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Theories of Gravitation in the Twilight of Classical Physics

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1.1 The Unfolding of Alternative Theories of Gravitation

More than is the case for any other theory of modern physics, general relativity is usually seen as the work of one man, Albert Einstein. In taking this point of view, however, one tends to overlook the fact that gravitation has been the subject of controversial discussion since the time of Newton. That Newton's theory of gravitation assumes action at a distance, i.e., action without an intervening mechanism or medium, was perceived from its earliest days as being problematical. Around the turn of the last century, in the twilight of classical physics, the problems of Newtonian gravitation theory had become more acute, also due to the rise of field theory suggesting alternative perspectives. Consequently, there was a proliferation of alternative theories of gravitation which were quickly forgotten after the triumph of general relativity. Yet in order to understand this triumph, it is necessary to compare general relativity to its contemporary competitors. General relativity owes much to this competition. The proliferation of theories of gravitation provides an exemplary case for studying the role of alternative pathways in the history of science. Thus, from this perspective, the emergence of general relativity constitutes an ideal topic for addressing longstanding questions in the philosophy of science on the basis of detailed historical evidence.

Different subdisciplines of classical physics generated different ways of approaching the problem of gravitation. The emergence of special relativity further increased the number of possible approaches and created new requirements that all approaches had to come to terms with. In this paper we will survey various alternative approaches to the problem of gravitation pursued around the turn of the last century and try to assess their potential for integrating the contemporary knowledge of gravitation.[1]

[1] This paper is closely based on our introduction to Renn 2007a, Vols. 3 and 4. There are numerous historical studies of the development of gravitation theories. In particular, we would like to mention (Roseveare 1982; Edward 2002; Whitrow and Morduch 1965; Lunteren 1991) and numerous contributions within the *Einstein Studies* series 1989–.

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From the perspective of an epistemologically oriented history of science, the unfolding of alternative theories of gravitation in the twilight of classical physics can be interpreted as the realization of the potential embodied in the *knowledge system* of classical physics to address the problem of gravitation, this *knowledge system* eventually being transformed by the first relativistic revolution. The dynamics of this unfolding was largely governed by internal tensions of the *knowledge system* rather than by new empirical knowledge, which at best played only a minor role. A central problem of the Newtonian theory of gravitation was, as already mentioned, that it assumes the interaction between two attracting bodies to be instantaneous and that it does not provide any explanation for the instantaneous propagation of such interactions through arbitrary distances. This characteristic feature of the Newtonian gravitational force, called action at a distance, became even more dubious after the mid-nineteenth century when it was recognized that electromagnetic forces do not comply with the idea of action at a distance. This internal tension of the *knowledge system* of classical physics was intensified, but not created, by the advent of the theory of special relativity, according to which the notion of an instantaneous interaction between two bodies as it appears in Newton's force law can no longer be accepted.

The attempts to resolve these kinds of tensions typically crystallized around *mental models* representing the gravitational interaction on the basis of other familiar physical processes and phenomena. A mental model is conceived here as an internal *knowledge representation structure* serving to simulate or anticipate the behavior of objects or processes, like imagining electricity as a fluid or a stream of particles. Mental models are flexible structures of thinking that are suitable for grasping situations about which no complete information is available. They do so by relying on default assumptions that result from prior experiences and can be changed if additional knowledge becomes available without having to give up the model itself.[2]

Thus, in what may be called the *gas model*, gravitation could be conceived as resulting from pressure differences in a gaseous aether. Or, in what may be called the *umbrella model*, the attraction of two bodies could be imagined to result from the mutual shielding of the two bodies immersed in an aether whose particles rush in random directions and, in collisions with matter atoms, push them in the direction of the particles' motions. Or one could think of gravitation in analogy to the successful description of electromagnetism by the *Lorentz model*, accepting a dichotomy of gravitational field on the one hand and charged particles—masses—that act as sources of the field on the other. The elaboration of these approaches, with the help of mathematical formalism, led typically to a further proliferation of alternative approaches and, at the same time, provided the tools to explore these alternatives to a depth that made it possible to reveal new tensions. The history of these alternative approaches can thus be read, in a way similar to the dynamics inherent in Einstein's

[2] The notions of mental model and default assumption are taken from cognitive science; see, e.g., (Gentner and Stevens 1983, Minsky 1988). They are here combined and adapted to interpret historical developments; for more extensive discussions, see (Renn and Sauer 2007, 127; Renn and Damerow 2007).

own work,[3] as an interaction between the physical meaning embodied in various models and the mathematical formalism used to articulate them.

