

# Contents

<b>1</b>	<b>Smooth Manifolds</b>	1
	Topological Manifolds	2
	Smooth Structures	10
	Examples of Smooth Manifolds	17
	Manifolds with Boundary	24
	Problems	29
<b>2</b>	<b>Smooth Maps</b>	32
	Smooth Functions and Smooth Maps	32
	Partitions of Unity	40
	Problems	48
<b>3</b>	<b>Tangent Vectors</b>	50
	Tangent Vectors	51
	The Differential of a Smooth Map	55
	Computations in Coordinates	60
	The Tangent Bundle	65
	Velocity Vectors of Curves	68
	Alternative Definitions of the Tangent Space	71
	Categories and Functors	73
	Problems	75
<b>4</b>	<b>Submersions, Immersions, and Embeddings</b>	77
	Maps of Constant Rank	77
	Embeddings	85
	Submersions	88
	Smooth Covering Maps	91
	Problems	95
<b>5</b>	<b>Submanifolds</b>	98
	Embedded Submanifolds	98
	Immersed Submanifolds	108

	Restricting Maps to Submanifolds . . . . .	112
	The Tangent Space to a Submanifold . . . . .	115
	Submanifolds with Boundary . . . . .	120
	Problems . . . . .	123
<b>6</b>	<b>Sard's Theorem</b> . . . . .	125
	Sets of Measure Zero . . . . .	125
	Sard's Theorem . . . . .	129
	The Whitney Embedding Theorem . . . . .	131
	The Whitney Approximation Theorems . . . . .	136
	Transversality . . . . .	143
	Problems . . . . .	147
<b>7</b>	<b>Lie Groups</b> . . . . .	150
	Basic Definitions . . . . .	151
	Lie Group Homomorphisms . . . . .	153
	Lie Subgroups . . . . .	156
	Group Actions and Equivariant Maps . . . . .	161
	Problems . . . . .	171
<b>8</b>	<b>Vector Fields</b> . . . . .	174
	Vector Fields on Manifolds . . . . .	174
	Vector Fields and Smooth Maps . . . . .	181
	Lie Brackets . . . . .	185
	The Lie Algebra of a Lie Group . . . . .	189
	Problems . . . . .	199
<b>9</b>	<b>Integral Curves and Flows</b> . . . . .	205
	Integral Curves . . . . .	206
	Flows . . . . .	209
	Flowouts . . . . .	217
	Flows and Flowouts on Manifolds with Boundary . . . . .	222
	Lie Derivatives . . . . .	227
	Commuting Vector Fields . . . . .	231
	Time-Dependent Vector Fields . . . . .	236
	First-Order Partial Differential Equations . . . . .	239
	Problems . . . . .	245
<b>10</b>	<b>Vector Bundles</b> . . . . .	249
	Vector Bundles . . . . .	249
	Local and Global Sections of Vector Bundles . . . . .	255
	Bundle Homomorphisms . . . . .	261
	Subbundles . . . . .	264
	Fiber Bundles . . . . .	268
	Problems . . . . .	268

<b>11</b>	<b>The Cotangent Bundle</b>	272
	Covectors	272
	The Differential of a Function	280
	Pullbacks of Covector Fields	284
	Line Integrals	287
	Conservative Covector Fields	292
	Problems	299
<b>12</b>	<b>Tensors</b>	304
	Multilinear Algebra	305
	Symmetric and Alternating Tensors	313
	Tensors and Tensor Fields on Manifolds	316
	Problems	324
<b>13</b>	<b>Riemannian Metrics</b>	327
	Riemannian Manifolds	327
	The Riemannian Distance Function	337
	The Tangent–Cotangent Isomorphism	341
	Pseudo-Riemannian Metrics	343
	Problems	344
<b>14</b>	<b>Differential Forms</b>	349
	The Algebra of Alternating Tensors	350
	Differential Forms on Manifolds	359
	Exterior Derivatives	362
	Problems	373
<b>15</b>	<b>Orientations</b>	377
	Orientations of Vector Spaces	378
	Orientations of Manifolds	380
	The Riemannian Volume Form	388
	Orientations and Covering Maps	392
	Problems	397
<b>16</b>	<b>Integration on Manifolds</b>	400
	The Geometry of Volume Measurement	401
	Integration of Differential Forms	402
	Stokes’s Theorem	411
	Manifolds with Corners	415
	Integration on Riemannian Manifolds	421
	Densities	427
	Problems	434
<b>17</b>	<b>De Rham Cohomology</b>	440
	The de Rham Cohomology Groups	441
	Homotopy Invariance	443
	The Mayer–Vietoris Theorem	448
	Degree Theory	457

Proof of the Mayer–Vietoris Theorem . . . . .	460
Problems . . . . .	464
<b>18 The de Rham Theorem . . . . .</b>	<b>467</b>
Singular Homology . . . . .	467
Singular Cohomology . . . . .	472
Smooth Singular Homology . . . . .	473
The de Rham Theorem . . . . .	480
Problems . . . . .	487
<b>19 Distributions and Foliations . . . . .</b>	<b>490</b>
Distributions and Involutivity . . . . .	491
The Frobenius Theorem . . . . .	496
Foliations . . . . .	501
Lie Subalgebras and Lie Subgroups . . . . .	505
Overdetermined Systems of Partial Differential Equations . . . . .	507
Problems . . . . .	512
<b>20 The Exponential Map . . . . .</b>	<b>515</b>
One-Parameter Subgroups and the Exponential Map . . . . .	516
The Closed Subgroup Theorem . . . . .	522
Infinitesimal Generators of Group Actions . . . . .	525
The Lie Correspondence . . . . .	530
Normal Subgroups . . . . .	533
Problems . . . . .	536
<b>21 Quotient Manifolds . . . . .</b>	<b>540</b>
Quotients of Manifolds by Group Actions . . . . .	541
Covering Manifolds . . . . .	548
Homogeneous Spaces . . . . .	550
Applications to Lie Theory . . . . .	555
Problems . . . . .	560
<b>22 Symplectic Manifolds . . . . .</b>	<b>564</b>
Symplectic Tensors . . . . .	565
Symplectic Structures on Manifolds . . . . .	567
The Darboux Theorem . . . . .	571
Hamiltonian Vector Fields . . . . .	574
Contact Structures . . . . .	581
Nonlinear First-Order PDEs . . . . .	585
Problems . . . . .	590
<b>Appendix A Review of Topology . . . . .</b>	<b>596</b>
Topological Spaces . . . . .	596
Subspaces, Products, Disjoint Unions, and Quotients . . . . .	601
Connectedness and Compactness . . . . .	607
Homotopy and the Fundamental Group . . . . .	612
Covering Maps . . . . .	615

<b>Appendix B</b>	<b>Review of Linear Algebra</b>	617
Vector Spaces		617
Linear Maps		622
The Determinant		628
Inner Products and Norms		635
Direct Products and Direct Sums		638
<b>Appendix C</b>	<b>Review of Calculus</b>	642
Total and Partial Derivatives		642
Multiple Integrals		649
Sequences and Series of Functions		656
The Inverse and Implicit Function Theorems		657
<b>Appendix D</b>	<b>Review of Differential Equations</b>	663
Existence, Uniqueness, and Smoothness		663
Simple Solution Techniques		672
<b>References</b>		675
<b>Notation Index</b>		678
<b>Subject Index</b>		683



<http://www.springer.com/978-1-4419-9981-8>

Introduction to Smooth Manifolds

Lee, J.

2012, XVI, 708 p., Hardcover

ISBN: 978-1-4419-9981-8