

## Chapter 2

# The Arrow of Time In a Universe with a Positive Cosmological Constant $\Lambda$

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There is a mounting evidence that our universe is propelled into an accelerated expansion driven by Dark Energy. The simplest form of Dark Energy is a cosmological constant  $\Lambda$ , which is woven into the fabric of spacetime. For this reason it is often referred to as vacuum energy. It has the “strange” property of maintaining a constant energy density despite the expanding volume of the universe. Universes whose energy is made of  $\Lambda$  possess an event horizon with an eternally finite constant temperature and entropy, and are known as DeSitter geometries. Since the entropy of DeSitter spaces remains a finite constant, then the meaning of a thermodynamic arrow of time becomes unclear. Here we explore the consequences of a fundamental cosmological constant  $\Lambda$  for our universe. We show that when the gravitational entropy of a pure DeSitter state ultimately dominates over the matter entropy, then the thermodynamic arrow of time in our universe may reverse in scales of order a Hubble time. We find that due to the dynamics of gravity and entanglement with other domain, a finite size system such as a DeSitter patch with horizon size  $H_0^{-1}$  has a finite lifetime  $\Delta t$ . This phenomenon arises from the dynamic gravitational instabilities that develop during a DeSitter epoch and turn catastrophic. A reversed arrow of time is in disagreement with observations. Thus we explore the possibilities that: Nature may not favor a fundamental  $\Lambda$ , or else general relativity may be modified in the infrared regime when  $\Lambda$  dominates the expansion of the Universe.

We live in a world that has an arrow of time. Time is a fascinating enigma. We are intrinsically programmed to have a *feeling* of time and of the difference between past and future. Yet there is no consensus as to what the nature of time is and why it has a direction pointing from past to future? St. Augustine described this observation and his view on the nature of time in his book of Confessions: “nevertheless there is time past and future”. The echo of these views rings louder in the last decade than over the last two millenia. The renewed interest in the time’s

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enigma at present is fueled by two major advances in physics: the discovery of the acceleration of the universe attributed to a dominant mysterious energy, *Dark Energy*; and, a Copernican extension of physics to a multiverse framework. The latter is motivated by the discovery of a landscape multiverse from string theory and from Big Bang inflation believed to be eternally reproducing new universes.

The acceleration of the universe at present is very similar to what happened during Big Bang inflation. The main difference stands in the energy scale at which inflation occurred being 122 orders of magnitude larger than the present energy. The acceleration of the universe can be attributed to a simple constant energy density term allowed by Einstein equation, known as the cosmological constant or vacuum energy. Or in complete analogy with the mechanism driving inflation, a slowly rolling scalar field, mimicking a nearly constant energy density, can be equally responsible as the driving force. Collectively these models are known as Dark Energy. Presently Dark Energy dominates the expansion of the universe by making up for 70% of its total energy budget [2]. The future evolution of the universe is entirely determined by Dark Energy. The universe can continue its accelerated expansion forever, end up in a Big Crunch or, even meet the destiny of a Big Rip whereas the acceleration of the expansion increases with time, resulting in a “tearing” apart of the very fabric of spacetime. Either one of the three options is likely to occur if Dark Energy be dynamic in nature. But if it turns out that Dark Energy is a pure cosmological constant, then the universe continues its accelerated expansion at a constant rate eternally. At the classical level of calculation, in the latter case [3], the universe ends up in a cosmic heat death, cold and empty of structure, maintaining its entropy and temperature at constant values eternally. Such a universe is known as a DeSitter universe.

Let us see what happens to a DeSitter universe when quantum mechanics is taken into account. The second law of thermodynamics states that the amount of disorder, “the entropy”, tends to increase. A thermodynamic arrow of time is determined by the direction of entropy increase. When the second law of thermodynamics is applied to the whole Universe, then the immediate implication for the time’s arrow is that the Universe must have started in an extremely low entropy state that has been growing ever since its birth to its present value. Low entropy states imply high energies, which are a small subset of the general phase space for the possible initial conditions, a universe could be born with. They are the exception rather than the generic rule. On these basis, Sir Roger Penrose argued that starting the Universe at high energies seems to make the choice of our Initial Conditions very special indeed. Argument about the improbability of high energy states relies on equilibrium statistics, with a possible loophole of eliminating the possibility of some dynamic selection being at work. Statistical mechanics estimates, often used in literature, can quantify the Penrose statement as follows: the probability to have an initial patch inflate at some high energy scale  $\Lambda_i$  goes as  $P \simeq e^{S_i}$  where the entropy  $S_i = 3/\Lambda_i$ . This expression indicates that a GUT scale inflating patch like ours, is the most special and unlikely event to have started the universe, as likely as 1 part in  $10^{10^{123}}$ . Yet without this low entropy we can’t explain the observed arrow of time.

In [14] we proposed a new approach, based on the quantum dynamics of the gravitational degrees of freedom (DoF), for the investigation of the Initial Conditions. We showed how the dynamics of the combined, matter + gravity system, is out-of-equilibrium and how this dynamics breaks ergodicity. The compression of the phase space into a small region of initial conditions corresponding to low entropy and high energy inflation resulted in the emergence of a new superselection rule for Initial Conditions with very different probability estimates from the previous arguments stated above. The superselection of high energy patches from the non-ergodic phase space helped us to understand why the Universe had to start from extremely low entropies, thereby with an arrow of time.

Let us briefly highlight the dynamics of the combined system below as it becomes relevant again for the late time accelerating Universe. The basic mechanism that picks out the high energy “corner” in the phase space of initial states is the following: gravitational DoF, here corresponding to the degrees of freedom on the DeSitter horizon  $H_0 \simeq \sqrt{\Lambda}$ , comprise a “negative heat capacity” system. Therefore gravity tries to reach equilibrium by tending to expand the spacetime to infinity; matter is a “positive heat capacity” system and thus it goes towards equilibrium by collapsing; the dynamics of the combined system, matter + gravitational DoF, is an out-of-equilibrium system with two opposing tendencies “fighting” to expand or collapse the patch. Depending upon which one wins, it can result in a collapsing initial patch or a growing one, thus creating irrelevant Initial Conditions for starting a universe when the initial state can not grow. Or, if the expansion driven by the dynamics of the gravitational DoF’s “wins” over the crushing effect of the dynamics of matter DoF’s, the results is an expanding initial patch which result in a physically relevant Initial Condition that gives birth to a universe. We belong in the latter, the “survivor universes” case. Quantitative, the entropy of the “survivor universes” is given by the Euclidean action of the system which is modified in the presence of gravity. In fact the modified entropy is even less than its equilibrium expression, due to the out-of-equilibrium dynamics. Entropy is given roughly by log of the volume in phase space. The modified entropy is then a direct consequence of the reduction of the relevant phase space to a small “corner” of its initial volume that contains the “survivor universes” only.