1.2 The Potential of Classical Physics

The history of treatments of gravitation in the nineteenth century reflects the transition from an era in which mechanics constituted the undisputed fundamental discipline of physics to an era in which mechanics became a subdiscipline alongside electrodynamics and thermodynamics.[4]

From the time of its inception, the action-at-a-distance conception of Newtonian gravitation theory was alien to the rest of mechanics, according to which interaction always involved contact. This explains the early occurrence of attempts to interpret the gravitational force by means of collisions, for instance, by invoking the umbrella model described above. During these early days the comparison of the gravitational force to electric and magnetic forces had already been suggested as well. However, the analogy with electricity and magnetism became viable only after theories on these subjects had been sufficiently elaborated. There were even attempts at thermal theories of gravitation after thermodynamics had developed into an independent subdiscipline of physics. Besides providing new foundational resources for approaching the problem of gravitation, the establishment of independent subdisciplines and the questioning of the primacy of mechanics that resulted from it affected the development of the theoretical treatment of gravitation in yet another way, namely, through the emergence of revisionist formulations of mechanics. This *heretical mechanics*, as we shall call it, consisted in attempts to revise the traditional formulation given to mechanics by Newton, Euler, and others, and often amounted to questioning its very foundations. Further stimuli for rethinking gravitation came, as we shall see in the following, from the development of astronomy and mathematics.

1.2.1 The Mechanization of Gravitation

Before the advent of the special theory of relativity, the validity of Newton's law of gravitation was essentially undisputed in mainstream physics. Alternative laws of gravitation were, of course, conceivable but Newton's law proved to be valid to a high degree of precision. While the minute discrepancies between the observed celestial motions and those predicted by Newtonian theory, most prominently the advance of Mercury's perihelion, could be resolved by one of these alternatives, they could also be resolved by adjusting lower-level hypotheses such as those regarding the distribution of matter in the solar system. In any case, the empirical knowledge at that time did not force a revision of Newtonian gravitation theory. The more pressing problem of this theory was that it did not provide a convincing model for the propagation of the gravitational force.[5]

[3] See (Renn 2007a, vols. 1 and 2).
[4] For a contemporary assessment, see (Zenneck 1903, translated in Renn 2007a, vol. 4).
[5] See (Zenneck 1903).

The most elaborate theories to address this problem made use of the umbrella model. These theories start from the idea of an impact of aether particles on matter, as formulated by Le Sage in the late eighteenth century (Le Sage 1784). The gravitational aether is imagined to consist of particles that move randomly in all directions. Whenever such an aether atom hits a material body it pushes the body in the direction of its movement. A single body remains at rest since the net impact of aether particles from all sides adds up to zero. However, if two bodies are present, they partly shield each other from the stream of aether particles. As a result, the impact of aether particles on their far sides outweighs that on their near sides and the two bodies are driven toward each other.

Caspar Isenkrahe, Sir William Thomson (Lord Kelvin), and others developed different theories based on this idea in the late nineteenth century (Thomson 1873; Isenkrahe 1879). But regardless of the details, this approach suffers from a fundamental problem related to the empirical knowledge about the proportionality of the force of gravity with mass. In order to take this into account one needs to allow the aether particles to penetrate a material body in such a way that they can interact equally with all of its parts. This requirement is better fulfilled the more transparent matter is to the aether particles. But, the more transparent matter is, the less shielding it provides from the aether particles on which the very mechanism for explaining gravity is based. Hence, without shielding there is no gravitational effect; without penetration there is no proportionality of the gravitational effect to the total mass. Furthermore, in theories explaining gravitation by the mechanical action of a medium, the problem of heat exchange between the medium and ordinary matter arises (in analogy to electromagnetic heat radiation), in most approaches leading to an extreme heating of matter.

From a broader perspective, such attempts at providing a mechanical explanation of gravity had lost their appeal by the end of the nineteenth century after the successful establishment of branches of physics that could not be reduced to mechanics, such as Maxwell's electrodynamics and Clausius' thermodynamics. Nevertheless, this development led indirectly to a contribution of the mechanical tradition to solving the problem of gravitation by provoking the emergence of revised formulations of mechanics, referred to here as heretical mechanics.

1.2.2 Heretical Mechanics

A critical revision of mechanics, pursued in different ways by Carl Neumann, Ludwig Lange, and Ernst Mach among others, had raised the question of the definition and origin of inertial systems and inertial forces, as well as their possible relation to the distribution of masses in the universe (Neumann 1870; Lange 1886; Mach 1883). Through the latter issue, this revision of mechanics was also important for the problem of gravitation. It also gave rise to attempts at formulating mechanics in purely relational terms, that is, exclusively in terms of the mutual distances of the particles and derivatives of these distances. Such attempts are documented, for instance, in texts by Immanuel and Benedict Friedlaender and of August Föppl.[6] As becomes clear from these texts, heretical mechanics contributed to understanding the relation between gravitational and inertial forces as both are due to the interaction of masses. According to Föppl there must be velocity-dependent forces between masses although he did not think of these forces as being gravitational. The Friedlaender brothers also conceived of inertia as resulting from an interaction between masses and did speculate on its possible relation to gravitation. In spite of such promising hints, heretical mechanics remained marginal within classical physics, in part because it lacked a framework with which one could explore the relation between gravitation and inertia. This relation was established by Einstein within the framework of field theory, first in 1907 through his principle of equivalence (Einstein 1907), and more fully with the formulation of general relativity.

