Chapter 2
Distributed Parameter Systems: Controllability, Observability, and Identification

2.1 Mathematical Description

We introduce the class of systems to be considered in the framework of this monograph and definitions on configurations of sensors and actuators. Important concepts are defined for parameter identification and optimal experiment design.

This section introduces important concepts for the analysis of distributed parameter systems from the literature [8].

2.1.1 System Definition

Here, we consider a class of linear DPSs whose dynamics can be described by the given state equation

$$\begin{cases} \dot{y}(t) = Ay(t) + Bu(t), & 0 < t < T, \\ y(0) = y_0, \end{cases} \tag{2.1}$$

where the state space is given as $Y = L^2(\Omega)$, and the set Ω is a bounded open subset of $\mathbb{R}^n$ with a sufficiently regular boundary $\Gamma = \partial\Omega$. The considered domain Ω stands for the geometrical support of the system defined by (2.1). The operator A is a linear operator describing the dynamics of system (2.1). A generates a strongly continuous semigroup $(\Phi(t))_{t\geq 0}$ on Y. The operator $B \in \mathcal{L}(U, Y)$ (the set of linear maps from U to Y) is the input operator; $u \in L^2(]0, T[; U)$ (space of integrable functions $f :]0, T[\mapsto U$ such that $t \mapsto \|f(t)\|^p$ is integrable on $]0, T[$); U is a Hilbert control space. The considered system can be augmented by the output equation

$$z(t) = Cy(t), \tag{2.2}$$

where $C \in \mathcal{L}(L^2(\Omega), Z)$, and Z is a Hilbert observation space. While such a definition can be used for the analysis of distributed parameter systems, it is fairly abstract

C. Tricaud, Y.Q. Chen, *Optimal Mobile Sensing and Actuation Policies in Cyber-physical Systems*, DOI 10.1007/978-1-4471-2262-3_2, © Springer-Verlag London Limited 2012

when considering controls. That is why we introduce the notions of actuators and sensors, as well as notions of spatial distribution. These notions allow one to study the system not only with respect to the operators A, B, and C, but also with respect to the spatial distribution, location, and number of the actuators and sensors.

Sensors and actuators have two separate roles in a DPS. The actuators provide an excitation on the system, and the sensors give information (measurements) about the state of the system. Both sensors and actuators can be of different natures: zone or pointwise, internal or boundary, stationary or moving, communicating or noncommunicating, collocated or noncollocated.

An important notion of the framework of a DPS is the region. It is generally defined as a subdomain of Ω in which we are especially interested. Instead of considering a problem on the whole domain Ω, it is possible to consider only a subregion ω of Ω. This has allowed the generalization of the concepts, theorems, and results of the analysis of DPSs to any subdomain of Ω. In the following, we give the mathematical definitions for actuators and sensors.

2.1.2 Actuator Definition

Let Ω be an open regular bounded subset of $\mathbb{R}^n$ with a sufficiently regular boundary $\Gamma = \partial\Omega$. The set Ω stands for the geometrical support of a considered DPS [79].

Definition 2.1

1. An actuator is a couple (D, g) where D is the geometrical support of the actuator, $D = \text{supp}(g) \subset \Omega$, and g is its spatial distribution.
2. An actuator (D, g) is defined as:
   - A zonal actuator if D is a nonempty subregion of Ω.
   - A pointwise actuator if D is reduced to a point $b \in \Omega$. In that situation, we have $g = \delta_b$ where δ_b is the Dirac function concentrated at b. The actuator is then denoted as (b, δ_b).
3. An actuator (zonal or pointwise) is called a boundary actuator if its support $D \subset \Gamma$.

An illustration of actuator's support is given in Fig. 2.1. In the previous definition, g is assumed to be in $L^2(D)$. For p actuators $(D_i, g_i)_{1\le i\le p}$, the control space is $U = \mathbb{R}^p$, and

