

# Preface

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**T**HE OLDEST MATHEMATICAL tablets we have date from 2400 B.C., but there is no reason to suppose that the urge to create and use mathematics is not coextensive with the whole of civilization. In four or five millennia a vast body of practices and concepts known as mathematics has emerged and has been linked in a variety of ways with our day-to-day life. What is the nature of mathematics? What is its meaning? What are its concerns? What is its methodology? How is it created? How is it used? How does it fit in with the varieties of human experience? What benefits flow from it? What harm? What importance can be ascribed to it?

These difficult questions are not made easier by the fact that the amount of material is so large and the amount of interlinking is so extensive that it is simply not possible for any one person to comprehend it all, let alone sum it up and compress the summary between the covers of an average-sized book. Lest we be cowed by this vast amount of material, let us think of mathematics in another way. Mathematics has been a human activity for thousands of years. To some small extent, everybody is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wall paper or to decorate a ceramic pot with a regular pattern is doing mathematics. Further, everybody is to some small extent a philosopher of mathematics. Let him only exclaim on occasion: "But figures can't lie!" and he joins the ranks of Plato and of Lakatos.

In addition to the vast population that uses mathematics on a modest scale, there are a small number of people who are professional mathematicians. They practice mathemat-

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ics, foster it, teach it, create it, and use it in a wide variety of situations. It should be possible to explain to nonprofessionals just what these people are doing, what they say they are doing, and why the rest of the world should support them at it. This, in brief, is the task we have set for ourselves. The book is not intended to present a systematic, self-contained discussion of a specific corpus of mathematical material, either recent or classical. It is intended rather to capture the inexhaustible variety presented by the mathematical experience. The major strands of our exposition will be the substance of mathematics, its history, its philosophy, and how mathematical knowledge is elicited. The book should be regarded not as a compression but rather as an impression. It is not a mathematics book; it is a book about mathematics. Inevitably it must contain some mathematics. Similarly, it is not a history or a philosophy book, but it will discuss mathematical history and philosophy. It follows that the reader must bring to it some slight prior knowledge of these things and a seed of interest to plant and water. The general reader with this background should have no difficulty in getting through the major portion of the book. But there are a number of places where we have brought in specialized material and directed our exposition to the professional who uses or produces mathematics. Here the reader may feel like a guest who has been invited to a family dinner. After polite general conversation, the family turns to narrow family concerns, its delights and its worries, and the guest is left up in the air, but fascinated. At such places the reader should judiciously and lightheartedly push on.

For the most part, the essays in this book can be read independently of each other.

Some comment is necessary about the use of the word “I” in a book written by two people. In some instances it will be obvious which of the authors wrote the “I.” In any case, mistaken identity can lead to no great damage, for each author agrees, in a general way, with the opinions of his colleague.

# Preface to the Study Edition

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The first *Mathematical Experience* appeared in 1981. At that time, only a few years ago, it was commonly believed that it was impossible to make contemporary mathematics meaningful to the intelligent non-mathematician. Since then, dozens of popular books on contemporary mathematics have been published. James Gleick's *Chaos* was a long-run best seller. John Casti is producing a continuing series of such books.

In technology and invention, it's a commonplace that knowing what's possible is the most important ingredient of successful innovation. Perhaps the first *Mathematical Experience* changed people's idea about what's possible in exposition of advanced contemporary mathematics.

Alert readers recognized the book as a work of philosophy — a humanist philosophy of mathematics. It was far out, “maverick” (see Philip Kitcher), virtually out of contact with official academic philosophy of mathematics. In the past 15 years, humanist philosophy of mathematics has bloomed. There are anthologies, symposia, a journal. The far-out maverick of 15 years ago might be the mainstream in a few years.

The first *Mathematical Experience* was a trade book, not a textbook. It was sold in book stores, not in professor's offices. But we heard over and over of college teachers using it, in the United States, Europe, Australia, Hong Kong, Israel. It's used in two different ways: “Math for liberal arts students” in colleges of art and science, and courses for future teachers, especially secondary math teachers, in colleges of education.

In mathematics teaching, it's a commonplace that “Mathematics isn't a spectator sport.” You learn by doing, especially doing problems. Like all truisms, this is half true. Mathematics education as doing, doing, doing—no thinking, no conversation, no contemplation—can seem dreary. An artist isn't prohibited from occasional art appreciation—quite the contrary. You can't learn practical skill as a spectator, but you can learn good taste, among other things.

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The first edition invited the reader to appreciate mathematics, contemplate it, participate in a conversation about it. It contained no problems. If a teacher selected it, he/she had to supply what the book lacked. The study edition will be more convenient for both teacher and student. It aims for balance between doing and thinking. There are plenty of problems, mostly created by Professor Elena Anne Marchisotto, who also supplies generous discussion guides, essay topics, and bibliographies. We've also introduced "projects": connected sequences of problems, rising in difficulty from very easy to a little less easy. They provide extra problem-solving enjoyment, and they make points about the nature of mathematics. We've written a section on differential and integral calculus—a complete course in 15 pages—and a section on the fascinating topic of complex numbers—fascinating from both mathematical and philosophic viewpoints.

