

Chapter 2

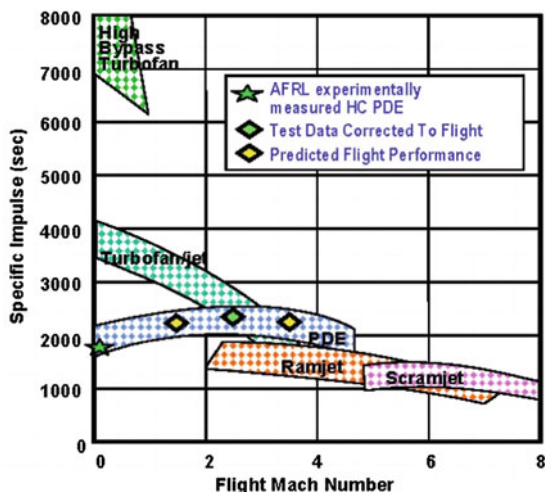
Introduction to Atmospheric Flight

2.1 Introduction to Propulsion for Airplanes

In evaluating the design and performance of airplanes for various atmospheric flight applications, we see the need for generating thrust F in order to counter the atmospheric aerodynamic drag D that is acting to slow a flight vehicle at a given airspeed V , and for a fixed-wing aircraft, the need for generating aerodynamic lift L to counter the weight W of the vehicle, in order to maintain a given altitude h . We note the correlation on the performance parameter F/W (thrust-to-weight, ~ 0.2 to 0.4 for most conventional fixed-wing aircraft, where thrust F in this case is typically the maximum sea level static value, F_o , and weight is the nominal maximum takeoff weight at sea level of the airplane, W_o) in sizing airplanes. Thus, the bigger the airplane, the more thrust required from the propulsion system, and thus more fuel consumed. One will encounter takeoff requirements that further establish the influence of the above important performance ratio, in this case on meeting the required runway length. Further in regard to thrust, we note the use of various propulsion systems to deliver thrust for a conventional fixed-wing aircraft, namely the propeller [driven by a piston, rotary or turboprop (TP) engine], turbojet (TJ) and turbofan (TF). In the case of conventional rotary-winged aircraft (helicopters), thrust and lift are primarily delivered by one or more large main rotors [driven by a turboshaft (TS) engine in most higher power applications].

Brake or thrust specific fuel consumption, or specific impulse (essentially the inverse of thrust specific fuel consumption), are common measures of aircraft engine performance and flight economy. In comparing one propulsion system to another, one often sees the use of charts such as the example of Fig. 2.1. In that example, high-bypass-ratio turbofans produce the best (highest) specific impulse, but these systems only function economically up to the high subsonic flight region for the most part ($Ma_\infty < 0.9$), up to about 13 km in altitude at best. On the other end of the spectrum, if one needs to fly quickly, as fast as Ma_∞ approaching 8 (at that speed, one would likely need to be cruising at a fairly high altitude, on the order of 31 km or higher) then a scramjet would be the only available option,

Fig. 2 1 Performance comparison (in terms of specific impulse, I_{sp}) of various air-breathing engines that might be used for airplanes, at differing flight Mach numbers. In practice, altitude will also play a role. AFRL refers to the Air Force Research Laboratory (US), and HC PDE refers to pulse detonation engine employing a conventional hydrocarbon fuel. Graph courtesy of DARPA



according to the example chart, unfortunately at the lowest peak specific impulse of the various air-breathing engines.

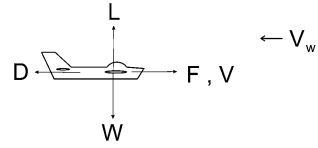
In order to provide some background to the larger picture that surrounds airplanes and the propulsion systems that are integrated to them to allow for a successful mission, an example set of mathematical equations describing airplane cruise performance reveals the close connection that exists between engine and airframe.

2.2 Example Mission Requirement: Range and Endurance of Fixed-Wing Airplanes

The actual overall range of a fixed-wing aircraft will include mission components such as takeoff, climb to altitude, cruise at altitude, descend, loiter, further descent and land. This accumulated distance, if including a substantial distance component for maneuvering and loitering, may be significantly greater than the nominal trip distance from points A to B. Cruising at altitude is a major component of a typical conventional flight mission profile, and performance assessment for this state is a good indicator of a given aircraft's range and endurance capability. We will focus on cruise performance for this example airplane mission requirement.

We note that turbojet and turbofan engine performance is largely ascertained as a function of thrust F , while propellered engine performance is determined as a function of power (shaft power delivered by engine, P_S ; and occasionally, one might use thrust power, FV). Given this correlation, let us separate the aircraft's range and endurance performance assessment into two categories: jet aircraft and propellered aircraft.

Fig. 2.2 Free-body force diagram for airplane in steady level flight. The diagram assumes that the airplane's angle of attack α is relatively small, for mathematical convenience



2.2.1 Jet Aircraft

One can define a pertinent performance parameter, thrust specific fuel consumption (TSFC), as follows:

$$\begin{aligned}\text{TSFC} &= \text{kg/h of fuel/N of thrust delivered, kg/(h N)} \\ &= \text{lb/h of fuel/lb of thrust delivered, lb/(h lb)}\end{aligned}$$

The corresponding fuel consumption rate in terms of weight for a jet engine would thus be:

$$\begin{aligned}\dot{w}_f = \frac{dw_f}{dt} &= -\frac{dW}{dt} = g \cdot \text{TSFC} \cdot F, \quad \text{N/h, where } F \text{ in newtons} \\ &= \text{TSFC} \cdot F, \quad \text{lb/h, where } F \text{ in pounds}\end{aligned}\tag{2.1}$$

The aircraft's overall weight, W , would drop as fuel is consumed, so that for a small time increment, for the S.I. unit case:

$$dt = -\frac{dW}{g \cdot \text{TSFC} \cdot F}\tag{2.2}$$

noting that the weight increment above is negative in value, in order for the time increment to be positive. One can integrate, to get an estimate of the time elapsed over a cruise segment, or in other words, the endurance of the aircraft for a given amount of fuel burned (as reflected below by the difference between the aircraft's weight at the start of the given cruise segment, W_{ini} , and at the end of the flight segment, W_{fin}):

$$t_e = \int dt = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{1}{\text{TSFC} \cdot F} \cdot dW\tag{2.3}$$

For the corresponding range of the cruise segment,

$$R = \int V dt = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{V}{\text{TSFC} \cdot F} \cdot dW\tag{2.4}$$

One recalls that for steady level flight (SLF), as per Fig. 2.2, one can correlate to the airplane's overall lift and drag coefficients, via:

$$F = D = W \frac{C_D}{C_L} \quad (2.5)$$

where $L = C_L \frac{1}{2} \rho V^2 S$ and $D = C_D \frac{1}{2} \rho V^2 S$, S being the aircraft's reference wing area, ρ the local air density and V the aircraft's true airspeed.

Substituting into the integral,

$$R = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{V \cdot \frac{C_L}{C_D}}{\text{TSFC} \cdot W} \cdot dW \quad (2.6)$$

If one assumes constant or mean values for V , C_L/C_D , and TSFC for the given cruise flight segment, integration produces the well-known Breguet range equation [1], in this case for jet aircraft and S.I. units:

$$R = \frac{1}{g} \frac{V}{\text{TSFC}} \frac{C_L}{C_D} \cdot \ln\left(\frac{W_{\text{ini}}}{W_{\text{fin}}}\right), \quad \text{km, where } g = 9.81 \text{ m/s}^2, \quad (2.7)$$

V in km/h, TSFC in kg/(hN)

In British (Imperial) units, the Breguet range equation for jets may appear as follows:

$$R = \frac{V}{\text{TSFC}} \frac{C_L}{C_D} \cdot \ln\left(\frac{W_{\text{ini}}}{W_{\text{fin}}}\right), \quad \text{nm (nautical miles),} \quad (2.8)$$

where $1 \text{ nm} = 1.1508 \text{ statute miles} = 6076 \text{ ft},$
 V in kt (i.e., nm/h), and TSFC in lb/h · lb

The best estimate of mean weight for a given cruise segment (possibly needed for calculating a mean C_L/C_D or V) may be found from inspection of the Breguet equation:

$$\overline{\ln(W)} = \frac{\ln(W_{\text{ini}}) + \ln(W_{\text{fin}})}{2} = \ln \sqrt{W_{\text{ini}}} + \ln \sqrt{W_{\text{fin}}} = \ln \sqrt{W_{\text{ini}} \cdot W_{\text{fin}}}$$

which in essence says:

$$\overline{W} = \sqrt{W_{\text{ini}} \cdot W_{\text{fin}}} \quad (2.9)$$

One can in turn establish that the corresponding endurance for a jet aircraft, for the S.I. unit case described for range above, may be found via:

$$t_e = \frac{R}{V} = \frac{1}{g} \frac{C_L/C_D}{\text{TSFC}} \cdot \ln\left(\frac{W_{\text{ini}}}{W_{\text{fin}}}\right), \quad \text{h} \quad (2.10)$$

Here, V would be the mean or constant airspeed for the cruise flight segment, in km/h. Inspection of the above endurance equation for jets suggests that one should maximize C_L/C_D for maximum endurance (time airborne). This requirement conforms with flying for minimum drag:

$$C_{L,me} = C_{L,md} = \sqrt{\frac{C_{Do}}{K}}, \quad C_{D,me} = C_{D,md} = 2C_{Do}, \quad V_{md} = \sqrt{\frac{2\bar{W}}{\rho S C_{L,md}}} \quad (2.11)$$

where one has the standard subsonic flight (subtransonic) drag polar expression, summing zero-lift drag and lift-induced drag components, as follows:

$$C_D = C_{Do} + KC_L^2 \quad (2.12)$$

Inspection of the range equation above suggests that for jets, one would want to maximize $V \cdot C_L/C_D$, or correspondingly, maximize the following:

$$V \cdot \frac{C_L}{C_D} = \sqrt{\frac{2\bar{W}}{\rho S C_L}} \cdot \frac{C_L}{C_D} = \sqrt{\frac{2\bar{W}}{S}} \cdot \frac{1}{\rho^{1/2}} \cdot \frac{C_L^{1/2}}{C_D} \quad (2.13)$$

