frequencies. In general, the results show that an increase in moisture content causes a nonlinear decrease in natural frequencies and, consequently, in elastic moduli. Good agreement between

reference solutions and the results obtained have been observed, proving that the global dynamic behavior of the wooden specimen have been accurately described under the assumptions considered.

References

1. Alfano, M., Pagnotta, L.: Determining the elastic constants of isotropic materials by modal vibration testing of rectangular thin plates. *J. Sound Vib.* (2006). doi:[10.1016/j.jsv.2005.10.021](https://doi.org/10.1016/j.jsv.2005.10.021)
2. Araújo, A.L., Mota Soares, C.M., Moreira de Freitas, M.J.: Characterization of material parameters of composite plate specimens using optimization and experimental vibrational data. *Compos. Part B Eng.* (1996). doi:[10.1016/1359-8368\(95\)00050-X](https://doi.org/10.1016/1359-8368(95)00050-X)
3. Ayorinde, E.O., Gibson, R.F.: Elastic constants of orthotropic composite materials using plate resonance frequencies, classical lamination theory and an optimized three-mode Rayleigh formulation. *Compos. Eng.* (1993). doi:[10.1016/0961-9526\(93\)90077-W](https://doi.org/10.1016/0961-9526(93)90077-W)
4. Ayorinde, E.O., Yu, L.: On the elastic characterization of composite plates with vibration data. *J. Sound Vib.* (2005). doi:[10.1016/j.jsv.2004.04.026](https://doi.org/10.1016/j.jsv.2004.04.026)
5. Bathe, K.J.: *Finite Element Procedures*. Prentice Hall, New York (1996)
6. Bell, J.F.: *Mechanics of Solids: Volume 1—The Experimental Foundations of Solid Mechanics*. Springer, Berlin (1973)
7. Cousins, W.J.: Elastic modulus of lignin as related to moisture content. *Wood Sci. Technol.* (1976). doi:[10.1007/BF00376380](https://doi.org/10.1007/BF00376380)
8. Cousins, W.J.: Young's modulus of hemicellulose as related to moisture content. *Wood Sci. Technol.* (1978). doi:[10.1007/BF00372862](https://doi.org/10.1007/BF00372862)
9. Deobald, R.L., Gibson, R.F.: Determination of elastic constants of orthotropic plates by a modal analysis/Rayleigh-Ritz technique. *J. Sound Vib.* (1988). doi:[10.1016/S0022-460X\(88\)80187-1](https://doi.org/10.1016/S0022-460X(88)80187-1)
10. De Visscher, J., Sol, H., De Wilde, W.P., Vantomme, J.: Identification of the damping properties of orthotropic composite materials using a mixed numerical experimental method. *Appl. Compos. Mat.* (1997). doi:[10.1007/BF02481386](https://doi.org/10.1007/BF02481386)
11. Ewins, D.J.: *Modal Testing: Theory and Practice*. Research Studies Press, Baldock (1986)
12. Fällström, K.E., Jonsson, M.A.: Determining material properties in anisotropic plates using Rayleigh's method. *Polym. Compos.* (1991). doi:[10.1002/pc.750120503](https://doi.org/10.1002/pc.750120503)
13. Felippa, C.: Introduction to finite element methods. <http://www.colorado.edu/engi-neering/cas/courses.d/IFEM.d/> (2009). Cited Jun 2010
14. Ferreira, A.J.M.: *MATLAB Codes for Finite Element Analysis*. Springer, Berlin (2009). doi:[10.1007/978-1-4020-9200-8](https://doi.org/10.1007/978-1-4020-9200-8)
15. Grédiac, M., Paris, P.A.: Direct identification of elastic constants of anisotropic plates by modal analysis: theoretical and numerical aspects. *J. Sound Vib.* (1996). doi:[10.1006/jsvi.1996.0434](https://doi.org/10.1006/jsvi.1996.0434)
16. Grédiac, M., Fournier, N., Paris, P.A., Surret, Y.: Direct identification of elastic constants of anisotropic plates by modal analysis: experimental results. *J. Sound Vib.* (1998). doi:[10.1006/jsvi.1997.1304](https://doi.org/10.1006/jsvi.1997.1304)
17. Haines, D.W.: On musical instrument wood, Part I. *J. Catgut. Acoust. Soc.* (1979). Published in [22], paper 88
18. Haines, D.W.: On musical instrument wood, Part II: surface finishes, plywood, light and water exposure. *J. Catgut. Acoust. Soc.* (1980). Published in [22], paper 89
19. Harris, C.M., Piersol, A.G.: *Harris' Shock and Vibration Handbook*. McGraw-Hill, New York (2002)
20. Häglund, M.: Parameter influence on moisture induced eigen-stresses in timber. *Eur. J. Wood Prod.* (2009). doi:[10.1007/s00107-009-0377-2](https://doi.org/10.1007/s00107-009-0377-2)