The *Standards* of the National Council of Teachers of Mathematics appeared after the first *Mathematical Experience*. To a large extent, they validated our enterprise. We were following the *Standards* before they were written. The study edition does so even more than the first.

No longer are "critical thinking" and "problem solving" just features of mathematics. They've become catchwords in American classrooms. The study edition of *The Mathematical Experience* is a part of the dominant trend in American education.

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P. J. DAVIS  
R. HERSH





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DEDICATED TO MARK KAC  
*"oh philosophie alimentaire!"*  
—Sartre

AT THE TURN OF THE CENTURY, the Swiss historian Jakob Burckhardt, who, unlike most historians, was fond of guessing the future, once confided to his friend Friedrich Nietzsche the prediction that the Twentieth Century would be "the age of oversimplification".

Burckhardt's prediction has proved frighteningly accurate. Dictators and demagogues of all colors have captured the trust of the masses by promising a life of bread and bliss, to come right after the war to end all wars. Philosophers have proposed daring reductions of the complexity of existence to the mechanics of elastic billiard balls; others, more sophisticated, have held that life is language, and that language is in turn nothing but strings of marble-like units held together by the catchy connectives of Fregean logic. Artists who dished out in all seriousness checkerboard patterns in red, white, and blue are now fetching the highest bids at Sotheby's. The use of such words as "mechanically" "automatically" and "immediately" is now accepted by the wizards of Madison Avenue as the first law of advertising.

Not even the best minds of Science have been immune to the lure of oversimplification. Physics has been driven by the search for one, only one law which one day, just around the corner, will unify all forces: gravitation and

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electricity and strong and weak interactions and what not. Biologists are now mesmerized by the prospect that the secret of life may be gleaned from a double helix dotted with large molecules. Psychologists have prescribed in turn sexual release, wonder drugs and primal screams as the cure for common depression, while preachers would counter with the less expensive offer to join the hosannahing chorus of the born-again.

It goes to the credit of mathematicians to have been the slowest to join this movement. Mathematics, like theology and all free creations of the Mind, obeys the inexorable laws of the imaginary, and the Pollyannas of the day are of little help in establishing the truth of a conjecture. One may pay lip service to Descartes and Grothendieck when they wish that geometry be reduced to algebra, or to Russell and Gentzen when they command that mathematics become logic, but we know that some mathematicians are more endowed with the talent of drawing pictures, others with that of juggling symbols and yet others with the ability of picking the flaw in an argument.

Nonetheless, some mathematicians have given in to the simplistics of our day when it comes to the understanding of the nature of their activity and of the standing of mathematics in the world at large. With good reason, nobody likes to be told what he is really doing or to have his intimate working habits analyzed and written up. What might Senator Proxmire say if he were to set his eyes upon such an account? It might be more rewarding to slip into the Senator's hands the textbook for Philosophy of Science 301, where the author, an ambitious young member of the Philosophy Department, depicts with impeccable clarity the ideal mathematician ideally working in an ideal world.

We often hear that mathematics consists mainly in "proving theorems". Is a writer's job mainly that of "writing sentences"? A mathematician's work is mostly a tangle of guesswork, analogy, wishful thinking and frustration, and proof, far from being the core of discovery, is more often than not a way of making sure that our minds are not playing tricks. Few people, if any, had dared write this out loud before Davis and Hersh. Theorems are not to

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mathematics what successful courses are to a meal. The nutritional analogy is misleading. To master mathematics is to master an intangible view, it is to acquire the skill of the virtuoso who cannot pin his performance on criteria. The theorems of geometry are not related to the field of Geometry as elements are to a set. The relationship is more subtle, and Davis and Hersh give a rare honest description of this relationship.

After Davis and Hersh, it will be hard to uphold the *Glasperlenspiel* view of mathematics. The mystery of mathematics, in the authors' amply documented account, is that conclusions originating in the play of the mind do find striking practical applications. Davis and Hersh have chosen to describe the mystery rather than explain it away.

Making mathematics accessible to the educated layman, while keeping high scientific standards, has always been considered a treacherous navigation between the Scylla of professional contempt and the Charybdis of public misunderstanding. Davis and Hersh have sailed across the Strait under full sail. They have opened a discussion of the mathematical experience that is inevitable for survival. Watching from the stern of their ship, we breathe a sigh of relief as the vortex of oversimplification recedes into the distance.

GIAN-CARLO ROTA

August 9, 1980



*“The knowledge at which geometry aims is the knowledge of the eternal.”*

PLATO, REPUBLIC, VII, 527

*“That sometimes clear . . . and sometimes vague stuff . . . which is . . . mathematics.”*

IMRE LAKATOS, 1922–1974

*“What is laid down, ordered, factual, is never enough to embrace the whole truth: life always spills over the rim of every cup.”*

BORIS PASTERNAK, 1890–1960



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