While one cannot do much as a pilot about the aircraft weight W or reference wing area S , according to the above, a lower air density ρ (higher altitude h) would be helpful to increase range, in addition to maximizing $C_L^{1/2}/C_D$. Let us now consider the latter term, but for convenience, look at minimizing the inverse of that term:

$$\frac{C_D}{C_L^{1/2}} = \frac{C_{Do} + KC_L^2}{C_L^{1/2}} = C_{Do} C_L^{-1/2} + KC_L^{3/2} \quad (2.14)$$

Differentiate to find the trough point:

$$\frac{d}{dC_L} [C_{Do} C_L^{-1/2} + KC_L^{3/2}] = 0 = -\frac{1}{2} C_{Do} C_L^{-3/2} + \frac{3}{2} KC_L^{1/2} \quad (2.15)$$

Rearranging, one arrives at the following for maximum jet air range:

$$C_{L,mr} = \sqrt{\frac{C_{Do}}{3K}}, \quad C_{D,mr} = C_{Do} + \frac{KC_{Do}}{3K} = \frac{4}{3} C_{Do} \quad (2.16)$$

The corresponding airspeed for best air range by a jet aircraft would in turn be:

$$V_{mr} = \sqrt{\frac{2\bar{W}}{\rho S C_{L,mr}}} = \frac{1}{\rho^{1/2}} \cdot \sqrt{\frac{2\bar{W}}{S}} \cdot \left(\frac{3K}{C_{Do}}\right)^{1/4} = 1.32 V_{md} \quad (2.17)$$

The above estimate holds until the airplane begins to appreciably enter *drag divergence*, where C_{Do} begins to rise with further increases in flight Mach number Ma_∞ . Of course, if such is the case, one is violating the original premise that

produced the above result (i.e., the assumption of a constant C_{Do}), and thus are in a transonic domain where the above guideline is invalid. One can also note at this juncture that TSFC for turbofan and turbojet engines also tends to worsen as the airplane moves above a certain threshold altitude. This could be in the neighborhood of 40,000–50,000 ft (12–15 km) for an airplane employing a conventional commercial jet engine.

A tailwind (V_w negative in value) is beneficial for gaining extra distance, while a headwind is detrimental, as per the correlation between ground range R_g as a function of air or still-air range R :

$$R_g = R - V_w t_e \quad (2.18)$$

where groundspeed V_g correlates to airspeed V via:

$$V_g = V - V_w \quad (2.19)$$

The endurance t_e will remain the same value, regardless of the magnitude of the wind. From pilot observation, it has been ascertained that one should fly at a bit faster airspeed than the still-air criterion when in a headwind to attain the maximum possible ground range, and the reverse (fly a bit slower, airspeed wise) when a tailwind is present. One can demonstrate this mathematically. For maximum ground range, observation of the Breguet range equation suggests that one needs to maximize the following:

$$(V - V_w) \frac{C_L}{C_D} = \sqrt{\frac{2W}{\rho S}} \cdot \frac{C_L^{1/2}}{C_D} - V_w \cdot \frac{C_L}{C_D} \quad (2.20)$$

Differentiation of the above allows one to find a peak point:

$$\frac{d}{dC_L} \left[\sqrt{\frac{2W}{\rho S}} \cdot \frac{C_L^{1/2}}{C_D} - V_w \cdot \frac{C_L}{C_D} \right] = \frac{d}{dC_L} \left[\frac{1}{C_{Do} + KC_L^2} \left\{ \sqrt{\frac{2W}{\rho S}} \cdot C_L^{1/2} - V_w C_L \right\} \right] = 0 \quad (2.21)$$

which ultimately reduces to the following for best C_L to fly at in a wind:

$$0 = C_{Do} - 3KC_L^2 - 2\sqrt{\frac{\rho S}{2W}} \cdot V_w C_{Do} C_L^{1/2} + 2\sqrt{\frac{\rho S}{2W}} \cdot V_w KC_L^{5/2} \quad (2.22)$$

2.2.2 Propellered Aircraft

In the case of propellered airplanes, fuel consumption by the engine driving the propeller is proportional to power, rather than thrust. As a result, one defines brake specific fuel consumption (BSFC) as a suitable performance parameter, as follows:

$$\begin{aligned}\text{BSFC} &= \text{kg/h of fuel/W of power delivered, kg/(h W)} \\ &= \text{lb/h of fuel/bhp of power delivered, lb/(h hp)}\end{aligned}$$

where bhp refers to brake horsepower (1 hp = 550 ft lb/s) as measured from the drive shaft of the engine. The corresponding fuel consumption rate in terms of weight for a shaft power (P_S) producing engine would thus be:

$$\begin{aligned}\dot{w}_f = \frac{dw_f}{dt} &= -\frac{dW}{dt} = g \cdot \text{BSFC} \cdot P_S, & \text{N/h, where } P_S \text{ in watts} \\ &= \text{BSFC} \cdot P_S, & \text{lb/h, where } P_S \text{ in horsepower}\end{aligned} \quad (2.23)$$

The aircraft's overall weight, W , would drop as fuel is consumed, so that for a small time increment, for the S.I. unit case:

$$dt = -\frac{dW}{g \cdot \text{BSFC} \cdot P_S} \quad (2.24)$$

One can integrate to arrive at the propellered aircraft's endurance for a given flight segment:

$$t_e = \int dt = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{1}{\text{BSFC} \cdot P_S} \cdot dW \quad (2.25)$$

Similarly, for air range, one has:

$$R = \int V dt = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{V}{\text{BSFC} \cdot P_S} \cdot dW \quad (2.26)$$

One notes the following correlations for shaft power, for the S.I. unit case, for an aircraft in level flight:

$$P_S = \frac{FV}{3.6\eta_{\text{pr}}} = \frac{DV}{3.6\eta_{\text{pr}}} = \frac{WC_D}{C_L} \cdot \frac{V}{3.6\eta_{\text{pr}}}, \quad \text{watts, where } V \text{ in km/h} \quad (2.27)$$

The symbol η_{pr} is the propeller propulsive efficiency, and the 3.6 factor arises from there being 3,600 s in one hour. Substituting,

$$R = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{V}{\text{BSFC} \cdot \frac{WC_D}{C_L} \cdot \frac{V}{3.6\eta_{\text{pr}}}} \cdot dW = \frac{1}{g} \int_{W_{\text{fin}}}^{W_{\text{ini}}} \frac{3.6\eta_{\text{pr}}C_L/C_D}{\text{BSFC} \cdot W} \cdot dW \quad (2.28)$$

Assuming constant or mean values for η_{pr} , BSFC and C_L/C_D over a given cruise flight segment, integration produces the Breguet range equation for propellered aircraft, with S.I. units:

$$R = \frac{1}{g} \cdot \frac{3.6\eta_{pr}C_L/C_D}{BSFC} \cdot \ln\left(\frac{W_{ini}}{W_{fin}}\right), \text{ km, where } g = 9.81 \text{ m/s}^2, \quad (2.29)$$

BSFC in kg/h · W

In British (Imperial) units, the Breguet range equation for propellered aircraft may appear as follows:

$$R = \frac{326\eta_{pr}C_L/C_D}{BSFC} \cdot \ln\left(\frac{W_{ini}}{W_{fin}}\right), \text{ nm, where BSFC in lb/h · hp} \quad (2.30)$$

The 326 factor arises from the combination of conversion factors for horsepower, distance and time as they appear in the above equation, to produce distance in nautical miles (nm). Note that the above is for air range (we will account for wind later).

Similarly, for endurance of a propellered aircraft over a given flight segment, S.I. units as for the corresponding range equation above,

$$t_e = \frac{R}{V} = \frac{1}{g} \cdot \frac{3.6\eta_{pr}C_L/C_D}{V \cdot BSFC} \cdot \ln\left(\frac{W_{ini}}{W_{fin}}\right), \text{ h} \quad (2.31)$$

noting that V is the constant or mean airspeed in km/h over the cruise flight segment.

By inspection of the above, maximum endurance for a propellered airplane would require the maximization of:

$$\frac{1}{V} \cdot \frac{C_L}{C_D} = \sqrt{\frac{\rho S}{2W}} \cdot \frac{C_L^{3/2}}{C_D} \quad (2.32)$$

One can observe that the above requirement corresponds to flying at minimum power:

$$C_{L,me} = C_{L,mp} = \sqrt{\frac{3C_{Do}}{K}} \quad (2.33a)$$

$$= C_{L,max}, \quad \text{if } \sqrt{\frac{3C_{Do}}{K}} \geq C_{L,max} \quad (2.33b)$$

Note the requirement that the aircraft cannot fly at a C_L value above the aerodynamic stall limit, $C_{L,max}$.

For maximum air range, inspection of the Breguet range equation suggests a propellered aircraft should fly at a maximum value for C_L/C_D :

$$C_{L,mr} = C_{L,md} = \sqrt{\frac{C_{Do}}{K}}, \quad V_{mr} = \sqrt{\frac{2W}{\rho S C_{L,mr}}} = \frac{1}{\rho^{1/2}} \cdot \sqrt{\frac{2W}{S}} \cdot \left(\frac{K}{C_{Do}}\right)^{1/4}, \quad (2.34)$$

The above equations suggest that there is some benefit in going to a higher altitude (lower ρ), to increase cruise airspeed and thus shorten the trip time in going from

point A to point B. The practical economic cruise altitude for a conventional turboprop aircraft is in the neighborhood of 15,000–25,000 ft (4.5–7.6 km), a bit lower for an airplane with a turbo-supercharged piston engine, and substantially lower still for an aircraft employing a normally-aspirated (i.e., non-supercharged) piston engine to drive the propeller (10,000 ft or less). Shaft power for turboprop engines does decrease with increasing altitude above a certain altitude, and propeller performance degrades as air density gets too low; these are factors that come into play with respect to practical economic cruise airspeeds and altitudes.