$$B : \mathbb{R}^p \to L^2(\Omega)$$

$$u(t) \to Bu(t) = \sum_{i=1}^{p} g_i u_i(t)$$

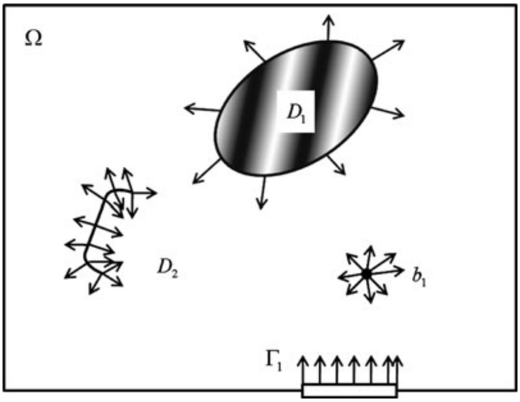


Fig. 2.1 Illustration of actuator's support

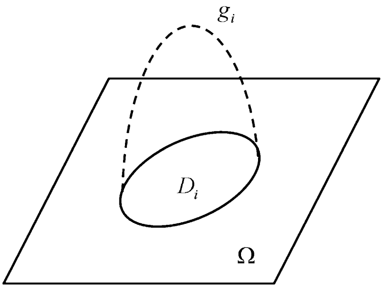


Fig. 2.2 Illustration of the geometrical support and spatial distribution of an actuator

where $u = (u_1, \dots, u_p)^T \in L^2(]0, T[; \mathbb{R}^p)$ and $g_i \in L^2(D_i)$ with $D_i = \mathrm{supp}\,(g_i) \subset \Omega$ for $i = 1, \dots, p$ and $D_i \cap D_j = \emptyset$ for $i \neq j$, and we have

$$B^\star y = \big(\langle g_1, y\rangle, \dots, \langle g_p, y\rangle\big)^T \quad \text{for } z \in L^2(\Omega),$$

where M^T is the transpose of M, and $\langle\cdot,\cdot\rangle = \langle\cdot,\cdot\rangle_Y$ is the inner product in Y, and for $v \in Y$, if $\mathrm{supp}\,(v) = D$, we have

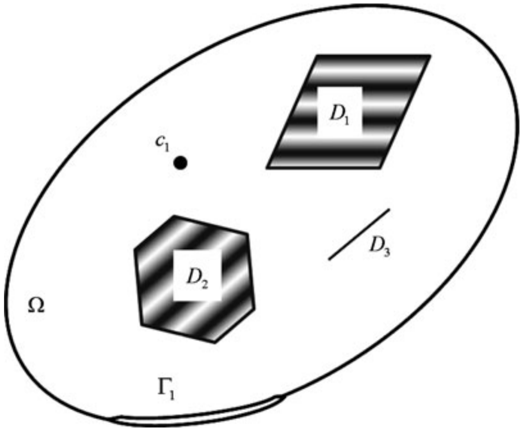
$$\langle v, \cdot\rangle = \langle v, \cdot\rangle_{L^2(D)}.$$

If D is independent of the time instant t, the actuator (D, g) is defined as fixed or stationary. If D varies with t, it is called a moving or mobile actuator denoted by (D_t, g_t), where D_t and g_t are, respectively, the geometrical support and the spatial distribution of the actuator at time t. An illustration of the geometrical support and spatial distribution of an actuator is given in Fig. 2.2.

2.1.3 Sensor Definition

Let us provide some concepts and definitions for sensors in DPSs [79].

Fig. 2.3 Illustration of sensor's support



Definition 2.2 A sensor is defined as a couple (D, h), where D is the spatial support of the sensor, $D = \text{supp}(h) \subset \Omega$, and h is its spatial distribution.

An illustration of sensor's support is given in Fig. 2.3. It is generally assumed that $h \in L^2(D)$. In a similar fashion, we can define zonal, pointwise, internal, boundary, fixed, or moving sensors. If the output of the system is given by means of q zonal sensors $(D_i, h_i)_{1\le i\le q}$ with $h_i \in L^2(D_i)$, $D_i = \text{supp}\,(h_i) \subset \Omega$ for $i = 1, \dots, q$ and $D_i \cap D_j = \emptyset$ if $i \neq j$, then in the case of a zonal output, the DPS's output operator C is defined by

$$\begin{aligned} C : L^2(\Omega) &\to \mathbb{R}^p \\ y \to Cy &= \big(\langle h_1, y\rangle, \dots, \langle h_q, y\rangle\big)^T \end{aligned}$$

and the output of the sensors is given by

$$z(t) = \begin{bmatrix} \langle h_1, y\rangle_{L^2(D_1)} \\ \langle h_2, y\rangle_{L^2(D_2)} \\ \vdots \\ \langle h_q, y\rangle_{L^2(D_q)} \end{bmatrix}. \tag{2.3}$$

