

# Contents

<b>Introduction to Shearlets</b> .....	1
Gitta Kutyniok and Demetrio Labate	
1 Introduction .....	1
2 The Rise of Shearlets .....	3
2.1 The Role of Applied Harmonic Analysis .....	3
2.2 Wavelets and Beyond .....	4
3 Notation and Background Material .....	6
3.1 Fourier Analysis .....	6
3.2 Modeling of Signal Classes .....	7
3.3 Frame Theory .....	9
3.4 Wavelets .....	10
3.5 Wavelets for Multivariate Data and Their Limitations ...	13
4 Continuous Shearlet Systems .....	15
4.1 Continuous Shearlet Systems and the Shearlet Group ...	17
4.2 The Continuous Shearlet Transform .....	17
4.3 Cone-Adapted Continuous Shearlet Systems .....	20
4.4 The Cone-Adapted Continuous Shearlet Transform .....	21
4.5 Microlocal Properties and Characterization of Singularities .....	22
5 Discrete Shearlet Systems .....	23
5.1 Discrete Shearlet Systems and Transforms .....	24
5.2 Cone-Adapted Discrete Shearlet Systems and Transforms	25
5.3 Compactly Supported Shearlets .....	28
5.4 Sparse Approximations by Shearlets .....	30
5.5 Shearlet Function Spaces .....	32
5.6 Extensions and Generalizations .....	32
6 Algorithmic Implementations of the Shearlet Transform .....	33
6.1 Fourier-Based Implementations .....	34
6.2 Spatial-Domain-Based Implementations .....	34
7 Shearlets in Applications .....	35
References .....	36

<b>Shearlets and Microlocal Analysis</b> .....	39
Philipp Grohs	
1 Introduction .....	39
1.1 Notation .....	40
1.2 Getting to Know the Wavefront Set .....	41
1.3 Contributions .....	49
1.4 Other Ways to Characterize the Wavefront Set .....	49
2 Reproduction Formulas .....	50
3 Resolution of the Wavefront Set .....	55
3.1 A Direct Theorem .....	55
3.2 Properties of the Wavefront Set .....	59
3.3 Proof of the Main Result .....	61
References .....	67
<b>Analysis and Identification of Multidimensional Singularities</b>	
<b>Using the Continuous Shearlet Transform</b> .....	69
Kanghui Guo and Demetrio Labate	
1 Introduction .....	69
1.1 Example: Line Singularity .....	70
1.2 General Singularities .....	75
2 Analysis of Step Singularities (2D) .....	76
2.1 Shearlet Analysis of Circular Edges .....	78
2.2 General 2D Boundaries .....	81
2.3 Proofs of Theorems 2 and 3 .....	83
2.4 Extensions and Generalizations .....	97
3 Extension to Higher Dimensions .....	98
3.1 3D Continuous Shearlet Transform .....	99
3.2 Characterization of 3D Boundaries .....	100
References .....	103
<b>Multivariate Shearlet Transform, Shearlet Coorbit Spaces</b>	
<b>and Their Structural Properties</b> .....	105
Stephan Dahlke, Gabriele Steidl, and Gerd Teschke	
1 Introduction .....	106
2 Multivariate Continuous Shearlet Transform .....	107
2.1 Unitary Representations of the Shearlet Group .....	107
2.2 Square Integrable Representations of the Shearlet Group .....	110
2.3 Continuous Shearlet Transform .....	113
3 General Concept of Coorbit Space Theory .....	113
3.1 General Coorbit Spaces .....	115
3.2 Atomic Decompositions and Banach Frames .....	116
4 Multivariate Shearlet Coorbit Theory .....	117
4.1 Shearlet Coorbit Spaces .....	117
4.2 Shearlet Atomic Decompositions and Shearlet Banach Frames .....	118
4.3 Nonlinear Approximation .....	119