As was the case for jet aircraft, propellered aircraft endurance is independent of the wind levels present. Ground range, as before, is ascertained via

$$R_g = R - V_w t_e \quad (2.35)$$

In addition, it can be shown that airspeed must be increased above the still-air criterion above, in the presence of a headwind, for maximum ground range attainment. Conversely, in a tailwind, the airspeed must be decreased below the still-air case. For maximum ground range, observation of the Breguet range equation suggests that one needs to maximize the following:

$$\frac{C_L}{C_D} - \frac{V_w}{V} \cdot \frac{C_L}{C_D} = \frac{C_L}{C_{Do} + KC_L^2} - \frac{V_w}{\sqrt{\frac{2W}{\rho S}}} \cdot \frac{C_L^{3/2}}{(C_{Do} + KC_L^2)} \quad (2.36)$$

Differentiating the above with respect to C_L , and setting the resulting equation to zero, one ultimately arrives at the following:

$$0 = C_{Do} - KC_L^2 + \frac{V_w}{\sqrt{\frac{2W}{\rho S}}} \left[\frac{1}{2} KC_L^{5/2} - \frac{3}{2} C_{Do} C_L^{1/2} \right] \quad (2.37)$$

It is hoped that the above example of airplane cruise performance analysis, allied to engine performance (here, largely as influenced by BSFC or TSFC), gives the reader a sense of importance that the propulsion system plays in making a particular aircraft viable. Please refer to the solution for Example Problem 2.1 at the end of this chapter, for a completed sample analysis of the cruise performance of a particular propellered aircraft. For students learning this material as part of a course, I would encourage them to attempt the example problem first, and then check the solution to see if any mistakes were made.

2.3 Example Mission Requirement: Takeoff of Fixed-Wing Airplanes

In order to provide further background to the larger picture that surrounds airplanes and the propulsion systems that are integrated to them to allow for a successful mission, an example set of mathematical equations describing airplane

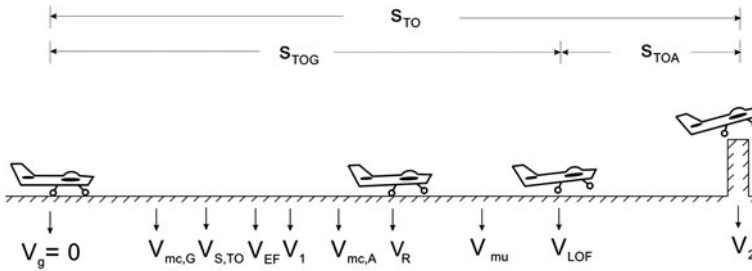


Fig. 2.3 Takeoff profile for fixed-wing aircraft

takeoff performance reveals further the close connection that exists between engine and airframe.

2.3.1 Introduction to Takeoff Performance

Government airworthiness regulations, like the Canadian Aviation Regulations (CARs) by Transport Canada, the Federal Aviation Regulations (FARs) by the US Federal Aviation Administration (see www.faa.gov for online reading of FARs), and the Joint Aviation Regulations (JARs) by the European Aviation Safety Agency (EASA), stipulate the minimum standards for safe aircraft operation, including those for takeoff and landing. Here, reference will be made to FAR Part 23 for small (light) airplanes, and FAR Part 25 for heavier ($>12,500$ lb, $5,670$ kg) transport aircraft (typically multi-engined; see FAR 25.105–25.115 for takeoff info).

Referring to Fig. 2.3 for a fixed-wing aircraft's takeoff profile, one can define a number of pertinent parameters describing the aircraft's takeoff performance (some parameters applicable only to multi-engined FAR 25 airplanes):

s_{TO}	overall takeoff distance, from a standing start (groundspeed $V_g = 0$, airspeed $V > 0$ if a headwind present) to clearing a specified obstacle (screen) height (FAR 25, $h_{sc} = 35$ ft (10.67 m); FAR 23, 50 ft (15.25 m)) at airspeed V_2
s_{TOG}	takeoff ground roll distance
s_{TOA}	takeoff airborne distance; also called climb segment #1 distance
$V_{mc, G}$	minimum control airspeed on ground; should a critical engine fail for a multi-engined airplane, aircraft will not yaw or bank appreciably given adequate application of rudder and aileron by the pilot
$V_{S, TO}$	stall airspeed, aircraft in takeoff configuration (partial flaps typical)
V_{EF}	critical engine failure airspeed, $>V_{mc, G}$ (otherwise, brake to full stop on runway)
V_1	critical engine failure recognition airspeed, $>V_{mc, G}$ and $<V_{LOF}$, sufficient field length for pilot to continue takeoff with critical engine inoperative

	(multi-engined aircraft); if engine fails at $V < V_1$, brake to full stop on runway; $V_1 < V_{1, MBE}$, i.e., do not exceed <i>maximum brake energy limit</i> (above which, catastrophic failure of overheated brakes or tires may be anticipated)
$V_{mc, A}$	minimum control airspeed in air, $< 1.2V_{S, TO}$; should a critical engine fail for a multi-engined aircraft, the pilot will have sufficient rudder, aileron and elevator so as not to yaw or bank appreciably
V_R	takeoff rotation airspeed, $> 1.05V_{mc, A}$; pilot begins rotation of aircraft in pitch to desired angle-of-attack (α) and corresponding C_L in preparation for liftoff; at pilot's discretion, the nose landing gear may begin to be retracted beyond this speed (to lower drag)
V_{mu}	minimum unstick airspeed; with critical engine inoperative, aircraft could still liftoff under adequate control
V_{LOF}	liftoff airspeed, $> 1.1V_{mu}$ with all engines up, or $> 1.05V_{mu}$ with critical engine inoperative; $\approx 1.2V_{S, TO}$
V_2	takeoff climb airspeed, $> 1.2V_{S, TO}$ and $> 1.1V_{mc, A}$; must be sufficiently high to satisfy climb segment #2 (35 ft (10.67 m) to 400 ft (122 m) AGL, FAR 25) minimum climb gradient requirement with critical engine inoperative; main landing gear can begin to be retracted (to lower drag)

A headwind in takeoff reduces the aircraft's groundspeed while maintaining the specified airspeed ($V = V_g + V_w$), thus producing a shorter takeoff distance. Conversely, a tailwind will result in a longer takeoff distance. Note, FAR 25 specifies the use of only 50% of the predicted headwind in preparations/calculations undertaken by the pilot prior to takeoff, and 150% of the predicted tailwind magnitude, to be conservative and allow for some safety margin.

Below a height h_{AGL} equivalent to the aircraft's wing span b , the airplane will be, to some degree, in ground effect. The ground imposes a reflecting boundary condition that inhibits downward flow of air (downwash from wings, primarily), associated with the production of lift. With respect to takeoff and landing, the primary effects of interest are an increase in the lift-curve slope, $C_{L\alpha}$, and lift coefficient, C_L , and a decrease in the lift-induced drag coefficient, C_{Di} . Recall for the aerodynamics of airplanes, the following:

$$\text{wing aspect ratio, } AR = \frac{b^2}{S} \quad (2.38)$$

$$\text{lift-induced drag factor, } K = \frac{1}{\pi \cdot AR \cdot e} \quad (2.39)$$

and the drag polar expression for overall drag coefficient,

$$C_D = C_{D0} + C_{Di} = C_{D0} + \phi K C_L^2 \quad (2.40)$$

Note that S in this case is the reference wing area, e is the Oswald efficiency factor, C_{D0} is the base (zero-lift) drag coefficient and ϕ is the ground effect factor on lift-induced drag ($0 < \phi < 1$). Once the airplane is out of ground effect as it gains

altitude, ϕ will go to unity. One expression for ϕ , proposed by Lan and Roskam [2], compares reasonably well with experimental data (h is the mean height of the wing over the ground, for this formula):

$$\phi = 1 - \frac{(1 - 1.32 \frac{h}{b})}{(1.05 + 7.4 \frac{h}{b})}, \quad 0.033 < h/b < 0.25 \quad (2.41)$$

Unless indicated otherwise, the airplane would be assumed to be out of ground effect for calculation purposes, once the screen height is reached or exceeded (again, to be conservative). At a height ratio of 0.1, the value for ϕ is about 0.52 according to the above, a result that is similar to experimental data.

One understands that the thrust generated from propellers, turbojets (at lower altitudes) or turbofans typically drops substantially with increasing airspeed from a standing start. We will have to account for this in any takeoff and early climb-out analysis. Do not make the mistake of using maximum static thrust throughout the ground roll and initial climb segment (this is not realistic). Also, for propellered airplanes, do not assume peak propeller propulsive efficiency η_{pr} applies at low speeds. In practice, more propeller thrust is available at lower speeds (a good thing for shortening takeoff distance), but at the price of a lower η_{pr} . A rough estimate of η_{pr} would be closer to 0.3–0.5 for the takeoff ground run's mean value, rather than the peak value seen at cruise conditions (0.8–0.85).

With a critical engine failure, note that there will be a drag increment due to propeller drag (at least until the propeller can be feathered) or turbine windmilling, and due to the asymmetric flight condition (rudder + aileron/spoiler deflection required to correct). Some yawing of the aircraft, as well as some accompanying roll, can occur with a thrust imbalance on a multi-engined airplane, and an additional roll component can occur due to engine torque imbalance.