It is possible that our Universe may be going through inflation again at late times due to the domination of Dark Energy in its energy budget. If true, then the dynamics of the gravitational DoF is expected to play a major role again at late times and a quantum treatment of DeSitter spaces becomes relevant. Within the framework of our theory for the dynamically selected birth of our universe from a multiverse, we would like to ask: What is the ultimate state of such a  $\Lambda$ -universe and can it achieve equilibrium?

Current astrophysical observations provide convincing evidence that our universe is indeed accelerating again at recent times, this time at an energy scale  $\Lambda \simeq (10^{-3}\text{eV})^4$ . Determining whether a pure vacuum energy  $\Lambda$  or some dynamic mechanism is at work is of paramount importance and intrinsically related to understanding and predicting the future evolution of our universe. But, a well known problem in cosmology is that the outcome of cosmological observables is observer

dependent. Thus we need to define our observer. A key point to keep in mind is that in order to *meaningfully* ask questions about the selection of our Initial and Boundary Conditions, quantum numbers or constants of nature, we need to rely on a super-observer, defined as the one that resides in the multiverse of the landscape or equivalently in the allowed relevant part of the phase space. In short we need an “out-of our box” observer in order to compare possible choices [15, 26, 32]. A local internal observer, defined as the one bound to our casual patch (the volume inside the horizon is the “box”), is always surrounded by the horizon thus it can never observe or be aware of the existence of other patches outside the horizon. It is impossible for the internal observer to ask questions about the selection of our initial and final conditions and our constants of nature because the observer is forbidden to ever know that possibilities other than the universe it observes, may exist. Using the long wavelength perturbations of inflation as the super-observer, let us now explore the implications of the proposal in [14] for the late time acceleration of the universe.

We show below how the final destiny of a DeSitter universe is to become quantum again, roughly in times of order a Hubble time. While in real spacetime universes are described by their spatial and time dimension, in the abstract phase space they are represented by localized quantum wavepacket that flow along well defined paths. These paths known as classical trajectories are determined by equations of physics and the decoupling among the various paths (disentanglement of wavepacket universes from each other) is achieved via a process known as decoherence that ensures the quantum to classical transition of the universe at birth. But should the wavepacket reach a turning point in its classical path, then the universe starts displaying its quantum nature again. We show, that is, what happens to a cold and empty DeSitter universe in which quantum fluctuations are the only mechanism left. As the universe becomes quantum thereby reverses the thermodynamic arrow of time at the turning point of its classical trajectory. We will refer to the transition to a quantum state as recoherence of the wavepacket. The details of the quantum state near the turning point depend on the boundary conditions chosen for the wavefunction which are important for the puzzle of the agreement between a thermodynamic and a cosmological arrow of time. We do not embark upon this puzzle here. The key point in this investigation is the fact that the Universe becomes quantum while it is trying to reach the thermal equilibrium of a pure classical DeSitter state. As a result it never reaches this equilibrium. We investigate how the universe transits into the quantum state when it approaches a pure DeSitter state and how this transition affects the thermodynamic arrow of time. We argue on these basis that within the framework of general relativity as the theory of gravity, nature may not favor the existence of Desitter spaces, i.e., of a pure fundamental constant  $\Lambda$ . We then speculate in the discussion section that should such an exclusion principle exist in nature, it would leave us to conclude: either the possibility of a temporary bleep of acceleration due to some dynamic [28] or transplanckian [27] mechanism or; to the consequence – *if we must have  $\Lambda$  then perhaps a new theory of gravity in the infrared regime could be taking over and replacing general relativity* [29].

## 2.1 The Problem with the Thermodynamic Arrow of Time in DeSitter Spaces

Suppose we do have a new fundamental scale at low energies in our Universe, the DeSitter horizon  $H_0 \simeq \sqrt{3/\Lambda}$ , given by the pure vacuum energy  $\Lambda \simeq (10^{-3}\text{eV})^4$ . Observations will soon pin down the equation of state  $w$  for Dark Energy, thus distinguishing between a pure  $\Lambda$  and a dynamical mechanism. Observationally we also know and agree that we have an arrow of time, pointing from past to future, determined by the direction of the entropy growth. What does the constant horizon entropy of the final state of the  $\Lambda$ CDM universe imply for the arrow of time and ultimately for the DeSitter universe?

Based on the assumption that Dark Energy is a fundamental  $\Lambda$ , it seems that we ultimately must end in a DeSitter space. The gravitationally driven expansion ultimately wins [3] for the following reason: in contrast to the entropy of DeSitter space which remains eternally a finite constant, not only does matter dilute with time,<sup>1</sup> but also as we accelerate, parts of our spacetime lose causal contact with each other, swallowing matter with them. Ad infinitum we lose all the matter from our universe in this manner and the entropy contribution associated with it. Ultimately the only entropy component left is the gravitational one associated with the DeSitter horizon. Unlike Black Holes, a DeSitter horizon does not evaporate away. Matter provides the environment that ensured decoherence. In terms of wavepackets, the entanglement among various wavefunction is described by the off-diagonal terms in their possible products, known as the density matrix. Thus the Gaussian suppression in the product of different wavefunctions, with width  $\Omega_R^{-1}$  in the off-diagonal elements of the density matrix  $\rho$ , ([4, 6, 16], see also (2.12)), describes how fast our (patch) wavepacket vacua  $\phi$  decoheres from the other wavefunctions and becomes classical,  $\rho \simeq e^{-\Omega_R a^4 (\phi - \phi')^2}$ . All the other wavefunction in the multiverse provide the environment with which our universe can entangle. Decoherence and the appearance of a classical world comes about from the backreaction of the environment on our branch of the wavefunction  $\Psi(a, \phi)$ . Losing causal contact with matter seems to imply that we reach thermal equilibrium by evolving in a pure DeSitter state since we lose the environment and the coarse-grained entropy growth provided by it. This reasoning implies that the energy level in the path in phase space for our DeSitter universe should broaden. Since the DeSitter horizon  $H_0$  classically is a constant then at the classical level so is its entropy, constant and finite. But the thermodynamic arrow of time is determined by the entropy growth. At the classical level, it is not clear at all what happens to the arrow of time during an eternal DeSitter epoch when its classical entropy has to remain a constant. These paradoxes indicate that we must search for an answer not at the classical level but by investigating the quantum dynamics of the gravitational degrees of freedom of the system.