Einstein's successful heuristic use of Machian ideas in his relativistic theory of gravitation encouraged the mechanical tradition to continue working toward a purely relational mechanics in the spirit of Mach. Attempts in this direction were made by Hans Reißner, Erwin Schrödinger, and, more recently, Julian Barbour and Bruno Bertotti.[7] The success of general relativity provided a touchstone for the viability of these endeavors. At the same time, the question of the extent to which the issues raised by heretical mechanics, such as a relational understanding of inertia, have been settled by general relativity is still being discussed today.[8]

1.2.3 From Peripheral Mathematics to a New Theory of Gravitation

The success or failure of a physical idea hinges to a large extent on the mathematical tools available for expressing it. In view of the crucial role of the mathematical concept of affine connection at a later state in the development of the general theory of relativity, it is interesting to consider the impact this tool might have had on the formulation of physical theories had it been part of mathematics by the latter half of the nineteenth century. That this counter-factual assumption is actually not that far-fetched can be seen from the work of Hermann Grassmann, Heinrich Hertz, Tullio Levi-Civita, and Elie Cartan.[9] Such a fictive development might have given rise to a kind of heretical gravitation theory driven by peripheral mathematics and formulated by some "Newstein" long before the advent of special relativity.[10] Perhaps the search for a different conceptualization of mechanics in which gravitation and inertia are treated alike, as is the case according to Einstein's equivalence principle, could have provided a physical motivation for such an alternative formulation of classical mechanics with the help of affine connections. Perhaps Heinrich Hertz's attempt to exclude forces from mechanics, replacing them by geometrical constraints, might

[6] (Friedlaender 1896; Föppl 1904); both texts are translated in (Renn 2007a, vol. 3). See further (Föppl 1905). See also (Hofmann 1904).

[7] See (Reißner 1914 and 1915; Schrödinger 1925; Barbour and Bertotti 1977, 1982).

[8] See, e.g., (Barbour and Pfister 1995).

[9] (Grassmann 1844), for a translation, see (Grassmann 1995; Levi-Civita 1916; Cartan 1986); these texts are partly reproduced in (Renn 2007a, vol. 4). See further (Hertz 1894).

[10] This idea has been developed in detail by John Stachel, see (Stachel 2007b).

have served as a starting point for such a development, triggering a geometrization of physics, had it not been so marginal to the mainstream of late nineteenth-century physics.

As with ordinary classical mechanics, Newstein's theory would have eventually conflicted with the tradition of electrodynamics and its implication of a finite propagation speed for physical interactions, which ultimately leads to the metrical structure of special relativity with its constraints on physical interactions. Then the problem that arose from this conflict could be—in contrast to the actual course of history—formulated directly in terms of the compatibility of two well-defined mathematical structures, the affine connection expressing the equality in essence of gravitation and inertia, and the metric tensor expressing the causal structure of space-time. This formulation of the problem would have smoothed the pathway to general relativity considerably since the heretical aspect of Einstein's work—the incorporation of the equality in essence of gravitation and inertia—would have already been implemented in Newstein's predecessor theory. General relativity might thus have been the outcome of mainstream research.

1.2.4 The Potential of Astronomy

Another field of classical science that might have contributed more than it actually did to the emergence of general relativity is astronomy. This is made evident by the sporadic interventions by astronomers such as Hugo von Seeliger, who questioned the seemingly self-evident foundations of the understanding of the universe in classical science.[11] Their work was stimulated by new mathematical developments such as the emergence of non-Euclidean geometries and by heretical mechanics insofar as it raised questions relevant to astronomy, for instance, concerning the definition of inertial systems. It was further stimulated by the recognition of astronomical deviations from the predictions of Newton's law (such as the perihelion advance of Mercury), or by the paradoxes resulting from applying classical physics to the universe-at-large when this is assumed to be infinite (such as the lack of definiteness in the expression of the gravitational force, or Olbers' paradox of the failure of the night sky to be as bright as the Sun).

Although the full extent to which these problems were connected became clear only after the establishment of general relativity, the astronomer Karl Schwarzschild, who was exceptional in his interdisciplinary outlook, addressed many of them and was even able to relate them to one another.[12] He explored, for instance, the cosmological implications of non-Euclidean geometry and considered the possibility of an anisotropic large-scale structure of the universe in which inertial frames can only be defined locally. With less entrenched disciplinary boundaries of late nineteenth-century classical science, such considerations could have had wider repercussions

[11] See, e.g., (Seeliger 1895 and 1909).

[12] See, for instance, (Schwarzschild 1897), (translated in Renn 2007a, vol. 3) and (Schwarzschild 1900). On Schwarzschild's prerelativistic work on foundational questions and its relation to his contribution to general relativity, see (Schemmel 2005), reproduced in (Renn 2007a, vol. 3).

on the foundations of physics, perhaps giving rise to the emergence of a nonclassical cosmology.