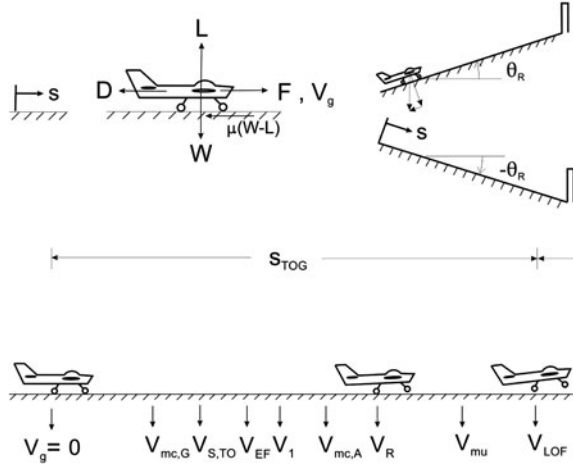
2.3.2 Takeoff Analysis: Ground Roll

Consider the free-body force diagram of an airplane undertaking a ground roll, as the first part of a takeoff run, as shown in Fig. 2.4. Newton's second law of motion ($F = m \cdot a$) stipulates the following with respect to the airplane moving in the longitudinal s -direction:

$$m \cdot a_{\text{long}} = m \frac{dV_g}{dt} = \sum F_s = F - D - \mu(W - L) - W \cdot \sin \theta_R \quad (2.42)$$

Since the angle of incline of the runway θ_R is typically small (usually well within $\pm 5^\circ$), one can apply small-angle assumptions in that regard (e.g., $\cos \theta_R \approx 1$). The coefficient of rolling friction μ with the brakes off is usually a small value, e.g., about 0.02 for dry asphalt. One can establish distance travelled s by the airplane via the following integration of groundspeed V_g with time:

Fig. 2.4 Forces acting on fixed-wing aircraft in ground roll



$$s = \int_0^t V_g dt, \quad m \quad (2.43)$$

Given aircraft mass $m = W/g$, and noting that longitudinal acceleration of the aircraft in the ground roll is given by:

$$a_{\text{long}} = \frac{dV_g}{dt} = \frac{g}{W} [F - D - \mu(W - L) - W \sin \theta_R] \quad (2.44)$$

then integrating the above provides

$$V_g = \int_0^t \frac{g}{W} [F - D - \mu(W - L) - W \sin \theta_R] dt \quad (2.45)$$

One would use airspeed V (recalling $V = V_g + V_w$) for estimating thrust F , lift L and drag D as follows:

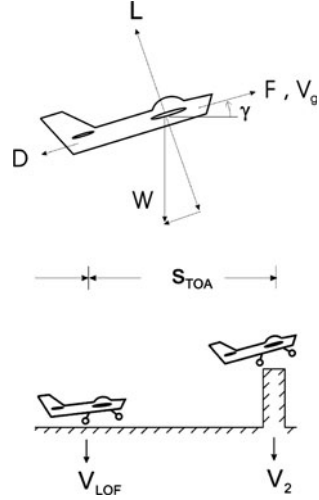
F , determined from empirical equations or charts, as a function of V or flight Mach number Ma

$L = C_{L,\text{TOG}} \frac{1}{2} \rho V^2 S$, where the lift coefficient is for the ground roll only

$D = C_{D,\text{TOG}} \frac{1}{2} \rho V^2 S = \frac{1}{2} \rho V^2 S (C_{D0} + \phi K C_{L,\text{TOG}}^2)$, where one finds the ground effect factor ϕ value at the wing's mean height above ground.

One can numerically integrate the appropriate equations above to arrive at an estimate of the airplane's ground roll distance s_{TOG} . For preliminary calculations, one could assume no aircraft rotation (in pitch) until the aircraft actually lifts off ($V = V_{\text{LOF}}$), rather than the actual case ($V = V_R$). Otherwise, one could allow for a transition phase from V_R to V_{LOF} , where the aircraft's lift coefficient is also

Fig. 2.5 Forces acting on fixed-wing aircraft in climb



changing from $C_{L,TOG}$ to $C_{L,TOA}$, in effect when the pilot is pulling back on the control column, to apply elevator in bringing the aircraft's nose up in preparation for liftoff and becoming airborne. The default conservative assumption is that the airplane is assumed to be out of ground effect, upon liftoff ($\phi = 1$).

The effective ground roll lift coefficient $C_{L,TOG}$ is set by the pitch attitude of the aircraft as it rolls along the ground, and by the flap setting (typically, a partial deflection of the flaps, held constant during the roll and early climb out). In order to maximize the longitudinal acceleration of the airplane in the ground roll (and thus shorten the ground roll distance), one would like to minimize the force sum of the drag and rolling resistance linked to $C_{L,TOG}$. Applying differential calculus (differentiation with respect to $C_{L,TOG}$) for estimating a trough point,

$$\frac{d}{dC_{L,TOG}} (C_{D_o} + \phi K C_{L,TOG}^2 - \mu C_{L,TOG}) = 0 \quad (2.46)$$

$$2\phi K C_{L,TOG} - \mu = 0 \quad (2.47)$$

so that one finally arrives at an expression for the nominal best choice:

$$C_{L,TOG} \big|_{\max.\text{accel.}} = \frac{\mu}{2\phi K} \quad (2.48)$$

The flap deflection, and the resulting actual $C_{L,TOG}$, may have to be a bit higher as a compromise, in order to get sufficient lift as the pilot commences applying elevator at rotation speed V_R . The influence of zero-lift drag coefficient C_{D_o} should not be ignored in this process, with respect to its value during the ground roll (landing gear fully extended; investigate means for reducing drag where possible) and its value as the nose and main gear are retracted as the airplane becomes airborne.

2.3.3 Takeoff Analysis: Climb Segment #1

At and beyond V_{LOF} , as the aircraft becomes airborne and gains height through Climb Segment #1 (see Fig. 2.5), the main landing gear can be retracted at the discretion of the pilot (thus lowering C_{Do}). Along the flight path at a given climb angle γ , Newton's law of motion stipulates:

$$F - D - W \sin \gamma = \frac{W}{g} \frac{dV_g}{dt} \quad (2.49)$$

such that

$$\frac{dV_g}{dt} = \frac{g}{W} [F - D - W \sin \gamma] = a_{\text{long}} \quad (2.50)$$

In this airborne transition, there is a curvilinear path being followed, such that the aircraft's pitch rotation rate $d\gamma/dt$ may be assumed to be governed by the pertinent forces normal to the flight path contributing to centripetal acceleration of the airplane:

$$L - W \cos \gamma = \frac{W}{g} V_g \frac{d\gamma}{dt} \quad (2.51)$$

such that

$$\frac{d\gamma}{dt} = \frac{g}{WV_g} [L - W \cos \gamma], \quad \text{rad/s} \quad (2.52)$$

Note that the airplane's angle-of-attack α is assumed to be small in these calculations, and that pitch rotation acceleration $d^2\gamma/dt^2$ due to pitching moment contributions is implicit in this simplified representation. We will also need the vertical distance travelled (height gained), via

$$\frac{dh}{dt} = V \sin \gamma, \quad \text{m/s} \quad (2.53)$$

so via integration with time,

$$h = \int_{t_{\text{LOF}}}^t V \sin \gamma \cdot dt, \quad \text{m} \quad (2.54)$$

One can also numerically integrate for groundspeed V_g :

$$V_g = V_{g, \text{LOF}} + \int_{t_{\text{LOF}}}^t \frac{g}{W} [F - D - W \sin \gamma] dt \quad (2.55)$$

and flight path angle γ :

$$\gamma = \gamma_o + \int_{t_{\text{LOF}}}^t \frac{g}{WV_g} [L - W \cos \gamma] dt, \quad \text{rad} \quad (2.56)$$

The initial flight path angle at liftoff, γ_o , may have a positive or negative value, depending on the incline angle θ_R of the runway. Here, V_g is assumed to be following the airplane's flight path, so ground distance covered is established via

$$s = s_{\text{TOG}} + \int_{t_{\text{LOF}}}^t V_g \cos \gamma \cdot dt \quad (2.57)$$

Accounting for the aircraft's pitch attitude relative to the Earth's surface below, airspeed V for thrust, lift and drag calculations is found from:

$$V = V_g + V_w \cos \gamma \quad (2.58)$$

where one is assuming the wind is horizontal, and V_w is the head or tailwind component acting on the airplane (i.e., not accounting for a crosswind component here). Unless indicated otherwise, simplifying assumptions for preliminary calculations of airborne ground distance covered can include assuming $V_R \approx V_{\text{LOF}}$ (i.e., if V_R is known, but not V_{LOF}), ground effect factor $\phi \approx 1$ and that the liftoff lift coefficient $C_{L,\text{TOA}}$ remains unchanged through the first climb segment (to reach the screen height h_{sc}). In practice, one may have a decrease in $C_{L,\text{TOA}}$ while one accelerates from V_{LOF} to V_2 , and in conjunction to reach and hold a desired initial climb angle.

For Climb Segment #1, one would continue the above integrations with time from liftoff ($t = t_{\text{LOF}}, h_{\text{AGL}} = 0$) up to reaching the screen height, $h_{\text{AGL}} = h_{\text{sc}}$. Upon reaching the screen height, one arrives at an airplane airspeed V_2 that one needs to confirm is acceptable under government guidelines (e.g., FAR 25, $V_2 > 1.2V_{S,\text{TO}}$), but also acceptable for further climbing out from the runway (segments #2 [h_{sc} to an altitude of 400 ft (122 m)] and #3 [to an altitude of 1,500 ft (457 m)]). If it turns out V_2 is too low in value, one would possibly need to increase V_{LOF} to get the value of V_2 up as well (alternatively, a lower initial climb angle might help in this respect, but will potentially extend the airborne transition ground distance). Note that one would have an upper limit value on V_{LOF} , with regards to the tire friction/heating threshold for the landing gear.

In completing the integrations up to the end of Climb Segment #1, one thus arrives at the overall takeoff ground distance, namely

$$s_{\text{TO}} = s_{\text{TOG}} + s_{\text{TOA}} \quad (2.59)$$

If the value for s_{TO} is higher than desired, one will need to revisit a number of parameters that contribute to that value, including $V_{S,\text{TO}}$, a critical variable that a number of parameters depend upon. One would not necessarily assume

immediately that more powerful engines (higher thrust) are the only means of resolution to address such a difficulty.

2.3.4 Approximation Model for Ground Roll

Recall from the earlier discussion on takeoff ground roll:

$$\frac{dV_g}{dt} = \frac{g}{W} [F - D - \mu(W - L) - W \sin \theta_R] \quad (2.60)$$

and

$$s = \int_0^t V_g dt \quad (2.61)$$

We can also consider the following relationships between groundspeed and longitudinal ground distance covered:

$$V_g = \frac{ds}{dt}, \quad V_g dV_g = \frac{ds}{dt} dV_g, \quad ds = \frac{V_g}{\frac{dV_g}{dt}} dV_g \quad (2.62)$$

Integrating for distance, one can make some substitutions:

$$s = \int ds = \int \frac{V_g}{\frac{dV_g}{dt}} \cdot dV_g = \int \frac{V_g \cdot dV_g}{\frac{g}{W} [F - D - \mu(W - L) - W \sin \theta_R]} \quad (2.63)$$

The above expression for distance covered during the ground roll portion of the takeoff is not, in general, readily solved analytically (i.e., more likely to be solved via application of a numerical method, e.g., the finite difference approach). However, for preliminary design or quick estimation, one can introduce a few convenient assumptions to in turn produce a quick and relatively reasonable approximation for s_{TOG} .