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<sup>1</sup>(e.g., in Black Holes by radiating away).

We showed in [14] that during an inflationary phase higher multipoles develop gravitational Jeans-like instabilities which illustrates the importance of the dynamics of gravitational degrees of freedom. The entropy of the universe is lowered in the combined matter + gravity inflating system. At early time, only patches inflating at high energies  $\Lambda_i$ , survived this instability. We discussed in [14, 15] how the mixing in the initial state must survive at late times, due to unitarity (see also [13]). The reduced density matrix obtained by integrating out the environment and superHubble matter modes, gave a scale of mixing and decoherence for our patch at initial times. It also provided us with the evolution equation of this mixing/coherence length at late times namely,  $[\hat{H}, \rho] = i \frac{d\rho}{dt}$  where  $\hat{H}$  is the Hamiltonian operator, values of which describe the energy of the system.

The initial decoherence width described, how fast the quantum to classical transition for our universe occurred at early times. The fact that the density matrix evolves with time  $[H, \rho_r] \neq 0$ , [14, 15] illustrates why, with gravity switched on, we should not be tempted to use equilibrium expression like  $P \simeq e_E^S$  again, and why we can not use causal patch observers for investigating this problem. The universe is in a mixed state and out-of equilibrium even at late times. How can it evolve to a pure state to reach equilibrium?

Classically, the ultimate destiny of the  $\Lambda$  universe is a cold DeSitter space in thermal equilibrium, with finite final entropy  $S \simeq 3/\Lambda$ , a pure state. Can an ultimate pure state be allowed for the DeSitter space? The environment in our investigation of the dynamics of matter + gravity DoF, is made of the matter modes within our casual patch as well as the backreaction of higher multipoles with superHubble wavelengths. By losing matter inside our casual patch means we are losing part of the environment, which is very important in providing the Gaussian suppression for the off-diagonal elements of  $\rho_r$  that ensure decoherence and a classical world. Gravity winning over matter seems to imply that we are evolving into a pure state, rather than a mixed state on the landscape phase space. *But, if we assume a unitary evolution, we can not evolve in a pure state.* Perhaps, even with matter gone, some degree of mixing must remain. This entanglement can come only from interaction with superhorizon modes, i.e., from “out-of-the box”. In fact it does. In the DeSitter stage, the environment comes from the entanglement and interaction of our DeSitter patch with the higher multipoles [14]. The dynamics of this mixed state is highly nontrivial and we set to explore it below.

Many authors have discussed the discrepancies and apparent paradoxes that arise from a finite DeSitter entropy in the final state of the universe. In [7–9, 12] authors point out that a finite entropy implies a finite size Hilbert space which is incompatible with DeSitter symmetries  $SO(d, 1)$ . Meanwhile in [21] by geometric arguments, authors realized that DeSitter space may be unstable, in accord with our results in [14, 15]. Authors of [8] then argued that perhaps DeSitter state is metastable on the landscape and it either goes through a Poincare cycle or tunnels to lower energy vacua. The decay time for tunneling to lower states  $T_f$  seems shorter than the recurrence time  $T_r$ ,  $T_f \simeq e^{S_o - S_f} \ll T_r \simeq e^S$ , where  $S_o$  is the vacua entropy and  $S_f$  the fluctuation entropy that would take the state to the top of the potential and allow it to roll the other way into the lower energy vacua. This picture fits well with eternal inflation where many pocket universes at different energies are

created and transition among them is possible. The possibility that a DeSitter state is not eternal but rather metastable due to tunneling to lower energy states, is certainly plausible.

But, even if it sounds unlikely, what if  $\Lambda$  is a globally fundamental scale, i.e., suppose that this DeSitter state has no other lower state to tunnel to? What then?

Let us show that even without tunneling, a DeSitter state is not just metastable, but gravitationally it is a catastrophically unstable state. We show here that even before it becomes metastable from tunneling, the *growth of gravitational instabilities* force the DeSitter patch to reverse its arrow of time by hitting a turning point in its trajectory in phase space. We show this phenomenon occurs in time scales of order roughly horizon scale,  $T_{\text{crunch}} \simeq \sqrt{\Lambda}$  which is exponentially shorter than tunneling time  $T_f$ . This result would be in a huge disagreement with observations. We do not observe a reversal of the arrow of time in our DeSitter universe, in time scales of the order of the age of our universe.

### 2.1.1 *Big Crunch Through Recoherence or a New Exclusion Principle?*

Suppose we are in the far future where we have lost almost all matter contact from our casual patch. As we discussed above, the only environment we are left with now comes from the backreaction of the superHubble matter modes, i.e., matter fluctuations of order DeSitter horizon  $H_0$  or larger. We would like to know the evolution of the wavefunction for our universe, and the evolution of the density matrix. The wavefunction of the universe propagates on a minisuperspace defined by the variables:  $\phi_k$  that labels the vacua with energy  $E_k$  on the landscape multiverse;  $a[t]$ , the scale factor that describes the geometry of the universes; and,  $f_k$  that describes the perturbation modes around each vacua, with wavelengths larger than the horizon. The density matrix describes the effect of the interaction of superhorizon modes on our DeSitter geometry. Proceeding as in [14], let us assume that within the WKB approximation, the wavefunction of the universe is given by,  $\Psi[a, f_n, \phi] \simeq e^S_E$ ,  $\Psi = \Psi_0 \Pi_n \psi_n$ ,  $S \simeq S_0 + \Sigma_n S_n \frac{1}{2} f_n^2$  where  $S_E$  is the Euclidean action equivalent to the entropy, and fluctuations  $f_n$  correspond to an expansion around zero of the matter field  $\phi = \phi(0) + \Sigma_n f_n(a) Q_n(x)$  which can be written as

$$\Psi \sim \Psi_0(a, \phi_0) \prod_n \psi_n(a, \phi_0, f_n). \quad (2.1)$$

The Wheeler–DeWitt Master Equation, that includes the backreaction  $\hat{H}_n$  of the matter higher multipoles  $f_n$ <sup>2</sup> is obtained from the quantized Hamiltonian

$$\hat{H} = \hat{H}_0 + \sum_n \hat{H}_n \quad (2.2)$$

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<sup>2</sup>(obtained by the expansion around zero of the landscape minisuperspace field  $\phi$ , see [14, 15]).