A sensor (D, h) is said to be a zonal sensor if D is a nonempty subregion of Ω. A sensor (D, h) is called pointwise if D is reduced to a point $c \in \Omega$, and $h = \delta_c$ is the Dirac function concentrated at c. The sensor is then denoted as (c, δ_c). For zonal or pointwise sensors, if $D \subset \Gamma = \partial\Omega$, a sensor (D, h) is said to be a boundary sensor. If D does not depend on time, the sensor (D, h) is said to be fixed or stationary; otherwise, it is said to be moving (or scanning) and is denoted as (D_t, h_t). In the case of q pointwise fixed sensors located in $(c_i)_{1\le i\le q}$, the output function is a vector defined as

$$z(t) = \begin{bmatrix} y(t, c_1) \\ y(t, c_2) \\ \vdots \\ y(t, c_q) \end{bmatrix}, \tag{2.4}$$

where c_i is the position of the ith sensor, and $y(t, c_i)$ is the value of the state of the system in c_i at time t.

2.2 Regional Controllability

Let Ω be an open regular bounded subset of $\mathbb{R}^n$, and $Y = L^2(\Omega)$ be the state space. In what follows, we denote $Q = \Omega \times]0, T[$ and $\Sigma = \partial\Omega \times]0, T[$, and we consider the system described by the state equation

$$\begin{cases} \dot{y}(t) = Ay(t) + Bu(t), \quad 0 < t < T, \\ y(0) = y_0 \in D(A), \end{cases} \tag{2.5}$$

where $D(A)$ is the domain of the operator A. The operator A generates a strongly continuous semigroup $(\Phi(t))_{t\geq 0}$ on Z, $B \in \mathcal{L}(\mathbb{R}^p, Y)$, and $u \in L^2(0, T; \mathbb{R}^p)$. The mild solution y of (2.5), denoted $y(\cdot, u)$, is given by

$$y(t, u) = \Phi(t)y_0 + \int_0^t \Phi(t - s)Bu(s)\,\mathrm{d}s, \tag{2.6}$$

and we have $y(\cdot, u) \in C[0, T; Y]$.

We consider a given region $\omega \subset \Omega$ of positive Lebesgue measure and a given desired state $y_d \in L^2(\omega)$ [55].

Definition 2.3

1. System (2.5) is said to be exactly regionally controllable (or exactly ω-controllable) if there exists a control $u \in L^2(]0, T[; \mathbb{R}^p)$ such that

$$p_\omega y(T, u) = y_d. \tag{2.7}$$

2. System (2.5) is said to be weakly regionally controllable (or weakly ω-controllable) if, given $\epsilon > 0$, there exists a control $u \in L^2(]0, T[; \mathbb{R}^p)$ such that

$$\left\| p_\omega y(T, u) - y_d \right\|_{L^2_\omega} \leq \epsilon, \tag{2.8}$$

where $y(\cdot, u)$ is given by (2.6), and $p_\omega y$ is the restriction of y to ω.

In the case of pointwise or boundary controls, $B \notin \mathcal{L}(\mathbb{R}^p, Z)$. We consider the operator

$$H : L^2\big(]0, T[; \mathbb{R}^p\big) \to Y$$

defined by

$$Hu = \int_0^T \Phi(T-\tau)Bu(\tau)\,\mathrm{d}\tau \tag{2.9}$$

and

$$p_\omega : L^2(\Omega) \to L^2(\omega) \tag{2.10}$$

defined by

$$p_\omega y = y|_\omega. \tag{2.11}$$

Then, from Definition 2.3, system (2.5) is exactly (respectively weakly) regionally controllable if

$$\mathrm{Im}(p_\omega H) = L^2(\omega)\big(\text{respectively } \overline{\mathrm{Im}\, p_\omega H} = L^2(\omega)\big). \tag{2.12}$$

We have equivalently

$$\overline{\mathrm{Im}(p_\omega H)} = L^2(\omega) \Leftrightarrow \mathrm{Ker}\big(H^\star i_\omega\big) = \{0\}, \tag{2.13}$$

where i_ω holds for the adjoint of p_ω. Characterizations (2.12) and (2.13) are often used in applications. We also have the following result [56].