A common approach for ground roll distance estimation is the so-called mean kinetic energy approximation. Equating work completed with kinetic energy developed during the ground roll, one can show that the following is approximately true for the ground distance covered:

$$s_{\text{TOG}} \approx \frac{V_{g,\text{LOF}}^2}{2 \left(\frac{dV}{dt} \right)} = \frac{V_{g,\text{LOF}}^2/2}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]} \quad (2.64)$$

where

$$V_{g,\text{LOF}} = V_{\text{LOF}} - V_w \quad (2.65)$$

noting that a headwind is treated as positive. As noted by the overbar on several parameters above, one needs to estimate thrust, drag and lift at a mean kinetic energy airspeed. Allowing for a headwind or tailwind, the airspeed at the mean kinetic energy point in the ground roll is:

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_w |V_w|}{2}} \quad (2.66)$$

In the case of a normal takeoff, the time elapsed for the ground roll can be estimated via

$$\Delta t_{\text{TOG}} = \frac{V_{g,\text{LOF}}}{\frac{dV}{dt}} = \frac{V_{g,\text{LOF}}}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]} \quad (2.67)$$

With one engine inoperative at some point in the ground roll,

$$\begin{aligned} \Delta t_{\text{TOG}} = & \frac{V_{g,\text{EF}}}{\left(\frac{dV}{dt}\right)_1} + \frac{V_{g,\text{LOF}} - V_{g,\text{EF}}}{\left(\frac{dV}{dt}\right)_2} = \frac{V_{g,\text{EF}}}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]_1} \\ & + \frac{V_{g,\text{LOF}} - V_{g,\text{EF}}}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]_2} \end{aligned} \quad (2.68)$$

The airspeed-based parameters at state 1 (period prior to engine failure) would be determined at:

$$\bar{V}_{1,\text{OEI}} = \sqrt{\frac{V_{\text{EF}}^2 + V_w |V_w|}{2}} \quad (2.69)$$

while at the state 2 condition (period after engine failure), they would be determined via:

$$\bar{V}_{2,\text{OEI}} = \sqrt{\frac{V_{\text{EF}}^2 + V_{\text{LOF}}^2}{2}} \quad (2.70)$$

The corresponding ground distance travelled over the 2-part ground roll would be:

$$\begin{aligned} s_{\text{TOG}} = & \frac{V_{g,\text{EF}}^2}{2 \left(\frac{dV}{dt}\right)_1} + \frac{V_{g,\text{LOF}}^2 - V_{g,\text{EF}}^2}{2 \left(\frac{dV}{dt}\right)_2} = \frac{V_{g,\text{EF}}^2/2}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]_1} \\ & + \frac{(V_{g,\text{LOF}}^2 - V_{g,\text{EF}}^2)/2}{\frac{g}{W} [\bar{F} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]_2} \end{aligned} \quad (2.71)$$

2.3.5 Approximation Model for Climb Segment #1

Let us now look at how we can estimate the distance covered over the latter part of the takeoff, when the aircraft is now off the ground. One can separate the airborne transition component of the takeoff run (Climb Segment #1) into two parts, for mathematical convenience:

- I. V increasing from V_{LOF} to V_2 , with wheels off the ground but little height gained
- II. Altitude above ground level, h_{AGL} , increasing, but with little velocity gained

In summary then, for Part I, assume acceleration from V_{LOF} to V_2 occurs at h_{AGL} near zero. For Part II, assume climb at constant airspeed of approximately V_2 , with a constant flight path angle γ , from zero height above ground to screen height h_{sc} . The conservative default assumption is that the airplane is out of ground effect for both parts, such that $\phi = 1$, unless indicated otherwise.

Part I:

The first part can be described by:

$$\frac{dV}{dt} \approx \frac{g}{W} (\bar{F} - \bar{D}) \quad (2.72)$$

For this flight phase, mean thrust and drag are estimated at:

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} \quad (2.73)$$

Distance covered for Part I is found via:

$$s_{\text{TOA},I} = \frac{V_2^2 - V_{\text{LOF}}^2}{2 \left(\frac{dV}{dt} \right)} = \frac{V_2^2 - V_{\text{LOF}}^2}{2 \frac{g}{W} (\bar{F} - \bar{D})} \quad (2.74)$$

Note that this is still-air distance, or air distance, for Part I (wind not considered on ground distance covered; we'll look at ground distance a bit later).

Time elapsed for Part I is:

$$\Delta t_{\text{TOA},I} = \frac{V_2 - V_{\text{LOF}}}{\left(\frac{dV}{dt} \right)} = \frac{V_2 - V_{\text{LOF}}}{\frac{g}{W} (\bar{F} - \bar{D})} \quad (2.75)$$

Part II:

For the second part, the airplane is assumed to be climbing at a constant angle γ . In this case, assuming the climb angle is not overly steep, one can estimate its value via the following:

$$\sin \gamma = \frac{F - D}{W} \approx \frac{F - D_{\text{SLF}}}{W} \approx \frac{F}{W} - \frac{C_D}{C_L} \Big|_{\text{SLF}} \quad (2.76)$$

The acronym SLF refers to steady level flight ($\gamma = 0^\circ$), noting that

$$C_{L, \text{SLF}} = \frac{2W}{\rho S V^2} \quad (2.77)$$

and

$$C_D = C_{D0} + K C_L^2 \quad (2.78)$$

For Part II, thrust and drag are ascertained at V_2 . The climb rate is found from:

$$\frac{dh}{dt} = V_2 \sin \gamma \quad (2.79)$$

Considering the triangle formed by the parameters associated with a climb, one can observe that

$$\Delta h = \Delta s \cdot \tan \gamma \quad (2.80)$$

which indicates that

$$s_{\text{TOA, II}} = \frac{h_{sc}}{\tan \gamma} \quad (2.81)$$

As for Part I, note that this is air distance for Part II (wind not considered). A bit later, we will account for a headwind or tailwind on distance covered.

Also from the climb's trigonometry,

$$\Delta t \cdot V_2 \cos \gamma = \Delta s \quad (2.82)$$

so that

$$\Delta t_{\text{TOA, II}} = \frac{s_{\text{TOA, II}}}{V_2 \cos \gamma} \quad (2.83)$$

Ground Distance with Wind Present:

Given the fact that the starting and ending criteria for Parts I and II are based on airspeeds (V_{LOF} , V_2), the times $\Delta t_{\text{TOA, I}}$ and $\Delta t_{\text{TOA, II}}$ noted above will remain the same regardless if there is a headwind or tailwind present. However, we know that air distance and ground distance tend to differ when V_w is finite, and that is certainly true here, as noted below (we will use the subscript g explicitly here for ground distance covered, to distinguish from the air distance estimate):

$$s_{g, \text{TOA, I}} = s_{\text{TOA, I}} - V_w \Delta t_{\text{TOA, I}} \quad (2.84)$$

and

$$s_{g, \text{TOA, II}} = s_{\text{TOA, II}} - V_w \Delta t_{\text{TOA, II}} \quad (2.85)$$

As a result, the overall ground distance covered during the airborne component of the takeoff is in the general case (allowing for wind):

$$s_{\text{TOA}} = s_{g,\text{TOA}, I} + s_{g,\text{TOA}, II} \quad (2.86)$$

2.3.6 Approximated Overall Takeoff Distance

In summarizing then, one arrives at an estimate of overall takeoff distance from a standing start at the beginning of the runway to the screen height h_{sc} :

$$s_{\text{TO}} = s_{\text{TOG}} + s_{\text{TOA}} \quad (2.87)$$

For some problems, one would continue estimating the distance and height covered as the airplane moves into Climb Segment #2 (h_{sc} to 400 ft AGL and then Climb Segment #3 (400–1,500 ft AGL) in climbing out and leaving the airport's control domain.

It is hoped that the above example of airplane takeoff performance analysis, allied to engine performance (here, largely as influenced by maximum takeoff thrust, that decreases with forward airspeed in the case of air-breathing engines), gives the reader a sense of importance that the propulsion system plays in making a particular aircraft viable. Please refer to the solution for Example Problem 2.5(b) at the end of this chapter, for a completed sample analysis of the takeoff performance of a particular jet aircraft.

In the next chapter, we will begin our study of propulsion systems with the propeller, a principal means of thrust delivery for lower speed airplanes. Later, we will discuss the various powerplants that can be used to drive the propeller's rotation.

2.4 Example Problems

2.1. A de Havilland DHC-8 (Dash 8) has a reference wing area S of 54.4 m^2 , a zero-lift drag coefficient C_{D_0} of 0.0255, and a value for the lift-induced drag factor K of 0.032. The regional transport airplane is powered by two Pratt and Whitney PW120A turboprops with propellers operating at 85% efficiency ($\eta_{pr} = 0.85$) at maximum cruise airspeed of 490 km/h, at an altitude of 4,600 m ASL (above sea level). Cruise BSFC is $2.2 \times 10^{-4} \text{ kg/(h W)}$. The airplane's maximum takeoff weight is 153.477 kN, maximum payload weight is 37.376 kN, maximum fuel weight is 25.261 kN and operating empty weight is 100.533 kN. For the current flight mission, the payload weight is *lower* than the maximum allowed, at 30.411 kN. Assume 1.374 kN of fuel is consumed in takeoff and climb to cruise, and 1.374 kN of fuel is consumed in descent from cruise and landing.

- (a) Determine cruise range for the above flight at max. cruise speed and altitude under still-air conditions, for the maximum possible fuel that can be loaded into the airplane (assume no fuel reserve in your calculations).
- (b) Repeat (a), but for a 56 km/h headwind.
- (c) Determine the maximum-range (minimum-fuel) airspeed at the mean aircraft weight for the cruise segment, for the still-air case.
- (d) Repeat (c), but for a 56 km/h headwind.