1.2.5 A Thermodynamic Analogy

In rejecting the assumption of an instantaneous propagation of gravitational interactions, it makes sense to modify classical gravitation theory by drawing upon analogies with other physical processes that have a finite propagation speed, such as the propagation of electromagnetic effects or the transport of heat in matter. Such analogies obviously come with additional conceptual baggage. A gravitational theory built according to the model of electrodynamic field theory, for instance, was confronted with the question of whether the gravitational analogue of electromagnetic waves really exists, or the question of why there is only one kind of charge (gravitational mass) in gravitation theory as opposed to two in electromagnetism (positive and negative charge). To avoid such complications, one could also consider amending Newtonian theory by extending the classical Poisson equation for the gravitational potential into a diffusion equation by adding a term with a first-order time derivative, exploiting the analogy with heat transport in thermodynamics. In 1911, such a theory was proposed by Gustav Jaumann without, however, taking into account the spacetime framework of special relativity (Jaumann 1911 and 1912). As a consequence, it had little impact.

1.2.6 Electromagnetism as a Paradigm for Gravitation

Since early modern times magnetism has served as a model for action at a distance as it apparently occurs between the constituents of the solar system. However, as long as there was no mathematical formulation describing magnetic forces, no quantitative description of gravitation could be obtained from this analogy. After Newton had established a quantitative description of gravitation, this could now conversely be used as a model for describing magnetic and electric forces, as realized in the laws of Coulomb, Ampère, and Biot-Savart. With its further development as represented by velocity-dependent force laws and Maxwellian field theory, electromagnetic theory regained its paradigmatic potential for understanding gravitation. After the striking success of Einstein's field theory of gravitation, which describes the gravitational force in terms of the geometry of spacetime, gravitation took the lead again as attempts were made that aimed at a geometrical description of electromagnetism and the other fundamental interactions with a view toward the unification of all natural forces. The successful development of a quantum theory of the electromagnetic field made electrodynamics a model in a number of attempts at a quantization of gravitation. It seems, however, that the successful geometrical description of gravitation on one hand and the successful quantum field theoretic description of electrodynamics on the other have driven gravitation and electromagnetism conceptually further apart than ever. It is an open and controversial issue today, how elements from the two traditions have to be combined in order to achieve a quantum theory of gravitation or, even more ambitiously, a unified theory of all fundamental interactions.

The motive of unification also underlay nineteenth-century attempts to reduce gravitation to electricity, such as those of Ottaviano Fabrizio Mossotti and Karl Friedrich Zöllner, who interpreted gravity as a residual effect of electric forces (Zöllner et al. 1882). They assumed that the attractive electric force slightly outweighs the repulsive one, resulting in a universal attraction of all masses built up from charged particles. Ultimately, however, this interpretation amounts to little more than the statement that there is a close analogy between the fundamental force laws of electrostatics and Newtonian gravitation.

The paradigmatic role of electromagnetism for gravitation theory was boosted dramatically when electrodynamics emerged as the first field theory of physics. A field-theoretic reformulation of Newtonian gravity modeled on electrostatics was provided by the Poisson equation for the Newtonian gravitational potential. Even though the Poisson equation was merely a mathematical reformulation of Newton's law, it had profound implications for the physical interpretation of gravitation and introduced new possibilities for the modification of Newtonian gravitation theory. The analogy with electromagnetism raised the question of whether gravitational effects propagate with a finite speed like electromagnetic effects. A finite speed of propagation further suggested the existence of velocity-dependent forces among gravitating bodies, amounting to a gravitational analogue to magnetic forces. It also suggested the possibility of gravitational waves. In short, a field theory of gravitation opened up a whole new world of phenomena that might or might not be realized in nature.

The uncertainty of the existence of such phenomena was in any case not the most severe problem that a field theory of gravitation was confronted with. If gravitation is conceived of as a field with energy content, the fact that like "charges" always attract has a number of problematic consequences. First and foremost, ascribing energy to the gravitational field itself leads to a dilemma that does not occur in the electromagnetic case. In the latter case, the work performed by two attracting charges equal in magnitude as they approach each other can be understood to be extracted from the field, and the field energy disappears when the charges meet at one point. In contrast, while work can also be performed by two approaching gravitating masses, the field energy is enhanced, rather than diminished, as they come together at one point. (Accordingly no equivalent of a black hole is known in electrodynamics.) As Gustav Mie explains in a paper on the gravitational potential (Mie 1915, translated in Renn 2007a, vol. 4), the gravitational field is peculiar in that it becomes stronger when work is released. While a similar effect occurs with the magnetic field of two current-bearing conductors, the source of the energy is obvious in this case. The energy comes from an external energy supply such as a battery. Such an external supply is missing in the case of gravitation. A plausible escape strategy was to assume that the energy of the gravitational field is negative so that, when the field becomes stronger, positive energy is released, which can be exploited as work. For the plausible option of formulating a theory of gravitation in strict analogy to electrodynamics by simply postulating Maxwell's equations with appropriately changed signs for the gravitational field, this negative energy assumption has dramatic consequences when considering dynamic gravitational fields. A minute deviation of a gravitating system from equilibrium will cause the field to release more and more energy, while the system deviates further and further from its original state of equilibrium. In fact, due to the reversed sign, gravitational induction, if conceived in analogy to electromagnetic induction, becomes a self-accelerating process. This will be referred to here as the *negative energy problem*.