Lemma 2.4 1. *System* (2.5) *is exactly regionally controllable if and only if*

$$\mathrm{Ker}(p_\omega) + \mathrm{Im}(H) = L^2(\Omega). \tag{2.14}$$

2. *System* (2.5) *is weakly regionally controllable if and only if*

$$\ker(p_\omega) + \overline{\mathrm{Im}(H)} = L^2(\Omega). \tag{2.15}$$

It is easy to show that (2.15) is equivalent to

$$\mathrm{Ker}(H^\star) \cap \mathrm{Im}(i_\omega) = \{0\}, \tag{2.16}$$

where $i_\omega = p_\omega^\star : L^2(\omega) \to L^2(\Omega)$ is given by

$$i_\omega z = \begin{cases} y(x), & x \in \omega, \\ 0, & x \in \Omega \backslash \omega. \end{cases}$$

2.3 Regional Observability

Let z be the state of a linear system with state space $Y = L^2(\Omega)$, and suppose that the initial state y_0 is unknown. Measurements are given by means of an output z depending on the number and the structure of the sensors. The problem to be studied here concerns the reconstruction of the initial state y_0 on the subregion ω. Let Ω be a regular bounded open set of $\mathbb{R}^n$ with boundary $\varGamma = \partial\varOmega$, ω be a nonempty subset of $\varOmega$, and $[0, T]$ with $T > 0$ be a time interval. We denote $Q = \varOmega \times]0, T[$ and $\sigma = \partial\varOmega \times]0, T[$, and we consider the autonomous system described by the state equation

$$\begin{cases} \dot{y}(t) = Ay(t), & 0 < t < T, \\ y(0) = y_0 & \text{supposed to be unknown}, \end{cases} \tag{2.17}$$

where A generates a strongly continuous semigroup $(\varPhi(t))_{t\geq 0}$ on the state space Y. An output function gives measurements of the state y by

$$z(t) = Cy(t), \tag{2.18}$$

where

$$C : y \in L^2(]0, T[; Y) \to z \in L^2\big(]0, T[; \mathbb{R}^q\big) \tag{2.19}$$

depends on the sensors' structure. In the case where the considered sensor is pointwise and located in $b \in \varOmega$, we have, with (2.18),

$$z(t) = \int_{\varOmega} y(x, t)\delta(x - b)\, \mathrm{d}x = y(b, t). \tag{2.20}$$

The problem consists in the reconstruction of the initial state, assumed to be unknown, in the subregion ω. We consider the following decomposition:

$$y_0 = \begin{cases} y^e, & x \in \omega, \\ y^u, & x \in \varOmega\backslash\omega, \end{cases} \tag{2.21}$$

where y^e is the state to be estimated, and y^u is the undesired part of the state. Then, the problem consists in reconstructing y^e with the knowledge of (2.17) and (2.18). As system (2.17) is autonomous, (2.18) gives

$$z(t) = C\varPhi(t)y_0 = K(t)y_0, \tag{2.22}$$

where K is an operator $Y \to L^2(]0, T[; \mathbb{R}^q)$. The adjoint $K^\star$ is given by

$$K^\star y = \int_0^T \varPhi^\star(s)C^\star z(s)\, \mathrm{d}s. \tag{2.23}$$

We recall that system (2.17) with the output (2.18) is said to be weakly observable if $\mathrm{Ker}(K) = \{0\}$. The associated sensor is then said to be strategic [56]. Consider now the restriction mapping

$$\chi_\omega : L^2(\varOmega) \to L^2(\omega) \tag{2.24}$$

defined by

$$\chi_\omega z = z|_\omega, \tag{2.25}$$

where $z|_\omega$ is the restriction of z to ω. For simplification, along this section we denote $\gamma = \chi_\omega$. Then, we introduce the following definition [80]:

Definition 2.5 System (2.17)–(2.18) is said to be regionally observable on ω (or ω-observable) if

$$\mathrm{Im}\big(\gamma K^\star\big) = L^2(\omega). \tag{2.26}$$

System (2.17)–(2.18) is said to be weakly regionally observable on ω (or weakly ω-observable) if

$$\overline{\mathrm{Im}\big(\gamma K^\star\big)} = L^2(\omega). \tag{2.27}$$

From the above definition we deduce the following characterization [56]:

Lemma 2.6 *System* (2.17)–(2.18) *is exactly ω-observable if there exists $\omega > 0$ such that, for all $z_0 \in L^2(\omega)$,*

$$\|\gamma y_0\|_{L^2(\omega)} \le \nu \big\| K\gamma^\star y_0 \big\|_{L^2(]0,T[;\mathbb{R}^q)}. \tag{2.28}$$

2.4 Parameter Identification and Optimal Experiment Design

2.4.1 System Definition

Due to the nature of the considered parameter identification problem, the abstract operator-theoretic formalism used in (2.1) to define the dynamics of a DPS is not convenient. In this section, the following PDE-based general definitions are given. Consider a DPS described by n partial differential equations of the following form:

$$\begin{aligned} &\mathcal{F}_1(x,t)\frac{\partial y(x,t)}{\partial t} = \mathcal{F}_2\big(x,t,y(x,t),\nabla y(x,t),\nabla^2 y(x,t),\theta\big),\\ &\quad (x,t) \in \Omega \times T \subset \mathbb{R}^{d+1} \end{aligned} \tag{2.29}$$

with initial and boundary conditions

$$\mathcal{B}(x,t,y) = 0, \quad (x,t) \in \partial\Omega \times T, \tag{2.30}$$

$$\mathcal{N}(x,t,y) = 0, \quad (x,t) \in \Omega \times \{0\}, \tag{2.31}$$

where

- $\Omega \subset \mathbb{R}^n$ is a bounded spatial domain with sufficiently smooth boundary $\Gamma = \partial\Omega$,
- t is the time instant,
- $T = [0, t_f]$ is a bounded time interval called observation interval,
- $x = (x_1, x_2, \ldots, x_d)$ is a spatial point belonging to $\overline{\Omega} = \Omega \cup \Gamma$,
- $y = (y_1(x,t), y_2(x,t), \ldots, y_n(x,t))$ stands for the state vector, and
- $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{B}$, and $\mathcal{N}$ are some known functions.

We assume that the system of equations (2.29)–(2.31) has a unique solution that is sufficiently regular. We can see that (2.29)–(2.31) contains an unknown set of parameters θ whose values belong to an admissible parameter space Θ_{ad}. Even though Θ_{ad} can have different forms, we make an assumption that the parameters are constant ($\theta \in \mathbb{R}^m$). The set of unknown parameters θ has to be determined based on observations made by N mobile pointwise sensors over the observation horizon T. We define $x_j : T \to \Omega_{\mathrm{ad}}$ as the trajectory of the jth mobile sensor, with $\Omega_{\mathrm{ad}} \subset \Omega$ being the region where measurements can be made. The observations are assumed to be of the form

$$z^j(t) = y\big(x^j(t), t\big) + \varepsilon\big(x^j(t), t\big), \quad t \in T,\ j = 1, \ldots, N. \tag{2.32}$$

The collection of measurements $z(t) = [z^1(t), z^2(t), \ldots, z^N(t)]^T$ is the N-dimensional observation vector, and ε represents the measurement noise assumed to be white, zero-mean, Gaussian, and spatial uncorrelated with the following statistics:

$$\mathrm{e}\big\{\varepsilon\big(x^j(t), t\big)\varepsilon\big(x^i(t'), t'\big)\big\} = \sigma^2 \delta_{ji} \delta(t - \tau), \tag{2.33}$$

where σ^2 stands for the standard deviation of the measurement noise, and δ_{ij} and $\delta(\cdot)$ are the Kronecker and Dirac delta functions, respectively.

2.4.2 Parameter Identification

According to this setup, the parameter identification problem is defined as follows. Given the model (2.29)–(2.31) and the measurements $z(t)$ along the trajectories (x^j), $j = 1, \ldots, N$, obtain an estimation $\hat{\theta} \in \Theta_{\mathrm{ad}}$ minimizing the following weighted least-squares criterion as in [18] and [109]:

$$\mathcal{J}(\theta) = \frac{1}{2} \int_0^T \big\| z(t) - \hat{y}(\mathbf{x}, t; \theta) \big\|^2 \, \mathrm{d}t, \tag{2.34}$$

where $\hat{y}(x, t; \theta)$ stands for the solution to (2.29)–(2.31) corresponding to a given set of parameters θ, and $\|\cdot\|$ stands for the Euclidean norm.

