

Preface

The study of integral equations is a thoroughly fascinating chapter in man's continuing search for mathematical understanding, and the outcome of this search is both strikingly beautiful and intrinsically interesting.

The greater part of the theory of integral equations was developed early in the twentieth century as a result of the efforts of many brilliant individuals, and most of the important integral equations fittingly bear their names.

One of the most important categories of integral equations is the Fredholm integral equation, which was named after the renowned Swedish mathematician Erik Ivar Fredholm (April 7, 1866 to August 17, 1927). His landmark paper, *Sur une classe d'équations fonctionnelles*, was published in *Acta Mathematica* in 1903. The first three chapters of this text are devoted to linear Fredholm integral equations.

Another important category of integral equations is the Volterra integral equation, which was named after the distinguished Italian mathematician Vito Volterra (May 3, 1860 to October 11, 1940). Chapter 4 is devoted to linear Volterra integral equations. Nonlinear Volterra integral equations are briefly discussed in Chap. 6, and singular Volterra integral equations are touched upon in Chap. 7.

The prolific German mathematician David Hilbert (January 23, 1862 to February 14, 1943) made huge contributions to the foundation of the general theory of integral equations in his tome *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen*. The Hilbert–Schmidt theorem in Chap. 3 and the Hilbert transform in Chap. 7 are essential tools in the field.

In little over a century, more than 11,000 articles and dozens of books and manuscripts have been written that concern various aspects of the theory of integral equations. Perhaps the best way to obtain an overview of this vast subject area would be to examine the 2010 Mathematics Subject Classification published by the American Mathematical Society. This classification appears in Appendix A for the convenience of the reader.

Although it is certain that the theory of integral equations is an important part of pure mathematics, it is also true that there are many applications of this theory to problems in the physical sciences. Furthermore, considerable interactions exist

between the area of integral equations and other areas within mathematics, such as linear algebra, operator theory, and ordinary differential equations.

The specific goal of the author of any textbook is to enrich the academic lives of students by constructing an effective tool for learning and teaching, as well as independent study. My primary intention in writing this text is to present a rigorous and systematic development of the classical theory of integral equations that is accessible to the advanced undergraduate or early graduate student. Since most students at this level and many practicing scientists are generally not familiar with the intricacies of Lebesgue integration or the theory of integral operators at a sophisticated level, no references are made to these theories here. Yet, it is still possible to present most of the main results by assuming only that the functions that appear in an integral equation are either continuous or integrable in the sense of Riemann. It is also possible to give a rather thorough treatment of many significant theorems involving integral operators without the intense sophistication required in an advanced course of study. Indeed, much of the theory was originally derived under these relaxed assumptions. Hopefully, our presentation will serve as a firm foundation for the theory as well as a springboard for further reading, exploration, and research.

Although no previous experience with the theory of integral equations is required, some prerequisites are essential. It is assumed that the reader has some expertise in reading as well as writing mathematics. It is also assumed that the reader is generally familiar with the important definitions and theorems in the subject areas of linear algebra and advanced calculus that are necessary to understand the development of the theory here. Undoubtedly, a concise review of the prerequisites would refresh the student's memory. In order to serve this purpose and to enhance the overall completeness of the presentation, a section entitled *Tools of the Trade* appears at the beginning of each chapter.

In preparing this text, I have striven for precision and completeness in every example, explanation, or exercise. Theorems and their proofs are presented in a rigorously analytical manner. No tricky, frustrating details are "easily verified" or "left to the reader." Furthermore, the reader is not asked to refer to other texts to search for results that are supportive of statements made in this text. Hopefully, the result is a self-contained, straightforward, and crystal-clear treatment of the theory.

There is considerable and justified emphasis on the methods of solution. Since it might be extremely difficult or even impossible to obtain an exact solution to an integral equation, numerical methods for computing approximate solutions assume great importance. Comprehensive examples are presented that reinforce underlying theories and illustrate computational procedures. When a particular numerical procedure is especially tricky or complicated, a chart entitled *A Concise Guide to Computation* is inserted into the text.

The problem sets are meant to be devices for enrichment. Since mathematics is learned by doing, not just reading and memorizing results, understanding is generally enhanced by struggling with a variety of problems, whether routine or challenging. Understanding may even be enhanced when a problem is not solved completely. Some problems are intended to reinforce the theory while others are

purely computational. A problem may also introduce new material or relate the material of that section to other areas of mathematics.

Some expertise in the use of Wolfram's *Mathematica* or a similar software package is assumed so that students can solve problems similar to those presented in the illustrative examples, verify solutions to the exercises that require computational intensity, and engage in creative investigations of their own.

In conclusion, I have endeavored to construct a completely comprehensible exposition of the theory, so that students who read it will be adequately prepared to solve a wide variety of problems. My hope is that the book will provide the background and insight necessary to facilitate a thorough understanding of the fundamental results in the classical theory of integral equations, as well as to provide the motivation for further investigation.

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The numerous calculations necessary to present the illustrative examples and exercises throughout the text were made with Wolfram's *Mathematica* 8.0 or previous versions of it.

I also wish to acknowledge the American Mathematical Society for granting permission to include the section of the 2010 Mathematics Subject Classification pertaining to integral equations in Appendix A. This section allows students to get an immediate overview of the organization of the subject.

Finally, portions of three previously published articles were reproduced in Appendix B with the kind permission of Springer Science+Business Media B.V. These three cited articles are available in their entirety on the SpringerLink web site.



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