Despite this problem, Hendrik Antoon Lorentz took up the thread of Mossotti and others and proposed to treat gravitation as a residual force resulting from electromagnetism.[13] While the electromagnetic approach to gravitation offered, in principle, the possibility to account for observed deviations from Newtonian gravitation theory, the field theories actually elaborated by Lorentz and others failed to yield the correct value for the perihelion advance of Mercury, a commonly used touchstone.

All in all, the analogy of gravitation with electromagnetism, promising as it must have appeared, could not be as complete as advocated by its proponents. The considerable potential of the tradition of field theory for formulating a new theory of gravitation still needed to be explored and the key to disclosing its riches had yet to be discovered.

The attempts to subsume gravitation under the familiar framework of electromagnetism were later followed by approaches that aimed at a unification of physics on a more fundamental level, still focusing, however, on gravitation and electromagnetism. The most prominent attempts along these lines, contemporary with the genesis of general relativity, were those of Gustav Mie and David Hilbert.[14] These attempts, however, only led to a formal integration of the two forces without offering any new insights into the nature of gravity.

The key to successfully exploiting the resources of field theory for a new theory of gravitation was only found when the challenge of formulating a gravitational field theory was combined with insights from heretical mechanics. Instead of attempting a formal unification of two physical laws, Einstein combined the field theoretic approach with the idea of an equality in essence of gravitation and inertia, and eventually achieved an integration of two knowledge traditions hitherto separated due to the high degree of specialization of nineteenth-century physics.

1.3 The Challenge of Special Relativity for Gravitation

The advent of special relativity in 1905 made the need for a revision of Newtonian gravitation theory more urgent since an instantaneous propagation of gravitation was incompatible with the new spacetime framework in which no physical effect can propagate faster than the speed of light. A revision of this kind could be achieved in various ways. One could formulate an action-at-a-distance law involving a finite time of propagation as had been developed in electromagnetism, e.g., by Wilhelm Weber. Or one could formulate a genuine field theory of gravitation. The four-dimensional

[13] Lorentz 1900, reproduced in Renn 2007a, vol. 3. See also Gans 1905 and 1912.

[14] Mie 1912, 1914, 1915; Hilbert 1916, 1917; all these sources are translated (Mie 1912 only in part) in Renn 2007a, vol. 4.

formulation of special relativity emerging from the work of Henri Poincaré, Hermann Minkowski, and Arnold Sommerfeld brought about a set of clearly distinguished alternative approaches for realizing such a field theory of gravitation. Eventually, however, due to the implications of special relativity not only for the kinematic concepts of space and time but also for the dynamic concept of mass, gravitation was bursting out of the framework of special relativity.

1.3.1 A New Law of Gravitation Enforced by Special Relativity

The simplest way to make gravitation theory consistent with special relativity was to formulate a new direct particle interaction law of gravitation in accordance with the conditions imposed by special relativity, e.g., that the speed of propagation of the gravitational force be limited by the speed of light. This kind of approach, which was pursued by Poincaré in 1906 and by Minkowski in 1908,[15] could rely on the earlier attempts to introduce laws of gravitation with a finite speed of propagation. However, the stricter condition of Lorentz invariance now had to be satisfied.

While the formulation of a relativistic law of gravitation could solve the particular problem of consolidating gravitation theory with the new theory of special relativity, it disregarded older concerns about Newtonian gravitation, such as those relating to action at a distance. Furthermore, questions concerning the fulfillment of fundamental principles of physics, such as the equality of action and reaction, emerged in these formulations. In any case, the extent to which the modified laws of gravitation could be integrated into the larger body of physical knowledge remained unclear.

1.3.2 Toward a Field Theory of Gravitation

More important and more ambitious than the attempts at a new direct-particle interaction law of gravitation was the program of formulating a new field theory of gravitation. As pointed out above, if gravitation—in analogy to electromagnetism—is transmitted by a field with energy content, the fact that in the gravitational case like "charges" (masses) attract has problematic consequences, such as the negative energy problem. A promising approach to the negative energy problem was the assumption that masses also have energy content defined in such a way that the energy content of two attracting masses decreases when the masses approach each other. This effect can in turn be ascribed to a direct contribution of the gravitational potential to the energy content of the masses. Hence, there is a way to infer a relation between mass and energy content by considering the negative energy problem of a gravitational field theory.

The above considerations on the negative energy problem suggest that the potential plays a greater role in a gravitational field theory than it does in classical electromagnetic field theory. How to represent the gravitational potential is further directly connected with the question of how to represent the gravitational mass, or, more

[15] See (Poincaré 1906) and the Appendix to (Minkowski 1908); see also (Lorentz 1910). All three texts are (partly) translated in (Renn 2007a, vol. 3).

generally, the source of the gravitational field, since both are related through the field equation. The following three mathematical types of potentials were considered before the establishment of general relativity with the corresponding implications for the field strengths and the sources.