The estimated values of the parameters $\hat{\theta}$ are influenced by the sensors' trajectories $x^j(t)$, and our objective is to obtain the best estimates of the system parameters. Therefore, deciding on the trajectory based on a quantitative measure related to the expected accuracy of the parameter estimates to be obtained from the data collected seems to be practically logical.

2.4.3 Sensor Location Problem

The Fisher information matrix (FIM) [119, 132] is a well-known performance measure when looking for best measurements and is widely used in optimum experimental design theory for lumped systems. Its inverse constitutes an approximation of the covariance matrix for the estimate of θ [16, 60, 162]. Let us give the following definition of the experiment:

$$s(t) = \left(x^1(t), \ldots, x^N(t)\right) \quad \forall t \in T, \tag{2.35}$$

and let $n = \dim(s(t))$. Under such conditions, the FIM can be written as [118]

$$M(s) = \sum_{j=1}^{N} \int_0^T g\left(x^j(t), t\right) g^T\left(x^j(t), t\right) \mathrm{d}t, \tag{2.36}$$

where $g(\mathbf{x}, t) = \nabla_\theta y(x, t; \theta)|_{\theta=\theta^0}$ is the vector made of the sensitivity coefficients, θ^0 being the previous estimate of the unknown parameter vector θ [146, 147].

By choosing s such that it minimizes a scalar function $\Psi(\cdot)$ of the FIM, one can determine the optimal mobile sensor trajectories. There are many candidates for such a function [16, 60, 162]:

- The A-optimality criterion suppresses the variance of the estimates

$$\Psi(M) = \operatorname{trace}\left(M^{-1}\right). \tag{2.37}$$

- The D-optimality criterion minimizes the volume of the confidence ellipsoid for the parameters

$$\Psi(M) = -\log\det(M). \tag{2.38}$$

- The E-optimality criterion minimizes the largest width of the confidence ellipsoid

$$\Psi(M) = \lambda_{\max}\left(M^{-1}\right). \tag{2.39}$$

- The sensitivity criterion' minimization increases the sensitivity of the outputs with respect to parameter changes

$$\Psi(M) = -\operatorname{trace}(M). \tag{2.40}$$

2.4.4 Sensor Clustering Phenomenon

The assumption on the spatial uncorrelation of the measurement noise can create a clustering of the sensors, which can be problematic in practice. We use an example from [147] to illustrate the sensor clustering problem.

Example 2.1 Consider the following parabolic partial differential equation:

$$\frac{\partial y(x,t)}{\partial t} = \theta_1 \frac{\partial^2 y(x,t)}{\partial x^2}, \quad x \in (0, \pi),\ t \in (0, 1),$$

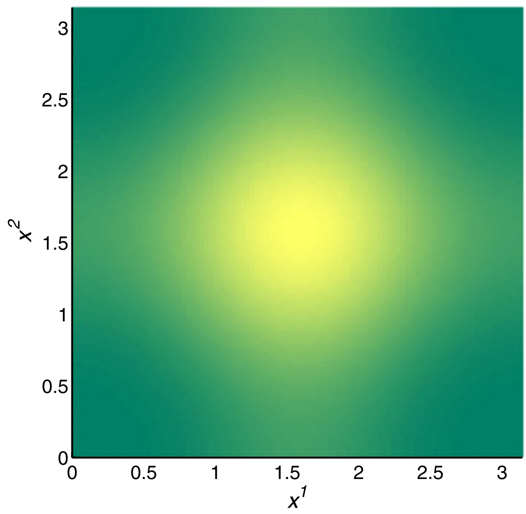


Fig. 2.4 Contour plot of $\det(M(x^1, x^2))$ versus the sensors' locations ($\theta_1 = 0.1$ and $\theta_2 = 1$)

with boundary and initial conditions

$$y(0,t) = y(\pi,t) = 0, \quad t \in (0,1),$$
$$y(x,0) = \theta_2 \sin(x), \quad x \in (0,\pi).$$