acting on the wavefunction, [4, 5]

$$\hat{H}_0 \Psi(a, \phi_0) = \left( - \sum_n \langle \hat{H}_n \rangle \right) \Psi(a, \phi_0), \quad (2.3)$$

where the angular brackets denote expectation values in the wavefunction  $\psi_n$  and  $\hat{H}_n$  denotes the backreaction correction from interaction with the superhorizon fluctuations  $f_n$ . The potential term of the backreaction corrections during a nearly DeSitter phase [4, 14], calculated from the inhomogeneous matter perturbations in [5], to second order, is

$$U_{\pm}(\alpha, \phi) \sim e^{6\alpha} \left[ \frac{n^2 - 1}{2} e^{-2\alpha} \pm \frac{m^2}{2} \right],$$

with  $m^2 \simeq \sqrt{\Lambda}$  at present and  $U_{\pm}$  sign corresponding to the interaction coming from subHubble(+) or superhorizon(−) multipoles respectively, as summarized in the appendix and shown in Fig. 2.3. During the DeSitter stage the backreaction energy density  $< U f_n^2 >$  becomes negative,  $U = U_-$ , since our environment contains only the modes with wavenumber  $n \leq aH_0$ , i.e., with superhorizon wavelengths [6, 14]. The index 0 refers to present day values and  $H_0 \simeq \sqrt{\Lambda}$  is the present DeSitter Hubble constant, with  $a$  the scale factor. This negative contribution yields an oscillatory correction to the action and it gives rise to tachyonic fluctuations in (2.3). The backreaction Hamiltonian, during the DeSitter stage with curvature  $\Lambda$ , is roughly

$$2e^{3\alpha} \hat{H}_n = - \frac{\partial^2}{\partial f_n^2} + e^{6\alpha} \left( -\sqrt{\Lambda} + e^{-2\alpha} (n^2 - 1) \right) f_n^2, \quad (2.4)$$

with  $\alpha = \text{Log}[a]$ ,  $\hat{H}_n \simeq \Sigma_n U_n f_n^2$  where matter fluctuations  $< f_n^2 > = O(H_0)$ . During the matter dominated phase the superhorizon modes are nearly frozen thus their kinetic term contribution can be ignored. During the DeSitter phase with vacuum energy  $\Lambda$ , our environment consisting of superhorizon matter fluctuations, contributes a negative backreaction,  $f_n^2 U_n = -(\sqrt{\Lambda} - n^2/a^2) f_n^2$ , as can be seen from the expression that  $U_n < 0$  is negative for the superhorizon higher multipoles  $n \leq aH_0$ , [6, 14]. This negative contribution drives the gravitational instability of the massive superhorizon size modes  $f_n$ , that is, during the DeSitter stage  $f_n$  modes start growing exponentially with time, (see the appendix and [14] for the  $f_n$  equation of motion).

The correction term  $\Psi_n$  to the wavefunction satisfies the following equation:

$$i \frac{\partial \Psi_n}{\partial t} = \hat{H}_n \Psi_n, \quad (2.5)$$



which for the modes  $n \leq aH_0$  with  $H_0$  our present DeSitter horizon, and  $U_n < 0$  gives a solution of the form,  $\Psi_n \simeq e^{-\frac{1}{2} f_n^2 S_n(a)}$  where, the WKB 2nd order correction term to the action is  $S_n \simeq i \int a^3 U_n \frac{d \ln(a)}{H}$ . Since  $U_n < 0$  and  $S_n$  is not real in this regime, then this correction breaks the finiteness condition.<sup>3</sup>

Perhaps a better way to see the role of the superhorizon size nonlocal entanglement of our DeSitter patch with  $f_n$ , or equivalently in the phase space description, the effect of the backreaction term on the classical path of the wavefunction is by integrating out the Friedman-like equation which includes the entanglement term. This equation is obtained from the generalized Hamiltonian in the Master equation (2.2), which contains the backreaction energy,  $\Sigma_n \hat{H}_n$

$$3M_p^2 \dot{a}^2 - \Lambda a^2 + aH_0^3 = 0, \quad (2.6)$$

where  $\Lambda = 3M_p^2 H_0^2$ . The last term comes from the backreaction of the higher multipoles  $k = n/a \leq H_0$  thus it is negative during the DeSitter stage as explained above. A rough estimate is obtained by summing over all the superhorizon modes,  $\Sigma_{aH_0}^0 < \hat{H}_n >$ , where  $\Sigma_{k=n/a=H_0}^0 (n/a)^2 dn f_n^2 \simeq aH_0^3$ . (Below we take  $8\pi G_N = 1$  unless explicitly stated). From (2.6) we can see that our classical trajectory reaches a turning point in phase space,  $\dot{a}_* = 0$ , during a finite time  $a_*$  given by:

$$a_*^2 \Lambda = a_* H_0^3, a_* = \sqrt{\Lambda}. \quad (2.7)$$

This means that due to the gravitational instability developed by the tachyonic superHubble modes,  $U_n < 0$ , our DeSitter space goes through a crunch in real spacetime or a turning point in phase space in a time given by the condition  $a_* = O(H_0)$ . Solutions to (2.6) allow us to estimate the time,  $t$ , that the transition takes to go, from the regime of a classical large  $\Lambda$  dominated DeSitter universe with scale factor  $a$  and Hubble expansion,  $H_0$ , to the quantum regime of the Universe as it approaches the crunching turning point, where the scale factor,  $a \rightarrow a_*$  becomes very small and the Hubble expansion rate,  $\dot{a} \rightarrow \dot{a}_* = 0$  approaches zero. This solution is

$$a(t) = \left( \frac{H_0^3}{\Lambda} \right) \text{Cosh}^2 \left[ \frac{1}{2} H_0 (t - t_*) \right]. \quad (2.8)$$