- *Scalar theories*. Potential and source are Lorentz scalars and the field strength is a (Lorentz) four-vector.
- *Vector theories*. Potential and source are four-vectors and the field is what was then called a "six-vector" (an antisymmetric second-rank tensor).
- *Tensor theories*. Potential and source are symmetric second-rank tensors and the field is represented by some combination of derivatives of the potential.

From what has been said above about a theory of gravitation construed in analogy with electrodynamics, the problems of a vector theory become apparent. In contrast to the electromagnetic case, where the charge density is one component of the four-current, the gravitational mass density is not one component of a four-vector. From this it follows in particular that no expression involving the mass is available to solve the negative energy problem by forming a scalar product of source and potential in order to adjust the energy expression.

Having thus ruled out vector theories, only scalar theories and tensor theories remain. Einstein's theories, in particular the *Entwurf* theory and his final theory of general relativity, belong to the latter class. Further alternative tensor theories of gravitation were proposed, but only after the success of general relativity, which is why they are not discussed here. As concerns scalar theories, a further branching of alternatives occurs as shall be explained in the following.

Every attempt to embed the classical theory of gravitation into the framework of special relativity had to cope not only with its kinematic implications, that is, the new spacetime structure which required physical laws be formulated in a Lorentz covariant manner, but also with its dynamical implications, in particular, the equivalence of energy and mass expressed by the formula $E = mc^2$. Since, in a gravitational field, the energy of a particle depends on the value of the gravitational potential at the position of the particle, the equivalence of energy and mass suggests that either the particle's mass or the speed of light (or both) must also be a function of the potential. Choosing the speed of light as a function of the potential immediately exits the framework of special relativity, which demands a constant speed of light. It thus may seem that choosing the inertial mass to vary with the gravitational potential is preferable since it allows one to stay within that framework.

According to contemporary evidence and later recollections,[16] Einstein in 1907 explored both possibilities, a variable speed of light and a variable mass. He quickly came to the conclusion that the attempt to treat gravitation within the framework of special relativity leads to the violation of a fundamental tenet of classical physics, which may be called *Galileo's principle*. It states that in a gravitational field all bodies fall with the same acceleration and that hence two bodies dropped from the same height with the same initial vertical velocity reach the ground simultaneously.

[16] For references to the historical sources, see (Renn 2007b and 2007c), and (Stachel 2007a).

The latter formulation generalizes easily to special relativity. If the inertial mass increases with the energy content of a physical system, as is implied by special relativity, a body with a horizontal component of motion will have a greater inertial mass than the same body without such a motion, and hence fall more slowly than the latter.

The same conclusion can be drawn by purely kinematic reasoning within the framework of special relativity. Consider two observers, one at rest, the other in uniform horizontal motion. When the two observers meet, they both drop identical bodies and watch them fall to the ground. From the viewpoint of the stationary observer, the body he has dropped will fall vertically, while the body the moving observer has dropped will fall along a parabolic trajectory. From the viewpoint of the moving observer, the roles of the two bodies are interchanged: the first body will fall along a parabolic trajectory while the second will fall vertically.

If one now assumes that, in the reference frame of the stationary observer, the bodies will touch the ground simultaneously, as is required by Galileo's principle in the above formulation, the same cannot hold true in the moving system due to the relativity of simultaneity. In other words, Galileo's principle cannot hold for both observers. Thus, the assumption of Galileo's principle leads to a violation of the principle of relativity. On the other hand, if one assumes, in accordance with the principle of relativity, that the two observers both measure the same time of fall for the body falling vertically in their respective frame of reference, the time needed for the body to fall along a parabolic path can be determined from this time by taking time dilation into account. It thus follows that the time needed for the fall along a parabolic path is longer than the time needed for the vertical fall, in accordance with the conclusion drawn from the dynamical assumption of a growth of inertial mass with energy content.

Each of the possibilities considered by Einstein, a dependence on the gravitational potential either of the speed of light or of the inertial mass, was later explored by Max Abraham and Gunnar Nordström, respectively. These theories represented the main competitors of Einstein's theories of gravitation.

1.3.3 The Problem of Gravitation as a Challenge for the Minkowski Formalism

The assumption of a dependence of the speed of light on the gravitational potential made it necessary to generalize the Minkowski formalism, although the full consequences of this generalization became clear only gradually. It was Max Abraham who took the first steps in this direction within this formalism by implementing Einstein's 1907 suggestion of a variable speed of light related to the gravitational potential.[17] Questioned by Einstein about the consistency of the modified formalism with Minkowski's framework, he introduced the variable line element of a nonflat four-dimensional geometry.[18]

Abraham's theory stimulated Einstein in 1912 to resume work on a theory of gravitation. Apart from developing his own theory, Abraham also made perceptive

[17] (Abraham 1912a, 1912b, 1912c, 1912d, 1912e); Abraham 1912a and 1912c are translated in (Renn 2007a, vol. 3).

