Preface

This volume[1], that adds to the four volumes[2] that already appeared, complements the study of ideas and techniques of the differential and integral calculus for functions of several variables with the presentation of several specific topics of particular relevance from which the calculus of functions of several variables has originated and in which it has its most natural context. Some chapters have to be seen as introductory to further developments that proceed autonomously and that cannot be treated here because of space and complexity. However, we believe that a discussion at an elementary level of some aspects is surely part of a basic mathematical education and helps to understand the context in which the study of abstract functions of many variables finds its true motivation.

Chapter 1 aims at illustrating in concrete situations the abstract treatment of the geometry of Hilbert spaces that we presented in [GM3]. After a short illustration of Lebesgue's spaces, in particular of L^2, and a brief introduction to Sobolev spaces, we present some complements to the theory of Fourier series, the method of separation of variables for the Laplace, heat and wave equations, and the Dirichlet principle and we conclude with some results concerning the Sturm–Liouville theory. Chapter 2 is dedicated to the theory of convex functions and to illustrating several instances in which it naturally shows. Among these, the study of inequalities, the Farkas lemma, and the linear and the convex programming with the theorem of Kuhn–Tucker and of von Neumann and Nash in the theory of games. Chapter 3 is an introduction to calculus of variations. Our aim is of just presenting the Lagrangian and Hamiltonian formalism, hinting at some of its connections with geometrical optics, mechanics, and some geometrical examples. Chapter 4 deals with the general theory of differential

[1] This book is a translation and a revised edition of M. Giaquinta, G. Modica *Analisi Matematica, V. Funzioni di più variabili: ulteriori sviluppi*, Pitagora Ed., Bologna, 2005.

[2] M. Giaquinta, G. Modica, *Mathematical Analysis, Functions of One Variable*, Birkhauser, Boston, 2003,
M. Giaquinta, G. Modica, *Mathematical Analysis, Approximation and Discrete Processes*, Birkhauser, Boston, 2004,
M. Giaquinta, G. Modica, *Mathematical Analysis, Linear and Metric Structures and Continuity*, Birkhauser, Boston, 2007,
M. Giaquinta, G. Modica, *Mathematical Analysis, An Introduction to Functions of several variables*, Birkhauser, Boston, 2009.
We shall refer to these books as to [GM1], [GM2], [GM3] and [GM4], respectively.

forms with the Stokes theorem, the Poincaré lemma, and some applications of geometrical character. The final two chapters, 5 and 6, are dedicated to the general theory of measure and integration, only outlined in [GM4], and includes the study of Borel, Radon and Hausdorff measures and of the theory of derivation of measures.

The study of this volume requires a strong effort compared to the one requested for the first four volumes, both for the intrinsinc difficulties and for the width and varieties of the topics that appear. On the other hand, we believe that it is very useful for the reader to have a wide spectrum of contexts in which the ideas have developed and play an important role and some reasons for an analysis of the formal and structural foundations that at first sight might appear excessive. However, we have tried to keep a simple style of presentation, always providing examples, enlightening remarks and exercises at the end of each chapter. The illustrations and the bibliographical note provide suggestions for further readings.

We are greatly indebted to Cecilia Conti for her help in polishing our first draft and we warmly thank her. We would also like to thank Paolo Acquistapace, Timoteo Carletti, Giulio Ciraolo, Roberto Conti, Giovanni Cupini, Matteo Focardi, Pietro Majer and Stefano Marmi for their comments and their invaluable help in catching errors and misprints, and Stefan Hildebrandt for his comments and suggestions concerning especially the choice of illustrations. Our special thanks also go to all members of the editorial technical staff of Birkhäuser for the excellent quality of their work and especially to Katherine Ghezzi and the executive editor Ann Kostant.

Note: We have tried to avoid misprints and errors. However, as most authors, we are imperfect. We will be very grateful to anybody who is willing to point out errors or misprints or wants to express criticism or comments. Our e-mail addresses are

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Pisa and Firenze
March 2011

http://www.springer.com/978-0-8176-8309-2

Mathematical Analysis
Foundations and Advanced Techniques for Functions of Several Variables
Giaquinta, M.; Modica, G.
2012, XIII, 405 p. 66 illus., Hardcover
ISBN: 978-0-8176-8309-2
A product of Birkhäuser Basel