Clearly when  $t \rightarrow t_*$  we have  $a \rightarrow a_*$ , i.e., The universe's scale factor is reducing to a very small size and the turning point is approached at time  $t_*$ . Prior to the instabilities growth, in the regime where the universe is a classical  $\Lambda$  dominated DeSitter space, i.e., for  $t \ll t_*$ , we see that the scale factor  $a$  is nearly exponentially large, and corresponding to that of a large DeSitter space. *We can readily estimate*

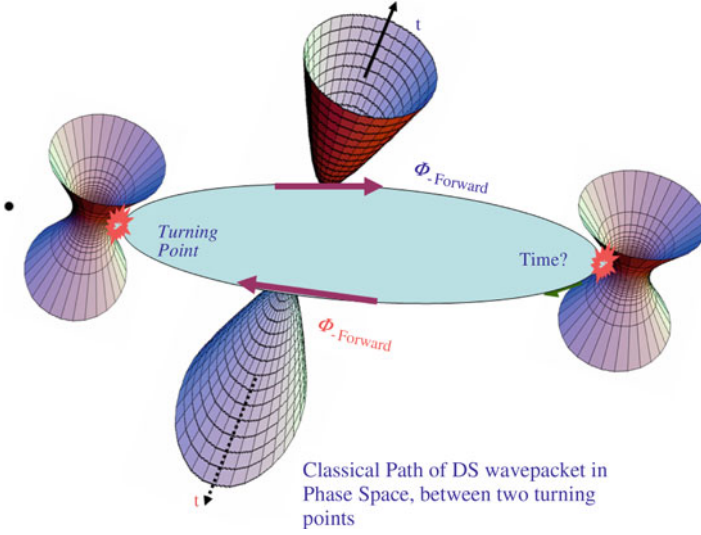
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<sup>3</sup>We can treat the contribution of each mode  $f_n$  separately since they are decoupled from each other.

from our solution, (2.8) that the time  $\Delta t = (t - t_*)$  the Universe takes to transit from a large classical regime with size given by the Hubble radius,  $a = r_{H_0}$ , to a small quantum regime  $a = a_*$  is  $\Delta t \simeq \frac{122}{H_0}$ , roughly about 122 times the DeSitter Hubble time. This is an amazing result, nothing short of an uncertainty principle! In this work, by taking into account the quantum dynamics of gravity, (2.7), leads us to an uncertainty principle for our quantum cosmological “intrinsic clocks” given by  $a(t)$  for the DeSitter system with size  $H_0^{-1}$  namely,  $a(t)H_0^{-1} \simeq l_p^2$  where Planck length  $l_p$  is unity in Planck dimensions. Using the solution for  $a(t)$  in (2.8) this result can be translated into an uncertainty principle for the physical time,  $t$ , of the DeSitter patch, given by  $\Delta t H_0 = \Delta N \simeq \text{Log}[M_p^4/\Lambda]$  where  $\Delta N$  is the number of efoldings that have crossed the horizon since the inflationary time, i.e., defined by  $a = e^N$ . It is reassuring that although DeSitter space is eternal at the classical level, our findings for a catastrophic collapse of DeSitter space at the quantum level during the time interval  $\Delta t$  due to nonlocal entanglement and the dynamics of gravity, are consistent with the uncertainty principle. Furthermore, the interpretation we offer above for the nonlocal entanglement of the superhorizon modes  $f_n$  performing a quantum measurement on the DeSitter patch is now justified by this uncertainty principle. (Note that in the above (2.6) we have not included the matter and radiation contribution to the energy density of the Universe, as we are interested here in investigating the transition regime when the Universe is becoming a DeSitter one, i.e., from the time when  $\Lambda$  dominates over the other components, and onwards. Subsequently the solution found for  $a(t)$ , (2.8), reflect that fact through the nearly exponential time dependence, the  $\text{Cosh}^2$  term.).

Since our classical trajectory  $\phi(a)$  goes through a turning point induced by the gravitational instabilities of backreaction in the DeSitter epoch,  $\dot{\Psi} = 0$ , it follows that after losing casual contact with the internal environment, the universe becomes quantum again as we approach the turning point in a time  $a_*$ . If observers bound to the causal patch survived while the universe is transiting to a quantum state, they would perceive the arrow of time as being reversed near the turning point. However note that very close to the turning point, the WKB approximation breaks down, therefore the concept of time and any statement about what could be observed becomes very fuzzy. A better estimate as to when effects related to “recoherence” display a significant effect in our universe, is obtained from the density matrix below, (2.11). Physically, the conditions placed on the wavefunction near its turning point, imply a mixing of “future” and “past” branches of the trajectory. That is, somehow the universe is “aware” of its future evolution since the forward and backward direction contribute into making the wavepacket of the universe. Near the turning point, the two branches merge resulting in a zero speed for the wavefunction there, (thus the name “turning point”), Fig. 2.1. The behavior of the wavefunction can not be described classically anylonger and the intrinsic quantum nature of the wavefunction takes over as it approaches the turning point.

We would like to explore the evolution of the density matrix  $\rho$  and the Gaussian suppression width, in order to understand better the effect of the interaction of superHubble modes with the DeSitter geometry and the (re-)appearance of a quantum world during the reversal of the arrow of time near the turning point.



**Fig. 2.1** The classical path of the wavefunction of the universe  $\Psi[\phi, a]$  between its two turning point, in phase space. The wavepacket is a mixture of the forward and backward branches in order to have a vanishing speed near its turning point,  $\dot{\Psi} = 0$

We exhibited in [14] that the phase space is not conserved due to these interactions. The equation that governs the evolution of  $\rho$  is

$$[\hat{H}, \rho] = i \frac{d\rho}{dt}. \quad (2.9)$$

After exit from inflation, during a matter dominated universe, the matter environment within our casual patch provides a strong suppression in the off-diagonal elements of  $\rho$  [16], with width given by  $\Omega_R^{-1}$ ,

$$\rho \simeq e^{-\Omega_R a^4 (\phi - \phi')^2}. \quad (2.10)$$

(Note that a large  $\Omega_R$  means high squeezing in  $\phi$  but a broad wavepacket in the conjugate momenta  $p_\phi$  of  $\phi$ ). We can now obtain the evolution of  $\rho$  by solving (2.9) for late times, when the small vacuum energy  $\Lambda$  starts dominating the total energy density in the Universe,  $\hat{H}$ , by including the effect of backreaction term. Solutions to (2.9) give the following:

$$\rho \simeq e^{-\left[\frac{\Omega_R}{a^2} + \mu^2\right] a^6 (\phi - \phi')^2}. \quad (2.11)$$