[18] See the "Correction" to (Abraham 1912a), (*Physikalische Zeitschrift* 13: 176).

observations on alternative options for developing a relativistic theory of gravity, and on internal difficulties as well as on physical and astronomical consequences such as energy conservation in radioactive decay or the stability of the solar system.[19]

1.3.4 A Field Theory of Gravitation in the Framework of Special Relativity?

While Abraham explored the implications of a variable speed of light, Nordström pursued the alternative option of a variable mass.[20] Nordström thus remained within the kinematic framework of special relativity. As in all such approaches, however, he did so at the price of violating to some extent Galileo's principle.

More importantly, Nordström also faced the problem that in a special relativistic theory of gravitation the dynamical implications of special relativity need to be taken into account as well. These dynamical consequences suggested, for example, ascribing to energy not only an inertial but also a gravitational mass, which immediately implies that light rays are curved in a gravitational field. This conclusion, however, is incompatible with special relativistic electrodynamics in which the speed of light is constant.

Another implication of the dynamic aspects of special relativity concerns the source of the gravitational field. If any quantity other than the energy-momentum tensor of matter is chosen as a source-term in the gravitational field equation, as is the case in all scalar theories including Nordström's, gravitational mass cannot be fully equivalent to inertial mass, whose role has been taken in special relativistic physics by the energy-momentum tensor. However, while such conceptual considerations cast doubt on the viability of special relativistic theories of gravitation, they were not insurmountable hurdles for such theories. In fact, Nordström's final version of his theory remained physically viable as long as no counter-evidence was known. Einstein's successful calculation of Mercury's perihelion advance on the basis of general relativity in late 1915 undermined Nordström's theory, which did not yield the correct value.[21] This, however, did not constitute a fatal blow as long as other astrophysical explanations of Mercury's anomalous motion remained conceivable. The fatal blow only came when the bending of light in a gravitational field was observed in 1919. Nordström's theory did not predict such an effect. For the final version of his theory this can easily be seen by observing that it can be reformulated in a conformally flat space-time. Indeed, Einstein and Adriaan Fokker showed that Nordström's theory can be viewed as a special case of a metric theory of gravitation with the additional condition that the speed of light is a constant, thus excluding a dispersion of light waves that gives rise to the bending of light rays (Einstein and Fokker 1914).

Before Nordström's theory matured to its final version, which constitutes a fairly satisfactory special relativistic theory of gravitation, several steps were necessary in which the original idea was elaborated, in particular regarding the choice of an

[19] See, in particular, (Abraham 1913 and 1915), both translated in (Renn 2007a, vol. 3).
[20] (Nordström 1912, 1913a, 1913b); for translations of these texts, see (Renn 2007a, vol. 3).
[21] Planetary motion according to Nordström's theory was discussed in (Behacker 1913).

appropriate source expression. The most obvious choice and the first considered by Nordström is the rest mass density. The problem with this quantity, however, is that it is not a Lorentz scalar. Nordström's second choice was the Lagrangian of a particle. This, however, leads to a violation of the equality of gravitational and inertial mass. While according to special relativity, kinetic energy (e.g., the thermal motion of the particles composing a body) adds to the body's inertial mass, it is subtracted from the potential energy in the Lagrangian. If that Lagrangian hence describes the gravitational mass, the difference between the two masses increases as more kinetic energy is involved. In his final theory Nordström chose, at Einstein's suggestion, the trace of the energy-momentum tensor, the Laue scalar, thus extending the validity of the equivalence principle from mass points at rest to "complete static systems." A complete static system is a system for which there exists a reference frame in which it is in static equilibrium. In such a frame, the mechanical behavior of the system is essentially determined by a single scalar quantity. In fact, since in special relativity the inertial behavior of matter is determined by the energy-momentum tensor, the requirement of equality of inertial and gravitational mass implies that a scalar responsible for the coupling of matter to the gravitational field must be derived from the energy-momentum tensor.

The problem in choosing the Laue scalar as a source expression is how to deal with the transport of stresses in a gravitational field while maintaining energy conservation. Einstein argued that—unless appropriate provisions are taken—such stresses may be used to construct a *perpetuum mobile*, since it seems that one is able to switch gravitational mass on and off, so to speak, by creating or removing stresses. In other words, while the work required for creating a stress can simply be recovered by removing it, the gravitational mass created by the stress can meanwhile be used to perform work in the presence of a gravitational field. Given that stresses depend on the geometry of the falling object under consideration, a solution can be found by appropriately adjusting the geometry, as Nordström showed. Thus, the assumption that gravitational mass can be generated by stresses led, in conjunction with the requirement of energy-momentum conservation, to the conclusion that the geometry has to vary with the gravitational potential.

According to Einstein's assessment of Nordström's final theory in his Vienna lecture, the theory satisfies all one can require from a theory of gravitation based on contemporary knowledge, which did not yet include the observation of light deflection in a gravitational field.[22] At that time no known gravitation theory was able to explain Mercury's perihelion advance. Einstein's only remaining objection concerned the fact that what he considered to be Mach's principle—the assumption that inertia is caused by the interaction of masses—appears not to be satisfied in Nordström's theory.