5	Structure of Shearlet Coorbit Spaces	121
5.1	Atomic Decomposition of Besov Spaces	125
5.2	A Density Result	126
5.3	Traces on the Real Axes	127
5.4	Embedding Results	131
6	Analysis of Singularities	134
6.1	Hyperplane Singularities	134
6.2	Tetrahedron Singularities	137
	References	142
	<b>Shearlets and Optimally Sparse Approximations</b>	145
	Gitta Kutyniok, Jakob Lemvig, and Wang-Q Lim	
1	Introduction	146
1.1	Choice of Model for Anisotropic Features	146
1.2	Measure for Sparse Approximation and Optimality	147
1.3	Why is 3D the Crucial Dimension?	147
1.4	Performance of Shearlets and Other Directional Systems	148
1.5	Band-Limited Versus Compactly Supported Systems	148
1.6	Outline	149
2	Cartoon-Like Image Class	149
3	Sparse Approximations	151
3.1	(Nonlinear) $N$ -term Approximations	151
3.2	A Notion of Optimality	155
3.3	Approximation by Fourier Series and Wavelets	158
4	Pyramid-Adapted Shearlet Systems	161
4.1	General Definition	162
4.2	Band-Limited 3D Shearlets	164
4.3	Compactly Supported 3D Shearlets	166
4.4	Some Remarks on Construction Issues	169
5	Optimal Sparse Approximations	170
5.1	Optimal Sparse Approximations in 2D	170
5.2	Optimal Sparse Approximations in 3D	191
	References	196
	<b>Shearlet Multiresolution and Multiple Refinement</b>	199
	Tomas Sauer	
1	Introduction	199
2	Filters and Filterbanks	201
2.1	Filterbanks	201
2.2	Symbols and Transforms	204
2.3	Filterbanks by Matrix Completion	208
2.4	Subbands and Multiresolution	210
3	Subdivision and Refinability	212
3.1	Convergence and Basic Properties	212
3.2	Interpolatory Subdivision and Filterbanks	214
3.3	Multiresolution	216

4	Multiple Subdivision and Multiple Refinability .....	218
4.1	Basic Properties .....	219
4.2	The Multiple MRA .....	221
4.3	Filterbanks, Cascades, Trees .....	223
4.4	Things Work Along Trees .....	226
4.5	A Canonical Interpolatory Construction .....	227
5	Shearlet Subdivision and Multiresolution .....	229
5.1	Shears and Scaling .....	230
5.2	Shears of Codimension 1: Hyperplane Shearlets .....	231
5.3	Orthogonal Shearlets by Tensor Product .....	233
5.4	Implementation .....	234
	References .....	236
	<b>Digital Shearlet Transforms .....</b>	<b>239</b>
	Gitta Kutyniok, Wang-Q Lim, and Xiaosheng Zhuang	
1	Introduction .....	240
1.1	A Unified Framework for the Continuum and Digital World .....	240
1.2	Band-Limited vs. Compactly Supported Shearlet Transforms .....	241
1.3	Related Work .....	242
1.4	Framework for Quantifying Performance .....	242
1.5	ShearLab .....	243
1.6	Outline .....	243
2	Digital Shearlet Transform Using Band-Limited Shearlets .....	244
2.1	Pseudo-Polar Fourier Transform .....	245
2.2	Density-Compensation Weights .....	248
2.3	Digital Shearlets on Pseudo-Polar Grid .....	251
2.4	Algorithmic Realization of the FDST .....	259
3	Digital Shearlet Transform Using Compactly Supported Shearlets .....	261
3.1	Digital Separable Shearlet Transform .....	262
3.2	Digital Non-separable Shearlet Transform .....	270
4	Framework for Quantifying Performance .....	272
4.1	Algebraic Exactness .....	273
4.2	Isometry of Pseudo-Polar Transform .....	273
4.3	Parseval Frame Property .....	274
4.4	Space-Frequency-Localization .....	275
4.5	Shear Invariance .....	277
4.6	Speed .....	278
4.7	Geometric Exactness .....	279
4.8	Stability .....	280
	References .....	282

<b>Image Processing Using Shearlets</b> .....	283
Glenn R. Easley and Demetrio Labate	
1 Introduction .....	283
2 Image Denoising .....	284
2.1 Discrete Shearlet Transform .....	286
2.2 Shearlet Thresholding .....	289
2.3 Denoising Using Shearlet-Based Total Variation Regularization .....	292
2.4 Complex-Valued Denoising .....	294
2.5 Other Shearlet-Based Denoising Techniques .....	295
3 Inverse Problems .....	296
3.1 Inverting the Radon Transform .....	296
3.2 Deconvolution .....	299
3.3 Inverse-Halftoning .....	303
4 Image Enhancement .....	305
5 Edge Analysis and Detection .....	309
5.1 Edge Analysis Using Shearlets .....	310
5.2 Edge Detection Using Shearlets .....	311
5.3 Edge Analysis Using Shearlets .....	313
6 Image Separation .....	315
6.1 Image Model .....	315
6.2 Geometric Separation Algorithm .....	316
7 Shearlets Analysis of 3D Data .....	318
8 Additional Applications .....	320
References .....	321
<b>Index</b> .....	327

Shearlets

Multiscale Analysis for Multivariate Data

Kutyniok, G.; Labate, D. (Eds.)

2012, XIX, 328 p. 50 illus., 19 illus. in color., Hardcover

ISBN: 978-0-8176-8315-3

A product of Birkhäuser Basel