The second term is the correction coming from the backreaction correction,  $S_n$ , where  $\mu^2 \simeq < \frac{n^2 a}{H_0} [1 - \frac{1}{3} (\frac{m^2 a}{n})^2] f_n^2 >$ . The first term is much studied in literature, (see, e.g., [16]). For the sake of illustration, we can use as an approximate estimate

for the first term  $\Omega_R$  the results obtained in, e.g., [17]. Clearly,  $\mu^2 \leq 0$  for the backreaction of superhorizon modes  $n \leq \sqrt{2/3}aH_0$ , on the DeSitter Universe. We can therefore see that the total Gaussian suppression in (2.11),  $\xi = [\frac{\Omega_R}{a^2} + \mu^2]$  is decreasing and approaching zero fast as  $\mu^2 \simeq \Sigma_n S_n f_n^2$  becomes negative.

The reduced density matrix:

$$\begin{aligned} \rho &= \int \Psi(a, \phi, f_n) \Psi(a', \phi', f'_n) \Pi_n df_n df'_n \\ &\simeq \rho_0 e^{-\frac{(a\Pi)^6 H^2 \mu^2 (\phi - \phi')^2}{2}} \\ \rho_0 &\simeq \langle \Psi_0(a, \phi) \Psi_0(a', \phi') \rangle \\ &\simeq \rho(a, a') e^{-\xi a^6 (\phi - \phi')^2}. \end{aligned} \quad (2.12)$$

Note that since  $\mu^2 < 0$ , we effectively have  $\xi = \frac{\Omega_R}{a^2} - |\mu^2|$ . As  $\xi \rightarrow 0$  the classical trajectory in phase space  $\phi$  is spreading thus losing its classicality and becoming highly quantum. The increasing width  $\xi^{-1}$  of the Gaussian cross-term of the density matrix  $\rho$  describes the broadening of the wavepacket during the transition to a quantum universe. What effects would we observe as the universe is going towards the transition? This is a difficult question since near the turning point, the width of the Gaussian suppression goes to infinity, thus the WKB approximation breaks down at the turning point. Hence, if we were to speculate as to what would we observe away from the turning point where the semiclassical treatment is still valid, as the broadening of the wavepacket is becoming comparable to the difference between neighboring energy levels  $\phi, \phi'$ , then we would probably expect to see an inhomogeneous growth of anisotropies in the sky and probably different values of  $\Lambda$  and quantum numbers in different parts of the sky, i.e., the opposite of the decoherence effect in the early universe [11].

Varying the above action of matter + gravity,  $S = S_g + \Sigma_n S_{n, \text{backreac}}$  and using the  $\phi(a)$  trajectory equation of motion, we can obtain the turning point for  $\phi$  by the condition  $\xi = 0$  which gives  $a_* \simeq \sqrt{\frac{2\Lambda}{3}}$  using the expressions above for  $\mu$  and for the time evolved solution  $\rho$ . During a DeSitter phase in a time  $a_*$ , the Gaussian width that suppresses the off-diagonal element of  $\rho_r$  in  $\phi$  space is increasing due to the developing instability,  $\rho_r \simeq e^{-[\Omega_R - a^2 \mu^2] a_*^4 (\phi - \phi')^2}$ . The energy level in phase space is broadening in the landscape coordinate  $\phi$ . By the uncertainty principle a broadening in  $\phi$  means high squeezing in the space of its conjugate momenta  $p_\phi$  since  $\Delta\phi \Delta p_\phi \simeq 1$ . Thus the big crunch in real space and the quantum recoherence in our universe as it goes through its turning point in phase space. We can estimate the entropy of the DeSitter universe as it is heading through its turning point by noting that the Euclidean action is the entropy of the system. Roughly,  $S \simeq |r_{\text{DeSitter}} - r_{f_n}|^2 \simeq (\sqrt{\frac{3}{\Lambda}} - \frac{3a^2}{2\Lambda^{3/2}}) \rightarrow 0$ . This means that the total entropy for the

DeSitter state including its horizon size fluctuations goes to zero at  $a_* \simeq O(\sqrt{\Lambda})$ ,<sup>4</sup> as the universe becomes quantum through its turning point in phase space and big crunch in real space. This is to say that the allowed volume in phase space for this classical configuration shrinks to a point, i.e., nearly zero. And so is the dimension of the Hilbert space for it. This implies that there are no states available in the Hilbert space for a DeSitter universe.

### 2.1.2 What Have We Learned?

The analysis carried in this work illustrates the fact that superhorizon nonlocal entanglement and the dynamics of gravity can not be ignored during a DeSitter era. When the DeSitter Universe loses causal contact with the internal environment, its only interaction left is with the higher multipoles. During this stage the DeSitter patch recoheres and ends up with its quantum entropy, which is zero, due to developing Jeans-like instabilities. An immediate consequence of this statement is that the arrow of time is reversed within a time  $a_*$  during the DeSitter phase, due to the appearance of coherence and broadening of levels, as the Universe heads towards its gravitationally induced turning point. Although the universe becomes quantum by being superimposed to its own reversed copy and other patches at its turning point, we as internal observers perceive this reversal as a catastrophic crunch in our patch. The details at the turning point depend on the boundary conditions imposed on  $\Psi(a_*)$ . However at the turning point the quantum universe is superimposed to its own time reversed copy and possibly other patches such that its momenta in phase space squeezes to nearly zero. The instability time when this phenomena occurs is, as we have shown here,  $a_* \simeq O(H_0)$ . This time scale is exponentially shorter even than the tunneling time, if the metastable DeSitter space could lower its energy by decaying. Tunneling to other vacua may not help save the DeSitter Universe. Due to the quantum dynamics of gravity this unstable state crunches catastrophically by becoming quantum during the time scales that it would have reached its final equilibrium and settle into a pure state. The mixing does not allow DeSitter space to evolve and settle into a classical and pure equilibrium state with constant entropy at the end. Instead as the universe is trying to reach its DeSitter equilibrium, this entanglement induces a violent transition to a quantum universe for the final state of the DeSitter space. This recoherence occurs much before the universe can tunnel through to a lower energy configuration. As the DeSitter universe is becoming unstable, internal observers in this patch, perceive their thermodynamic arrow of time as being reversed during a time scale  $a_*$  of order a Hubble time. Clearly we

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<sup>4</sup>A result we could have expected physically, for a quantum DeSitter state but that is confirmed here by a different calculation.

don't observe this in our universe. This fact in combination with the result that the volume of phase space for DeSitter states is zero leads us to conclude: Nature forbids the existence of a pure cosmological constant.