The two parameters θ_1 and θ_2 are assumed to be constant but unknown. In addition, we assume that the measurements are taken by two static sensors whose locations are decided by maximizing the determinant of the FIM. The analytical solution of the PDE can be easily obtained as

$$y(x,t) = \theta_2 \exp(-\theta_1 t)\sin(x).$$

The assumption is made that the signal noise statistic $\sigma = 1$ does not change the optimal location of the sensors. The determinant of the matrix is given by

$$\det\bigl(M\bigl(x^1, x^2\bigr)\bigr) = \frac{\theta_2^2}{16\theta_1^4}\bigl(-4\theta_1^2\exp(-2\theta_1) - 2\exp(-2\theta_1) + \exp(-4\theta_1) + 1\bigr)$$
$$\times \bigl(2 - \cos^2(x_1) - \cos^2(x_2)\bigr)^2.$$

The results are shown in Fig. 2.4, and one quick observation allows one to determine that the best location for both sensors is at the center of the interval $(0, \pi)$.

2.4.5 Dependence of the Solution on Initial Parameter Estimates

Another serious issue in the FIM framework of optimal measurements for parameter estimation of DPS is the dependence of the solution on the initial guess on parameters. We illustrate the problem using an example from [111].

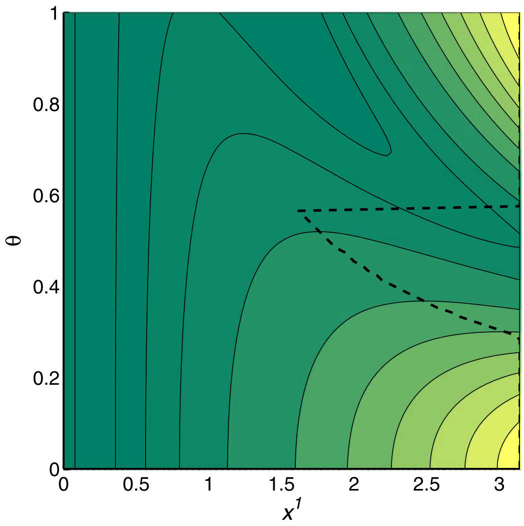


Fig. 2.5 Contour plot of $M(x^1;\theta)$

Example 2.2 Consider the following hyperbolic partial differential equation:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \theta \frac{\partial^2 y(x,t)}{\partial x^2}, \quad x \in (0,\pi),\ t \in (0,\pi),$$

with boundary and initial conditions

$$y(0,t) = \frac{1}{4}\cos(t), \qquad y(\pi,t) = \sin(\pi\theta)\sin(t) + \frac{1}{4}\cos(\pi\theta)\cos(t), \quad t \in (0,\pi),$$

$$y(x,0) = \frac{1}{4}\cos\theta x, \qquad \left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = \sin(\theta x), \quad x \in (0,\pi).$$

The parameter θ is assumed to be constant and unknown. In addition, we assume that the measurements are taken by one static sensor located at $x^1 \in (0,\pi)$. The analytical solution of the PDE can be easily obtained and is given as

$$\begin{aligned} M(x^1) &= \int_0^\pi \left(\frac{\partial y(x^1,t;\theta)}{\partial \theta}\right)^2 \mathrm{d}t \\ &= \frac{1}{2}x^2\pi\cos^2(\theta x) + \frac{1}{32}x^2\pi\sin(\theta x). \end{aligned}$$

The results are shown in Fig. 2.5 (the optimal location of the sensor is represented by a dashed line), and it is easy to observe that the optimal sensor location depends on the value of θ.

The dependence of the optimal location on θ is very problematic; however, some techniques called "robust designs" have been developed to minimize or elude the influence [132, 162]. We propose similar methodologies in Chap. 5.

2.5 Chapter Summary

In this chapter, we gave very important definitions in the framework of DPSs. We defined the dynamic equations of the system, the mathematical descriptions of a sensor and an actuator. From those definitions we introduced the concepts of regional controllability and observability. Then, we described the dynamics of the system in an appropriate way for the FIM framework of optimal sensor location for parameter estimation. We gave the definitions of the parameter estimation and optimal sensor location. Finally, we discussed two of the important issues of the FIM framework: the sensor clustering phenomenon and the dependence of the solution on initial parameter estimates.

Optimal Mobile Sensing and Actuation Policies in
Cyber-physical Systems
Tricaud, C.; Chen, Y.
2012, XVIII, 170 p., Hardcover
ISBN: 978-1-4471-2261-6