Preface

Mathematical analysis is central to mathematics curricula not only because it is a stepping-stone to the study of advanced analysis, but also because of its applications to other branches of mathematics, physics, and engineering at both the undergraduate and graduate levels. Although there are many texts on this subject under various titles such as “Analysis,” “Advanced Calculus,” and “Real Analysis,” there seems to be a need for a text that explains fundamental concepts with motivating examples and with a geometric flavor wherever it is appropriate. It is hoped that this book will serve that need. This book provides an introduction to mathematical analysis for students who have some familiarity with the real number system. Many ideas are explained in more than one way with accompanying figures in order to help students to think about concepts and ideas in several ways. It is hoped that through this book, both student and teacher will enjoy the beauty of some of the arguments that are often used to prove key theorems—regardless of whether the proofs are short or long.

The distinguishing features of the book are as follows. It gives a largely self-contained and rigorous introduction to mathematical analysis that prepares the student for more advanced courses by making the subject matter interesting and meaningful. The exposition of standard material has been done with extra care and abundant motivation. Unlike many standard texts, the emphasis in the present book is on teaching these topics rather than merely presenting the standard material. The book is developed through patient explanations, motivating examples, and pictorial illustrations conveying geometric intuition in a pleasant and informal style to help the reader grasp difficult concepts easily.

Each section ends with a carefully selected set of “Questions” and “Exercises.” The questions are intended to stimulate the reader to think, for example, about the nature of a definition or the fate of a theorem without one or more of its hypotheses. The exercises cover a broad spectrum of difficulty and are intended not only for routine problem solving, but also to deepen understanding of concepts and techniques of proof. As a whole, the questions and exercises provide enough material for oral discussions and written assignments, and working through them should lead to a mature knowledge of the subject presented.

Some of the exercises are routine in nature, while others are interesting, instructive, and challenging. Hints are provided for selected questions and exercises. Students are strongly encouraged to work on these questions and exercises and to discuss them with fellow students and teachers. They are also urged to prepare short synopses of various proofs that they encounter.

Content and Organization: The book consists of eleven chapters, which are further divided into sections that have a number of subsections. Each section includes a careful selection of special topics covered in subsections that will serve to illustrate the scope and power of various methods in real analysis. Proofs of even the most elementary facts are detailed with a careful presentation. Some of the subsections may be ignored based on syllabus requirements, although keen readers may certainly browse through them to broaden their horizons and see how this material fits in the general scheme of things. The main thrust of the book is on convergence of sequences and series, continuity, differentiability, the Riemann integral, power series, uniform convergence of sequences and series of functions, Fourier series, and various important applications.

Chapter 1 provides a gentle introduction to the real number system, which should be more or less familiar to the reader. Chapter 2 begins with the concept of the limit of a sequence and examines various properties of convergent sequences. We demonstrate the bounded monotone convergence theorem and continue the discussion with Cauchy sequences. In Chapter 3, we define the concept of the limit of a function through sequences. We then continue to define continuity and differentiability of functions and establish properties of these classes of functions, and briefly explain the uniformly continuity of functions. In Chapter 4, we prove Rolle's theorem and the mean value theorem and apply continuity and differentiability in finding maxima and minima. In Chapter 5, we establish a number of tests for determining whether a given series is convergent or divergent. Here we introduce the base of the natural logarithm e and prove that it is irrational. We present Riemann's rearrangement theorem for conditionally convergent series. We end this chapter with applications of Dirichlet's test and summability of series. There are two well-known approaches to Riemann integration, namely Riemann's approach through the convergence of arbitrary Riemann sums, and Darboux's approach via upper and lower sums. In Chapter 6, we give both of these approaches and show their equivalence, along with a number of motivating examples. After presenting standard properties of Riemann integrals, we use them in evaluating the limits of certain sequences. In this chapter, we meet the fundamental theorem of calculus, which "connects the integral of a function and its antiderivative." In Chapter 7, we discuss the convergence and the divergence of improper integrals and give interesting examples of improper integrals, namely, the *gamma function* and the *beta function*. Our particular application emphasizes the integral test, the convergence of harmonic p-series, and the Abel–Pringsheim divergence test. We deal with a number of applications of the Riemann integral, e.g., in finding areas of regions bounded by curves and arc lengths of plane curves.

Chapter 8 begins with the theory of power series, their convergence properties, and Abel's theorem and its relation to the Cauchy product. Finally, we present some methods of computing the interval of convergence of a given power series. Chapter 9 contains a systematic discussion of pointwise and uniform convergence of sequence of functions. Students generally find it difficult to understand the difference between pointwise and uniform convergence. We illustrate this difference with numerous examples. We examine the close relationship between uniform convergence and integration—on the interchange of the order of integration and summation in the limit process—followed by a similar relationship between uniform convergence and differentiation. In Chapter 10, we introduce Fourier series with their convergence properties. In addition, we present a number of examples to demonstrate the use of Fourier series, such as how a given function can be represented in terms of a series of sine and cosine functions. The reader is encouraged to make use of computer packages such as *Mathematica*® and Maple™ where appropriate. Finally, in Chapter 11, we introduce a special class of functions, namely functions of bounded variation, and give a careful exposition of the Riemann–Stieltjes integral.

Numbering: The various theorems, corollaries, lemmas, propositions, remarks, examples, questions, and exercises are numbered consecutively within a chapter, without regard to label, and always carry the number of the chapter in which they reside. The end of the proof of a theorem, corollary, lemma, or proposition is indicated by a solid square ■ and the end of a worked-out example or remark by a bullet ●.

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