## 2.2 Comments

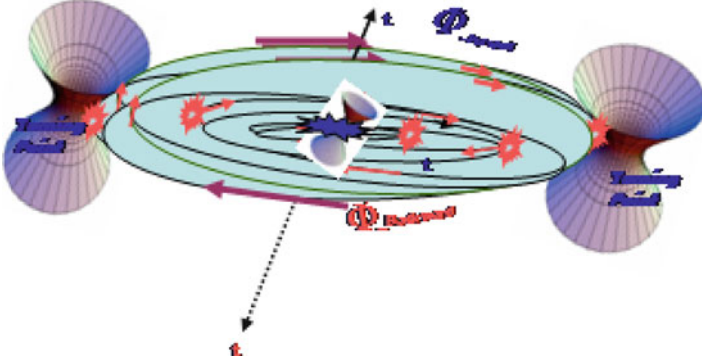
Then what are we going to make of the observed acceleration of our universe? Based on the above analysis, we would predict that this acceleration is a temporary bleep due to some dynamic or transplanckian field and not our final destiny due to a fundamental scale  $\Lambda$  in nature. Observations will soon tell us whether this is true or false. In our analysis we assume general relativity as the theory of gravity and the validity of quantum mechanics. If it turns out that we observe a pure  $\Lambda$  then this must signal the breakdown of the general relativity in the IR regime and the emergence of a new theory of gravity by which our arguments above would not hold. It is though entirely possible that, just like the second law of thermodynamics, we may have a new empirical law in nature, let us call it the "Entropy Exclusion Principle", that is: *Nature May Not Allow a Fundamental  $\Lambda$* .

How can these predictions be tested? The combined observations from SN1a, large scale structure LSS and CMB, from existing or upcoming experiments like Planck, SNAP and LISA, will soon be able to pin down whether the equation of state of Dark Energy is a constant or if it evolves with time [24]. We concluded here that if it turns out we do observe a cosmological constant then gravity must be modified in the ultralarge scales, the IR regime. A modified theory of gravity would leave a strong signature on large scale structure of the universe [22,23]. Dark Matter experiments, e.g., [25] and weak lensing tomography for LSS would reveal such deviations and test our predictions.

It is interesting to note that the initial and boundary states are selected by the same superselection rule in phase space. Without a precise knowledge of quantum gravity and the phase space for the wavefunctions, we can only speculate as to what happens next: if quantum mechanics remains valid deep into the quantum gravity realm then, after the turning point crunch, for example with a boundary condition  $\dot{\Psi} = 0$ , the "DeSitter" universe would travel its reversed path in the trajectory towards its initial point,<sup>5</sup> therefore completing one of its cycles between initial and final states. Such a condition on the wavefunction also implies a mixing of past and future, of

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<sup>5</sup>For a different choice of boundary conditions for the wavepacket in phase space, the reversed path can be very different from the forward one depending on what WKB wavefunctions in the wavepacket dominate the action in the reversed direction, in which case there may not even be a cyclic behavior at all. Subtleties related to the boundary conditions at the turning point were investigated by Hawking and Page [30] for the case of closed universes, pointed out to me by R. Holman. It would be interesting to investigate the role of the choice of boundary conditions for  $\Psi(a_*)$  on the cosmological arrow of time, for DeSitter space but this issue is beyond the scope of the present paper.



**Fig. 2.2** A schematic drawing of the cycles between the two turning points of the classical path for the DeSitter wavefunction. Each cycle gets shorter due to some energy of the wavepacket being lost through the interaction with the environment. After many such cycles, the shifting of the classical path and the shortening of the cycle results in spiraling into a merging event. However, the forward and backward branches are still mixed together for each cycle. The universe always seems “aware” of its future evolution

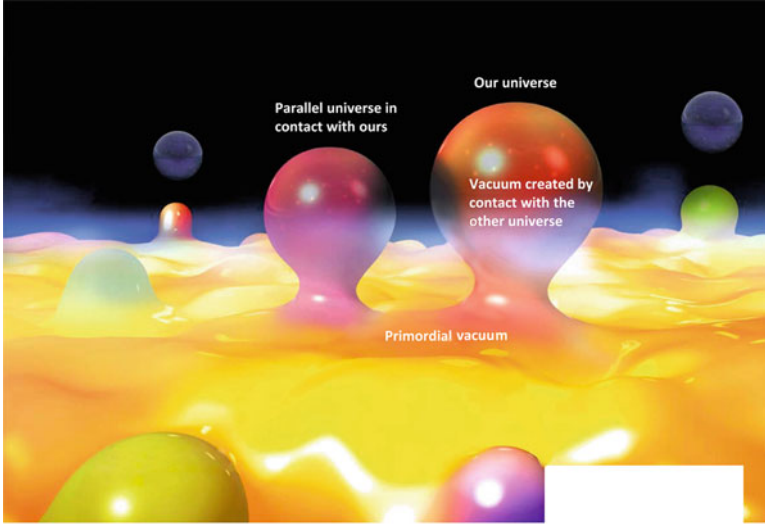
forward and backward paths, of a universe being aware of its future evolution as it is intrinsically build onto its past. In every cycle between two turning points, the recycling universe bounces between classical and quantum in each turning point, thus a classical world would be only an intermediate stage when the universe is away from the turning point in its trajectory. Near the turning point the wavepacket would spread out by disintegrating into its many components and thus the Universe would behave as a quantum system. However decoherence in each cycle takes its toll: each cycle gets shorter and shorter due to the shifting of the energies by the interaction with states, Fig. 2.2. (The shifting of the energy is a well known result in particle physics when a particle interacts with a field then its trajectory is modified due to its energy shift by interaction [17].) It is possible in this picture that after many such cycles, with shorter and shorter trajectories, the universe will spiral in to one point and disappear. Could this speculation provide us with a mechanism for DeSitter space evaporating away its  $\Lambda$  during its recycling in phase space?

