

Preface

This book is intended to be an advanced look at the basic theory of groups, suitable for a graduate class in group theory, part of a graduate class in abstract algebra or for independent study. It can also be read by advanced undergraduates. Indeed, I assume no specific background in group theory, but do assume some level of mathematical sophistication on the part of the reader.

A look at the table of contents will reveal that the overall topic selection is more or less standard for a book on this subject. Let me at least mention a few of the perhaps less standard topics covered in the book:

- 1) An historical look at how Galois viewed groups.
- 2) The problem of whether the commutator subgroup of a group is the same as the *set* of commutators of the group, including an example of when this is not the case.
- 3) A discussion of xY-groups, in particular,
 - a) groups in which all subgroups have a complement
 - b) groups in which all normal subgroups have a complement
 - c) groups in which all subgroups are direct summands
 - d) groups in which all normal subgroups are direct summands.
- 4) The subnormal join property, that is, the property that the join of two subnormal subgroups is subnormal.
- 5) Cancellation in direct sums: A group G is **cancellable in direct sums** if

$$A \boxplus G \approx B \boxplus H, \quad G \approx H \quad \Rightarrow \quad A \approx B$$

(The symbol \boxplus represents the external direct sum.) We include a proof that any finite group is cancellable in direct sums.

- 6) A complete proof of the theorem of Baer that a nonabelian group G has the property that all of its subgroups are normal if and only if

$$G = Q \rtimes A \rtimes B$$

where Q is a quaternion group, A is an elementary abelian group of exponent 2 and B is an abelian group all of whose elements have odd order.

- 7) A somewhat more in-depth discussion of the structure of p -groups, including the nature of conjugates in a p -group, a proof that a p -group with a unique subgroup of any order must be either cyclic (for $p > 2$) or else cyclic or generalized quaternion (for $p = 2$) and the nature of groups of order p^n that have elements of order p^{n-1} .
- 8) A discussion of the Sylow subgroups of the symmetric group (in terms of wreath products).
- 9) An introduction to the techniques used to characterize finite simple groups.
- 10) Birkhoff's theorem on equational classes and relative freeness.

Here are a few other remarks concerning the nature of this book.

- 1) I have tried to emphasize universality when discussing the isomorphism theorems, quotient groups and free groups.
- 2) I have introduced certain concepts, such as subnormality and chain conditions perhaps a bit earlier than in some other texts at this level, in the hopes that the reader would acclimate to these concepts earlier.
- 3) I have also introduced group actions early in the text (Chapter 4), before giving a more thorough discussion in Chapter 7.
- 4) I have emphasized the role of applying certain operations, namely intersection, lifting, quotient and unquotient to a "group extension" $H \leq G$.

A couple of random notes: Unless otherwise indicated, any theorem not proved in the text is an invitation to the reader to supply a proof. Also, sections marked with an asterisk are optional, meaning that they can be skipped without missing information that will be required later.

Let me conclude by thanking my graduate students of the past five years, who not only put up with this material in manuscript form but also put up with the many last-minute changes that I made to the manuscript during those years. In any case, if the reader should find any errors, I would appreciate a heads-up. I can be contacted through my web site www.romanpress.com.

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