2.2. A Boeing 747-100 has a reference wing area of 511 m^2 , a zero-lift C_{Do} of 0.02 and a lift-induced K of 0.065. The heavy airliner is powered by four Pratt and Whitney JT9D-7A turbofans with a cruise TSFC of $0.0694 \text{ kg}/(\text{h N})$ at a maximum cruise true Mach number of 0.8 at 9,150 m ASL. The airplane's maximum takeoff weight is 3,286 kN, maximum payload weight of 667.1 kN, maximum fuel weight of 1442.1 kN and operating empty weight of 1,687 kN (maximum zero-fuel weight of 2354.4 kN). For the current flight mission, the payload weight is *lower* than the maximum allowed at 402.2 kN. Assume 73.6 kN of fuel is consumed in takeoff and climb to cruise, and 49.1 kN of fuel in descent and landing.

- (a) Determine cruise range for the above flight at max. cruise Mach number and altitude under still-air conditions, for the maximum possible fuel that can be loaded into the airplane (assume no fuel reserve in your calculations).
- (b) Repeat (a), but for a 56 km/h tailwind.
- (c) Determine the mean maximum-range airspeed and corresponding flight Mach number, for the still air case. Note that drag divergence is not being accounted for, when you see the result.
- (d) Repeat (c), but for a 56 km/h tailwind.

2.3. A de Havilland DHC-5D (Buffalo) has a reference wing area S of 945 ft^2 , a zero-lift drag coefficient C_{Do} of 0.024 and a value for the lift-induced drag factor K of 0.04. The transport airplane is powered by two General Electric CT64-820-4 turboprops with propellers operating at 85% efficiency ($\eta_{pr} = 0.85$) at cruise at an altitude of 10,000 ft ASL. Cruise BSFC is $0.5 \text{ lb}/\text{h}\cdot\text{hp}$. The airplane's maximum takeoff weight is 43,000 lb, maximum payload weight is 12,000 lb, maximum fuel weight is 14,000 lb and operating empty weight is 25,000 lb. At the present time, the airplane is facing a 30-kt headwind in cruise at 10,000 ft, with the current aircraft weight being 41,000 lb. What would be the maximum-ground-range airspeed and flight Mach number at this juncture? How does this value compare to the still-air case?

2.4. Note the characteristics of the following aircraft:

- Aircraft:** Single-engined light recreational aircraft; wing span 32 ft; wing ref. area 180 sq ft; Oswald efficiency $0.75C_{L \text{ max}}$ 1.7 (partial flaps); aircraft weight 2,800 lb; $V_{LOF} = 1.15V_{S,L}$ C_{Do} 0.04, takeoff configuration; clearing 50-ft height, $V_2 = 1.3V_{S,L}$
- Engines** One 4-cylinder normally aspirated air-cooled piston engine
 Variable-pitch constant-speed propeller, $n = 2,700 \text{ rpm}$, $d_p = 6 \text{ ft}$
 Forward thrust, at sea level, where $J = V/(nd_p)$, static $F_{o,S/L} = 500 \text{ lbf}$,
 $F_{S/L}/F_{o,S/L} = 1 - 0.3 J$, $\text{lbf}; F(h)/F_{S/L} = \sigma(\text{thrust vs. altitude, } \sigma = \rho/\rho_{S/L})$

Airfield ISA conditions at specified altitudes; mean wing height $h_{w,g} = 8$ ft, $\mu = 0.05$, free roll on short-grass runway; upslope of 0° , $C_{L,TOG} = 0.9$; runway obstacle height 50 ft

Evaluate the following cases for takeoff distance, using an approximate method:

- (a) Sea level, no wind
- (b) Sea level, 10-kt headwind
- (c) 3,000 ft altitude, no wind
- (d) 3,000 ft altitude, 10-kt headwind.

2.5. Note the characteristics of the following aircraft:

Aircraft Boeing 747-100 heavy wide-body commercial airliner; wing span 196 ft; wing ref. area 5,600 sq. ft.; Oswald efficiency $0.7C_{L,max}$ 1.8 (partial flaps); aircraft weight 735,000 lb; $V_{LOF} = 1.18V_{S,L}C_{D_o}$ 0.036, takeoff configuration; clearing 35-ft screen height, $V_2 = 1.2V_{S,L}$

Engines Four Pratt and Whitney JT9D-7A turbofan engines; for a single engine, engine, max. forward thrust for regular takeoff given by $F_{eng, S/L} = 46100 - 46.7 V + 0.0467 V^2$, lbf (thrust at S/L, V in ft/s), and $F(h)/F_{S/L} = \sigma^{0.8}$ (variation of thrust as function of altitude);

Airfield ISA conditions at specified altitudes; mean wing height $h_{w,g} = 20$ ft; runway downslope of 0° ; free roll on asphalt, μ 0.02; $C_{L,TOG} = 1.0$ runway obstacle height 35 ft.

Evaluate the following cases for takeoff distance, using an approximate method:

- (a) Sea level, no wind
- (b) Sea level, 30-kt tailwind
- (c) 5,000 ft altitude, no wind.

2.5 Solution for Problems

2.1. (a) Cruise range, no wind:

Air density at 4,600 m ISA alt. is $\rho = 0.77 \text{ kg/m}^3$. Sound speed at 4,600 m alt. is $a = 322 \text{ m/s}$.

Given $K = 0.032$.

Breguet range equation for propellered aircraft:

$$R_{cr} = \frac{3.6}{g} \eta_{pr} \frac{C_L/C_D}{BSFC} \ln \left(\frac{W_{ini}}{W_{fin}} \right); \text{ note: weights, if given in kg, should be converted to newtons.}$$

verted to newtons.

$W_{ini} = W_o - W_{F,t/o+climb} = 153477 - 1374 = 152103 \text{ N} = 152.1 \text{ kN}$, weight at cruise's start

$W_{Fo} = W_o - W_E - W_{PL} = 153477 - 100553 - 30411 = 22513 \text{ N} = 22.5 \text{ kN}$, initial fuel weight loaded into aircraft.

$W_{fin} = W_{ini} - (W_{Fo} - W_{F,lo+climb} - W_{F,des+ldg} - W_{Fres})$, where W_{Fres} (reserve fuel) = 0 (not specified)

$$W_{fin} = 152103 - (22513 - 1374 - 1374 - 0) = 132338 \text{ N} = 132.3 \text{ kN}$$

$V_{cr} = 490 \text{ km/h} = 136.5 \text{ m/s}$ (specified, so unless told otherwise, do not change this cruise speed)

$\overline{W} = \sqrt{W_{ini} W_{fin}} = (152100 \cdot 132300)^{1/2} = 141850 \text{ N} = 141.9 \text{ kN}$, mean cruise weight

$$\overline{C_L} = \frac{\overline{W}}{\frac{1}{2} \rho V_{cr}^2 S} = 141900 / (0.5 \cdot 0.77 \cdot 136.5^2 \cdot 54.4) = 0.364$$

$$\overline{C_D} = C_{Do} + K \overline{C_L}^2 = 0.0255 + 0.032(0.364^2) = 0.03$$

$R_{cr} = 3.6/9.81 \cdot 0.85 \cdot (0.364/0.03)/2.2 \times 10^{-4} \ln(152.1/132.3) = 2400 \text{ km}$, still-air cruise range

(b) Cruise ground range, 56 km/h headwind:

$$R_{G,cr} = R_{cr} - V_w \cdot t_{cr}, \text{ where } t_{cr} = R_{cr}/V_{cr} = 2400/490 = 4.9 \text{ h}$$

$$R_{G,cr} = 2400 - 56(4.9) = 2126 \text{ km}, \text{ so shorter cruise range with a headwind.}$$

(c) Mean maximum-range velocity at 4,600 m altitude:

$$\text{Still air, propellered aircraft, } C_{L,mr} = \sqrt{\frac{C_{Do}}{K}} = (0.0255/0.032)^{1/2} = 0.89,$$

$$V_{mr} = \left(\frac{\overline{W}}{\frac{1}{2} \rho C_{L,mr} S} \right)^{1/2} = (141900 / (0.5 \cdot 0.77 \cdot 0.89 \cdot 54.4))^{1/2} = 87.3 \text{ m/s in still air.}$$

(d) With a 56 km/h (15.6 m/s) headwind, one knows to fly a bit faster than the still-air V_{mr} . Referring to the textbook equation:

$$0 = C_{Do} - K C_{L,mr}^2 + \frac{V_w}{\sqrt{\frac{2 \overline{W}}{\rho S}}} \left[\frac{1}{2} K C_{L,mr}^{5/2} - \frac{3}{2} C_{Do} C_{L,mr}^{1/2} \right]$$

$$0 = 0.0255 - 0.032 C_{L,mr}^2 + 0.189 \left[0.016 C_{L,mr}^{2.5} - 0.03825 C_{L,mr}^{0.5} \right]$$

Solving iteratively, one arrives at $C_{L,mr} = 0.806$, which says:

$$V_{mr} = \left(\frac{\overline{W}}{\frac{1}{2} \rho C_{L,mr} S} \right)^{1/2} = (141900 / (0.5 \cdot 0.77 \cdot 0.806 \cdot 54.4))^{1/2} \\ = 91.7 \text{ m/s in 56 km/h headwind.}$$

2.2. (a) Cruise range, no wind:

Air density at 9,150 m ISA alt. is $\rho = 0.459 \text{ kg/m}^3$. Sound speed at 9,150 m alt. is $a = 304 \text{ m/s}$.