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## 2.3 Appendix

Here we summarize the calculation for the backreaction potential term  $U_{-}(\alpha, \phi)$ , of (2.3). Based on [4, 5, 14], a time parameter  $t$  is defined for WKB wavefunctions of the universe such that the equation for the perturbation modes  $\psi_n$  in the





The landscape of vacua  $\{\Phi_k\}$ . A universe is born from the wavepacket  $\Psi_n[\Phi, a]$  in vacua  $\Phi_k$ . The geometry of the universe is described by its scale factor  $a_k[t]$ .

**Fig. 2.3** A schematic drawing of the birth of the universe from the multiverse. Only the high energy initial states in the multiverses are dynamically selected to give birth to a universe. Low energy states become “terminal”. Notice the minisuperspace defined by the landscape vacua  $\phi_k$  and the three-geometries  $a[t]$  of universes, is the abstract space in which the wavefunctions of the universe propagate

wavefunction can be written as a Schrödinger equation and be consistent with Einstein equations.

If  $S$  is the action for the mean values  $\alpha, \phi$  for a nearly DeSitter state with scale factor  $a = \text{Log}[\alpha]$  and vacuum energy given roughly by  $\Lambda \simeq 1/2(m^2\phi^2)$ , then by defining the parameter  $y \equiv (\partial S/\partial\alpha) / (\partial S/\partial\phi) \sim \dot{\alpha}/\dot{\phi}$ , one can write in the second order WKB approximation, [4, 5]:

$$\begin{aligned} \psi_n &= e^{\frac{\alpha}{2}} \exp\left(i \frac{3}{2y} \frac{\partial S}{\partial \phi} f_n^2\right) \psi_n^{(0)} \\ i \frac{\partial \psi_n^{(0)}}{\partial t} &= e^{-3\alpha} \left\{ -\frac{1}{2} \frac{\partial^2}{\partial f_n^2} + U(\alpha, \phi) f_n^2 \right\} \psi_n^{(0)} \\ U(\alpha, \phi) &= e^{6\alpha} \left\{ \left( \frac{n^2 - 1}{2} \right) e^{-2\alpha} + \frac{m^2}{2} + 9m^2 y^{-2} \phi^2 - 6m^2 y^{-1} \phi \right\}. \quad (2.13) \end{aligned}$$

During the nearly DeSitter stage,  $S \approx -1/3 m e^{3\alpha} \phi_{\text{inf}} \approx -1/3 e^{3\alpha} \sqrt{\Lambda}$ , where  $\phi_{\text{inf}}$  is the value of the field during inflation, so that  $y = 3\phi_{\text{inf}}$ . Thus long wavelength

matter fluctuations are amplified during a vacuum energy dominated universe, and driven away from their ground state [5]. This is not the case for a matter or radiation dominated universe. When the universe exits the DeSitter stage, the wavepacket for the universe is in an oscillatory regime,  $y$  is large so that the potential  $U(\alpha, \phi)$  changes from  $U_-(\alpha, \phi)$  to  $U_+(\alpha, \phi)$ , where

$$U_{\pm}(\alpha, \phi) \sim e^{6\alpha} \left[ \frac{n^2 - 1}{2} e^{-2\alpha} \pm \frac{m^2}{2} \right], \quad (2.14)$$

as can be seen from (2.13). It is straightforward to check from the equation of motion for the superHubble perturbation modes that in this case,  $U = U_+$ , as it was shown in [5], these modes are frozen in as in the conventional theory of perturbations. We can follow the evolution of these superHubble matter perturbation during the second stage of a vacuum energy domination phase, by using the Schrödinger equation above. During the nearly DeSitter state in the future, these modes develop gravitational instabilities because for reasons shown above, again  $U(\alpha, \phi) < 0$ . The matter perturbation modes  $f_n$  with  $n \leq aH_0$  are coupled to the background gravitational potential, therefore the main difference with the mechanism described in [14] for the instabilities during an early DeSitter stage, is that the scale of the curvature at late times, is of the order of the low energy DeSitter Hubble horizon,  $m^2 \simeq \sqrt{\Lambda}$ . From (2.13) we see that during a (nearly) DeSitter stage, the patches that have  $U(\alpha, \phi) < 0$ , which can happen for small enough physical wave vector  $k_n = ne^{-\alpha}$ , develop tachyonic instabilities due to the growth of perturbations:  $\psi_n \simeq e^{-\mu_n \alpha} e^{i\mu_n \phi}$ , where  $-\mu_n^2 = U(\alpha, \phi) f_n^2$ . Thus the solution for the wavefunction of the universe, in phase space given by  $\Psi(a, \phi, f_n) = \Psi_0(a, \phi) \Pi_n \psi_n(a, \phi, f_n)$  is damped in the intrinsic time  $\alpha$  rather than oscillatory. The damping of these wavefunctions is correlated with the tachyonic, Jeans-like instabilities that develop in real spacetime for the corresponding matter perturbation modes, consistent with Einstein equations for matter perturbation  $f_n$ ; when  $U(\alpha, \phi) < 0$ ,  $f_n \sim e^{\pm \mu_n t}$ , while for  $U(\alpha, \phi) > 0$ , the long wavelength matter perturbations  $f_n$  are frozen in.

The gravitational instabilities of matter fluctuation in real spacetime, can be seen from the equation of motion for  $\phi$ ,  $f_n$  obtained by varying the action with respect to these variables. For the tachyonic case  $U < 0$  universes, we have

$$\ddot{f}_n + 3H \dot{f}_n + \frac{U_-}{a^3} f_n = 0. \quad (2.15)$$

From (2.14) we can see that only during a DeSitter stage, when  $U < 0$  one obtains growing and decaying solution in spacetime roughly for  $f_n \simeq e^{\pm \mu t}$  and the growth of instabilities in the infinite number of superHubble matter modes  $f_n$  with  $0 < n < aH_0$  which can eventually overcome the pressure of the vacuum energy on spacetime and drive the wavefunction of the universe away from its classical trajectory.

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