But as we have seen, because of the role of stresses for gravitational mass, Nordström had to assume that the behavior of rods and clocks also depends on the gravitational potential. Indeed, as becomes clear with the hindsight provided by general relativity, it is arguable whether his theory really fits the special relativistic

[22] See also the discussion in (Laue 1917 and Giulini 2007).

framework, corresponding as it does to a spacetime theory that is only conformally flat, i.e., based on a metric that is flat up to a scalar factor. The relation in which Nordström's theory stands to general relativity in that it attributes transformations to material bodies, which in the later theory are understood as transformations of space-time, is reminiscent of the way that Lorentz's theory of the aether stands to special relativity.

Appendix: Is Special Relativistic Gravitation a Theoretically Viable Option?

The following is part of a correspondence between John Norton and Domenico Giulini with whom we discussed the status of Nordström's final theory and the question of the possibility of consistent special relativistic theories of gravitation. The two continued the discussion in an e-mail exchange which we think, in its dialogical form, clarifies some of the points raised in the last subsection of our paper.[23]

John Norton You are worried that Einstein asserts a violation of conservation of energy in a theory that demonstrably conforms to energy conservation [Nordström's final theory]. But such a theory can be completely messed up if you "add" an extra assumption incompatible with the theory. Let us say, for example, that the theory requires bodies in mechanical equilibrium to change their length with gravitational potential (just as Lorentz' theory requires bodies to change their length with motion, as a mechanical effect). If you now add the assumption that such a body does not change its length as the gravitational potential changes, then you have an inconsistent set of assumptions. That inconsistency could be manifested in many different ways, including a violation of energy conservation.

Domenico Giulini Sure, if you add an assumption about the dynamical behavior of clocks and rods that simply is inconsistent with their dynamical laws, that's the end of our discussion. But do you have indications that Einstein actually did this—tacitly perhaps? What I know from his writings is that he always urged us to regard clocks and rods as "solutions to differential equations," which I read as "obeying consistent dynamical laws."

Clearly, the universal scaling behavior of atomic scales in the scalar theory of gravity may suggest to use the conformally rescaled metric as fundamental field of gravity, thereby eliminating the Minkowski metric from the description altogether, similarly to the procedure in the "flat approach to general relativity," where one starts with a mass $= 0$, spin $= 2$ field in Minkowski space. This "curved" interpretation of the scalar theory is possible. So what? It remains true that this theory has a formally consistent interpretation in Minkowski space. The curved interpretation is certainly not necessary in order to avoid formal inconsistencies, though some may find it more "natural."

[23] For a more in-depth analysis, see (Giulini 2008), in which the discussion with John Norton is also acknowledged.

John Norton Concerning Einstein urging us to regard clocks and rods as "solutions to differential equations," which you read as "obeying consistent dynamical laws": He says that later on, but doesn't do it. You've read Einstein's statements of his thought experiments pertaining to Nordstroem's theory, so you know as much about them as I do. My reading is that he assumes that a rod moved about in a gravitational field has its length fixed by the Minkowski metric, not by what mechanical equilibrium of dynamical systems yields.

Two answers to your "so what": First, if the length of real rods (and times of real clocks) responds to the conformal metric and not the Minkowski metric, then the Minkowski metric has become a kind of unknowable ether. The standard move in the "flat approach to general relativity" is to abandon the flat background, which is exactly what I take Einstein to be doing here.

Second, what if Einstein is establishing that real dynamical rods must vary in length according to the gravitational field (my original suggestion)? For purposes of a *reductio* argument he assumes they don't and ends up with a contradiction. That there is a consistent Minkowski metric theory is compatible with this contradiction, for the contradiction is just telling us that the consistent Minkowski metric theory must harbor rods that change in length with the field—which is Einstein's conclusion.

Domenico Giulini O.k., that I understand and I think there is little disagreement left. Would you then agree that (except for its experimental impossibility) the *only* flaw of scalar gravity is not an internal inconsistency, but a redundancy of its primitive elements which is precisely given by the conformal representative of Minkowski metric, which is not the one rods and clocks respond to? (I am not certain how general one may take the meaning of "clocks" and "rods" beyond that of "electromagnetically bound systems.")

John Norton Yes, I do think we are agreeing on this. The upshot of Einstein's thought experiment was that consistency required rods to respond, in effect, to the conformal metric and not the Minkowski metric, but nothing in the thought experiment spoke against the consistency of the resulting theory.

Perhaps the only difference left is one of emphasis. That the Minkowski metric has become inaccessible is generally taken as strong grounds for discarding it. It is the final step from flat spacetime to curved spacetime in the spin-2 field pathway to general relativity. The analogous step is taken when the Newtonian space and time background of Lorentz's consistent electrodynamics is discarded in favor of the Minkowski metric to which rods and clocks respond.

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