$$AR = 59.8^2/511 = 7, K = 1/(\pi \cdot 7 \cdot 0.7) = 0.065$$

Breguet range equation for jet aircraft:

$$R_{cr} = \frac{1}{g} \frac{V_{cr}}{TSFC} \frac{C_L}{C_D} \ln \left(\frac{W_{ini}}{W_{fin}} \right); \text{ weights given in kg should be converted to newtons.}$$

$$W_{ini} = W_o - W_{F,t/o+climb} = 3.286 \times 10^6 - 73575 = 3.212 \times 10^6 \text{ N} = 3212 \text{ kN, weight at cruise's start}$$

$$W_E = W_{ZF} - W_{P/L,max} = 2.354 \times 10^6 - 667080 = 1.687 \times 10^6 \text{ N}$$

$$W_{Fo} = W_o - W_E - W_{P/L} = 3.286 \times 10^6 - 1.687 \times 10^6 - 402210 = 1.197 \times 10^6 \text{ N} = 1197 \text{ kN, initial fuel weight loaded into aircraft.}$$

$$W_{fin} = W_E + W_{P/L} + W_{F,des+ldg} + W_{Fres}, \text{ where } W_{Fres} \text{ (reserve fuel)} = 0 \text{ (not specified)}$$

$$W_{fin} = 1.687 \times 10^6 + 402210 + 49050 + 0 = 2.138 \times 10^6 \text{ N} = 2138 \text{ kN}$$

$$V_{cr} = a(Ma_{cr}) = 304 (0.8) = 243 \text{ m/s} = 875 \text{ km/h (specified, so unless told otherwise, do not change this cruise speed)}$$

$$\overline{W} = \sqrt{W_{ini} W_{fin}} = (3212 \cdot 2138)^{1/2} = 2621 \text{ kN, mean cruise weight}$$

$$\overline{C_L} = \frac{\overline{W}}{\frac{1}{2} \rho V_{cr}^2 S} = 2621000 / (0.5 \cdot 0.459 \cdot 243^2 \cdot 511) = 0.379$$

$$\overline{C_D} = C_{Do} + K \overline{C_L}^2 = 0.02 + 0.065 (0.379^2) = 0.0293$$

$$R_{cr} = 1/9.81 \cdot 875 / 0.0694 \cdot (0.379 / 0.0293) \ln(3212/2138) = 6767 \text{ km, still-air cruise range.}$$

(b) Cruise ground range, 56 km/h tailwind:

$$R_{G,cr} = R_{cr} - V_w \cdot t_{cr}, \text{ where } t_{cr} = R_{cr}/V_{cr} = 6767/875 = 7.73 \text{ h}$$

$$R_{G,cr} = 6767 + 56(7.73) = 7200 \text{ km, so greater cruise range with a tailwind.}$$

(c) Mean maximum-range velocity at 9,150 m altitude:

$$\text{Still air, jet aircraft, } C_{L,mr} = \sqrt{\frac{C_{Do}}{3K}} = (0.02/(3 \cdot 0.065))^{1/2} = 0.32,$$

$$V_{mr} = \left(\frac{\overline{W}}{\frac{1}{2} \rho C_{L,mr} S} \right)^{1/2} = (2621000 / (0.5 \cdot 0.459 \cdot 0.32 \cdot 511))^{1/2} = 264 \text{ m/s in still air.}$$

$Ma_{mr} = V_{mr}/a = 264/304 = 0.868$, which for this older airplane, is likely bringing it significantly into drag divergence. Likely would lower the airplane's cruise Mach number closer to 0.8.

(d) With a 56 km/h tailwind ($V_w = -15.6 \text{ m/s}$), one knows to fly a bit slower than the still-air V_{mr} . Referring to the textbook:

$$0 = C_{Do} - 3KC_{L, \text{mr}}^2 + 2\sqrt{\frac{\rho S}{2\bar{W}}} \cdot V_w \left[KC_{L, \text{mr}}^{5/2} - C_{Do}C_{L, \text{mr}}^{1/2} \right]$$

$$0 = 0.02 - 3(0.065)C_{L, \text{mr}}^2 + 2(0.459 \cdot 511 / (2 \cdot 2621000))^{0.5} (-15.6) [0.065C_{L, \text{mr}}^{2.5} - 0.02C_{L, \text{mr}}^{0.5}]$$

$$0 = 0.02 - 0.195C_{L, \text{mr}}^2 - 0.2087[0.065C_{L, \text{mr}}^{2.5} - 0.02C_{L, \text{mr}}^{0.5}]$$

Solving iteratively, one arrives at $C_{L, \text{mr}} = 0.333$, which says:

$$V_{\text{mr}} = \left(\frac{\bar{W}}{\frac{1}{2}\rho C_{L, \text{mr}} S} \right)^{1/2} = (2621000 / (0.5 \cdot 0.459 \cdot 0.333 \cdot 511))^{1/2} = 259 \text{ m/s, in}$$

56 km/h tailwind, giving a cruise Mach number of $0.852 < 0.868$.

2.3. Max. ground-range airspeed, 30-kt headwind:

Air density at 10,000 ft ISA alt. is $\rho = 0.001756 \text{ slug/ft}^3$. Sound speed at 10,000 ft alt. is $a = 1077 \text{ ft/s}$.

$$\text{AR} = 96^2 / 945 = 9.75, K = 1 / (\pi \cdot 9.75 \cdot 0.82) = 0.04$$

$$\text{Still air, prop aircraft, } C_{L, \text{mr}} = \sqrt{\frac{C_{Do}}{K}} = (0.024 / (0.04))^{1/2} = 0.775,$$

$$V_{\text{mr}} = \left(\frac{\bar{W}}{\frac{1}{2}\rho C_{L, \text{mr}} S} \right)^{1/2} = (41000 / (0.5 \cdot 0.001756 \cdot 0.775 \cdot 945))^{1/2} = 253 \text{ ft/s in still air (150 kt).}$$

$$Ma_{\text{mr}} = V_{\text{mr}} / a = 253 / 1077 = 0.235, \text{ a fairly low subsonic value.}$$

With a 30-kt headwind ($V_w = 50.7 \text{ ft/s}$), one knows to fly a bit faster than the still-air V_{mr} . Referring to the lecture notes:

$$0 = C_{Do} - KC_{L, \text{mr}}^2 + \frac{V_w}{\sqrt{\frac{2\bar{W}}{\rho S}}} \left[\frac{1}{2} KC_{L, \text{mr}}^{5/2} - \frac{3}{2} C_{Do} C_{L, \text{mr}}^{1/2} \right]$$

$$0 = 0.024 - 0.04C_{L, \text{mr}}^2 + 0.22808[0.02C_{L, \text{mr}}^{2.5} - 0.036C_{L, \text{mr}}^{0.5}]$$

Solving iteratively, one arrives at $C_{L, \text{mr}} = 0.69$, which says:

$$V_{\text{mr}} = \left(\frac{\bar{W}}{\frac{1}{2}\rho C_{L, \text{mr}} S} \right)^{1/2} = (41000 / (0.5 \cdot 0.001756 \cdot 0.69 \cdot 945))^{1/2} = 268 \text{ ft/s, in 30-kt headwind (159 kt), giving a cruise Mach number of 0.267.}$$

2.4. (a) Find the takeoff distance using an approximate method, in this case for sea level altitude ($\rho = 0.002378 \text{ slugs/ft}^3$) and no wind ($V_w = 0 \text{ ft/s}$).

General info:

$$\text{AR} = b^2 / S = 32^2 / 180 = 5.7, K = 1 / (\pi \text{AR} \cdot e) = 1 / (\pi \cdot 5.7 \cdot 0.75) = 0.0745, n = 2700 / 60 = 45 \text{ revs./s}$$

$$V_{S, \text{t/o}} = (2W_o / (\rho S C_{L, \text{max}}))^{1/2} = (2 \cdot 2800 / (0.002378 \cdot 180 \cdot 1.7))^{1/2} = 87.7 \text{ ft/s or 52 kt}$$

$V_{\text{LOF}} = 1.15 V_{S,t/o} = 101 \text{ ft/s}$, $V_2 = 1.30 V_{S,t/o} = 114 \text{ ft/s}$, $\theta_R = 0^\circ$ (no runway slope)

Ground roll:

Using textbook method, begin with $\bar{V} = V_{\text{LOF}}/\sqrt{2} = 71.4 \text{ ft/s}$, $\bar{J} = \frac{\bar{V}}{nd_p} = 71.4/$

$$(45 \cdot 6) = 0.264$$

$$\bar{T} = T_o(1 - 0.3\bar{J}) = 500(1 - 0.3(0.264)) = 460 \text{ lbf}$$

$$\phi = 1 - \frac{(1 - 1.32h/b)}{(1.05 + 7.4h/b)} = 1 - (1 - 1.32(8/32))/(1.05 + 7.4(8/32)) = 0.77$$

$$C_{D,GE} = C_{D_o} + \phi KC_{L_g}^2 = 0.04 + 0.77(0.0745)0.9^2 = 0.087$$

$$\bar{L} = \frac{1}{2}\rho\bar{V}^2 SC_{L_g} = 982 \text{ lbf}, \bar{D} = \frac{1}{2}\rho\bar{V}^2 SC_{D,GE} = 94 \text{ lbf}, V_{G,LOF} = V_{\text{LOF}} - V_w = 101 \text{ ft/s}$$

$$\begin{aligned} s_{\text{TOG}} &= \frac{V_{G,LOF}^2/2}{\frac{g}{W}[\bar{T} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]} \\ &= 101^2/2/(32.2/2800)/[460 - 94 - 0.05(2800 - 982) - 0] \\ &= 1615 \text{ ft} \end{aligned}$$

Early airborne transition (I):

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} = [(101^2 + 114^2)/2]^{1/2} = 108 \text{ ft/s},$$

$$\bar{J} = \frac{108}{45 \cdot 6} = 0.4, \bar{T} = 500[1 - 0.3(0.4)] = 440 \text{ lbf}$$

$$\bar{C}_L = \frac{W}{\frac{1}{2}\rho\bar{V}^2 S} = 2800/(0.5 \cdot 0.002378 \cdot 108^2 \cdot 180) = 1.12$$

$\bar{C}_D = 0.04 + 1.0(0.0745)(1.12^2) = 0.133$, noting $\phi = 1.0$ is assumed once airplane is airborne

$$\bar{D} = \frac{1}{2}\rho\bar{V}^2 S\bar{C}_D = 332 \text{ lbf}$$

$$s_{\text{TOA,I}} = \frac{(V_2^2 - V_{\text{LOF}}^2)/2}{\frac{g}{W}[\bar{T} - \bar{D}]} = (114^2 - 101^2)/2/(32.2/2800)/[440 - 332] = 1125 \text{ ft}$$