21. Hanhijärvi, A.: Advances in the knowledge of the influence of moisture changes on the long-term mechanical performance of timber structures. *Mater. Struct.* (2000). doi:[10.1007/BF02481695](https://doi.org/10.1007/BF02481695)
22. Hutchins, C.M., Benade, V.: *Research Papers in Violin Acoustics, 1973–1995*. Acoust. Soc. Amer., USA (1996)
23. Hurlbauss, S., Gaul, L., Wang, J.T.S.: An exact series solution for calculating the eigenfrequencies of orthotropic plates with completely free boundary. *J. Sound Vib.* (2001). doi:[10.1006/jsvi.2000.3541](https://doi.org/10.1006/jsvi.2000.3541)
24. Hurlbauss, S.: Calculation of eigenfrequencies for rectangular free orthotropic plates—an overview. *J. Appl. Math. Mech.* (2007). doi:[10.1002/zamm.200710349](https://doi.org/10.1002/zamm.200710349)
25. Hwang, S.F., Chang, C.F.: Determination of elastic constants of materials by vibration testing. *Compos. Struct.* (2000). doi:[10.1016/S0263-8223\(99\)00132-4](https://doi.org/10.1016/S0263-8223(99)00132-4)
26. Jones, R.M.: *Mechanics of Composite Materials*. Taylor & Francis, London (1999)
27. Kwon, Y.W., Bang, H.: *The Finite Element Method Using MATLAB*. CRC Press, Boca Raton (1997)
28. Larsson, D.: Using modal analysis for estimation of anisotropic material constants. *J. Eng. Mech.* (1999). doi:[10.1061/\(ASCE\)0733-9399\(1997\)123:3\(222\)](https://doi.org/10.1061/(ASCE)0733-9399(1997)123:3(222))
29. Lauwagie, T.: *Vibration-based methods for the identification of the elastic properties of layered materials*. Ph.D. thesis, Katholieke Universiteit Leuven, Belgium (2005)
30. Leissa, A.W.: *Vibration of Plates*. Acoust. Soc. Amer., USA (1993)
31. Liew, K.M., Wang, C.M., Xiang, Y., Kitipornchai, S.: *Vibration of Mindlin Plates: Programming the p-Version Ritz Method*. Elsevier, Amsterdam (1998)
32. Maia, N.M.M., Silva, J.M.M.: *Theoretical and Experimental Modal Analysis*. Research Studies Press, Baldock (1997)
33. McIntyre, M.E., Woodhouse, J.: On measuring the elastic and damping constants of orthotropic sheet materials. *Acta Metall.* (1988). doi:[10.1016/0001-6160\(88\)90209-X](https://doi.org/10.1016/0001-6160(88)90209-X)
34. Molin, N.E., Lindgren, L.E.: Parameters of violin plates and their influence on the plate modes. *J. Acoust. Soc. Am.* (1988). doi:[10.1121/1.396430](https://doi.org/10.1121/1.396430)
35. Mota Soares, C.M., Moreira de Freitas, M., Araújo, A.L., Pedersen, P.: Identification of material properties of composite plate specimens. *Compos. Struct.* (1993). doi:[10.1016/0263-8223\(93\)90174-O](https://doi.org/10.1016/0263-8223(93)90174-O)
36. Ohlsson, S., Perstorper, M.: Elastic wood properties from dynamic tests and computer modeling. *J. Struct. Eng.* (1992). doi:[10.1061/\(ASCE\)0733-9445\(1992\)118:10\(2677\)](https://doi.org/10.1061/(ASCE)0733-9445(1992)118:10(2677))
37. Pagnotta, L., Stigliano, G.: Elastic characterization of isotropic plates of any shape via dynamic tests: theoretical aspects and numerical simulations. *Mech. Res. Commun.* (2008). doi:[10.1016/j.mechrescom.2008.03.008](https://doi.org/10.1016/j.mechrescom.2008.03.008)
38. Qing, H., Mishnaevsky, L.: Moisture-related mechanical properties of softwood: 3D micromechanical modeling. *Comput. Mater. Sci.* (2009). doi:[10.1016/j.commatsci.2009.03.008](https://doi.org/10.1016/j.commatsci.2009.03.008)
39. Rayleigh, J.W.S.: *The Theory of Sound*, vol. 1. Macmillan Co., New York (1877) (Reprinted by Dover Publications, New York (1945))
40. Reddy, J.N.: *Theory and Analysis of Elastic Plates*. Taylor & Francis, London (1999)
41. Rodgers, O.E.: The effect of the elements of wood stiffness on violin plate vibration. *J. Catgut. Acoust. Soc.* (1988). Published in [22], paper 48
42. Silva, C.W.: *Vibration and Shock Handbook*. CRC Press, Boca Raton (2005)
43. Sol, H.: *Identification of anisotropic plate rigidities using free vibration data*. Ph.D. thesis, University of Brussels, Belgium (1986)
44. Thompson, R.: The effect of variations in relative humidity on the frequency response of free violin plates. *J. Catgut. Acoust. Soc.* (1979). Published in [22], paper 53
45. United States Department of Agriculture: *The Encyclopedia of Wood*. Skyhorse Publishing, New York (2007)
46. Voichita, B.: *Acoustics of Wood*. CRC Press, Boca Raton (1995)
47. Wegst, U.G.K.: Wood for sound. *Am. J. Bot.* **93**(10), 1439–1448 (2006)

Vibration and Structural Acoustics Analysis
Current Research and Related Technologies
Vasques, C.M.A.; Dias Rodrigues, J. (Eds.)
2011, XXX, 327 p., Hardcover
ISBN: 978-94-007-1702-2