Late airborne transition (II):

$$V = V_2 = 114 \text{ ft/s}, J = 114/(45 \times 6) = 0.42, T = 500(1 - 0.3(0.42)) = 437 \text{ lbf},$$

$$C_L = 2800/(0.5 \times 0.002378 \times 114^2 \times 180) = 1.0,$$

$$C_D = 0.04 + 0.0745(1.0^2) = 0.115$$

$$\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 437/2800 - 0.115/1.0 = 0.041 \text{ radians or } 2.4^\circ s_{\text{TOA,II}}$$

$$\Pi = \frac{h}{\tan \gamma} = 50/\tan(2.4^\circ) = 1195 \text{ ft}$$

$$\text{Overall: } s_{TO} = s_{\text{TOG}} + s_{\text{TOA,I}} + s_{\text{TOA,II}} = 1615 + 1125 + 1195 = 3935 \text{ ft}$$

Note: for a light aircraft, this is fairly long (around 2,000 ft would be more typical for an airplane of this size), suggesting the airplane is a bit underpowered

(b) Sea level, headwind V_w of 10 kt (17 ft/s): $V_{G,LOF} = V_{LOF} - V_w = 101 - 17 = 84$ ft/s

Ground roll, using textbook method:

$$\bar{V} = \sqrt{\frac{V_w^2 + V_{LOF}^2}{2}} = 72.4 \text{ ft/s}, \bar{J} = 72.4/(45 \cdot 6) = 0.268,$$

$$\bar{T} = 500(1 - 0.3(0.268)) = 460 \text{ lbf}$$

$$\bar{L} = 0.5(0.002378)72.4^2(180)0.9 = 1010 \text{ lbf},$$

$$\bar{D} = 0.5(0.002378)72.4^2(180)0.087 = 98 \text{ lbf}$$

$s_{TOG} = 84^2/2/(32.2/2800)/[460 - 98 - 0.05(2800 - 1010) - 0] = 1126$ ft ... use this estimate for later.

Early airborne (I):

$s_{TOA,I} = s_{TOA,I}(0) - V_w \Delta t_{TOA,I}$, where $s_{TOA,I}(0)$, no wind case, is found in (a) above. Given the same $\bar{V}$ as (a), mean thrust and drag are same as (a):

$$\Delta t_{TOA,I} = \frac{V_2 - V_{LOF}}{\frac{g}{W}[\bar{T} - \bar{D}]} = (114 - 101)/(32.2/2800)/[440 - 332] = 10.5 \text{ s}$$

$$s_{TOA,I} = 1125 - 17(10.5) = 947 \text{ ft}$$

Late airborne (II):

Same γ as (a), since calculated at same V_2 .

$$s_{TOA,II} = s_{TOA,II}(0) - V_w \Delta t_{TOA,II},$$

$$\Delta t_{TOA,II} = \frac{s_{TOA,II}(0)}{V_2 \cos \gamma} = 1195/(114 \cdot \cos 2.4^\circ) = 10.5 \text{ s}$$

$$s_{TOA,II} = 1195 - 17(10.5) = 1017 \text{ ft}$$

Overall:

$$s_{TO} = 1126 + 947 + 1017 = 3090 \text{ ft}$$

(c) 3,000 ft alt., headwind V_w of 0 kt:

$\sigma = 0.915$ given $\rho = 0.00218$ slugs/ft³ at 3000 ft ISA.

$V_{S,t/o} = (2W_o/(\rho S C_{L \max}))^{1/2} = (2 \cdot 2800/(0.00218 \cdot 180 \cdot 1.7))^{1/2} = 91.7$ ft/s or 54 kt

$$V_{LOF} = 1.15 V_{S,t/o} = 105.5 \text{ ft/s}, V_2 = 1.30 V_{S,t/o} = 119.2 \text{ ft/s}$$

$$T = \sigma T_{S/L}$$

Ground roll, using textbook method:

$$\bar{V} = \sqrt{\frac{V_w^2 + V_{LOF}^2}{2}} = 74.6 \text{ ft/s}, \bar{J} = 74.6/(45 \cdot 6) = 0.276,$$

$$\bar{T} = 458(1 - 0.3(0.276)) = 420 \text{ lbf}$$

$$\bar{L} = 0.5(0.00218)74.6^2(180)0.9 = 983 \text{ lbf},$$

$$\bar{D} = 0.5(0.00218)74.6^2(180)0.087 = 95 \text{ lbf}$$

$$s_{\text{TOG}} = 105.5^2/2/(32.2/2800)/[420 - 95 - 0.05(2800 - 983) - 0] = 2067 \text{ ft}$$

Early airborne (I):

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} = [(105.5^2 + 119.2^2)/2]^{1/2} = 112.6 \text{ ft/s}, \bar{J} = \frac{112.6}{45 \cdot 6} = 0.42,$$

$$\bar{T} = 458[1 - 0.3(0.42)] = 400 \text{ lbf}$$

$$\bar{C}_L = \frac{W}{\frac{1}{2}\rho \bar{V}^2 S} = 2800/(0.5 \cdot 0.00218 \cdot 112.6^2 \cdot 180) = 1.126$$

$$\bar{C}_D = 0.04 + 1.0(0.0745)(1.126^2) = 0.134$$

$$\bar{D} = \frac{1}{2}\rho \bar{V}^2 S \bar{C}_D = 333 \text{ lbf}$$

$$s_{\text{TOA,I}} = \frac{(V_2^2 - V_{\text{LOF}}^2)/2}{\frac{g}{W}[\bar{T} - \bar{D}]} = (119.2^2 - 105.5^2)/2/(32.2/2800)/[400 - 333] = 1998 \text{ ft}$$

Late airborne (II):

$$V = V_2 = 119.2, J = 119.2/(45 \times 6) = 0.442, T = 458(1 - 0.3(0.442)) = 397 \text{ lbf},$$

$$C_L = 2800/(0.5 \times 0.00218 \times 119.2^2 \times 180) = 1.0,$$

$$C_D = 0.04 + 0.0745(1.0^2) = 0.115$$

$$\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 397/2800 - 0.115/1.0 = 0.027 \text{ radians or } 1.55^\circ$$

$$s_{\text{TOA,II}} = \frac{h}{\tan \gamma} = 50/\tan(1.55^\circ) = 1848 \text{ ft}$$

$$\text{Overall: } s_{\text{TO}} = 2067 + 1998 + 1848 \approx 5913 \text{ ft}$$

(d) 3000 ft alt., headwind V_w of 10 kt (17 ft/s): $V_{G,\text{LOF}} = V_{\text{LOF}} - V_w = 105.5 - 17 = 88.5 \text{ ft/s}$

Ground roll, using textbook method:

$$\bar{V} = \sqrt{\frac{V_w^2 + V_{\text{LOF}}^2}{2}} = 72.4 \text{ ft/s}, \bar{J} = 72.4/(45 \cdot 6) = 0.268, \bar{T} = 500(1 - 0.3(0.268)) = 460 \text{ lbf}$$

$$\bar{L} = 0.5(0.002378)72.4^2(180)0.9 = 1010 \text{ lbf}, \bar{D} = 0.5(0.002378)72.4^2(180)0.087 = 98 \text{ lbf}$$

$$s_{\text{TOG}} = 88.5^2/2/(32.2/2800)/[419.5 - 97.6 - 0.05(2800 - 1009.2) - 0] = 1466 \text{ ft}$$

Early airborne (I):

$s_{\text{TOA,I}} = s_{\text{TOA,I}}(0) - V_w \Delta t_{\text{TOA,I}}$, where $s_{\text{TOA,I}}(0)$, no wind case, is found in (a) above. Same thrust and drag as (c) above, given the same mean airspeed:

$$\Delta t_{\text{TOA,I}} = \frac{V_2 - V_{\text{LOF}}}{\frac{g}{W}[\bar{T} - \bar{D}]} = (119.2 - 105.5)/(32.2/2800)/[400 - 333] = 17.8 \text{ s}$$

$$s_{\text{TOA,I}} = 1998 - 17(17.8) = 1695 \text{ ft}$$

Late airborne (II):

Same γ as (c), since at same V_2 .

$$s_{\text{TOA,II}} = s_{\text{TOA,II}}(0) - V_w \Delta t_{\text{TOA,II}},$$

$$\Delta t_{\text{TOA,II}} = \frac{s_{\text{TOA,II}}(0)}{V_2 \cos \gamma} = 1848 / (119.2 \cdot \cos(1.55^\circ)) = 15.5 \text{ s}$$

$$s_{\text{TOA,II}} = 1848 - 17(15.5) = 1585 \text{ ft}$$

Overall:

$$s_{\text{TO}} = 1466 + 1695 + 1585 \approx 4746 \text{ ft}$$

2.5. (a) Find the nominal takeoff distance using an approximate method, in this case for sea level altitude ($\rho = 0.002378 \text{ slugs/ft}^3$) and no wind ($V_w = 0 \text{ ft/s}$).

General info:

$$AR = b^2/S = 196^2/5600 = 6.9, K = 1/(\pi \cdot AR \cdot e) = 1/(\pi \cdot 6.9 \cdot 0.7) = 0.066$$

$$V_{S,t/o} = (2W_o/(\rho SC_{L_{\max}}))^{1/2} = (2 \cdot 735000 / (0.002378 \cdot 5600 \cdot 1.8))^{1/2} = 248 \text{ ft/s}$$

or 147 kt

$$V_{\text{LOF}} = 1.18 V_{S,t/o} = 293 \text{ ft/s}, V_2 = 1.20 V_{S,t/o} = 298 \text{ ft/s}, \theta_R = 0^\circ \text{ (no runway slope)}$$

$$\text{Ground roll: } \phi = 1 - \frac{(1 - 1.32h/b)}{(1.05 + 7.4h/b)} = 1 - (1 - 1.32(20/196)) / (1.05 + 7.4(20/196)) = 0.52$$

$$C_{D,GE} = C_{D_o} + \phi KC_{L_g}^2 = 0.036 + 0.52(0.066)1.0^2 = 0.07; \text{ normal t/o, } N = 4$$

engines operating $\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_w |V_w|}{2}} = \sqrt{\frac{293^2 + (0)(0)}{2}} = 207.2 \text{ ft/s},$

$$\bar{T} = 46100 - 46.7 \bar{V} + 0.0467 \bar{V}^2 = 38429 \text{ lbf for one engine, } 153715 \text{ lbf for 4 engines.}$$

$$\bar{L} = \frac{1}{2} \rho \bar{V}^2 SC_{L_g} = 285857 \text{ lbf}, \bar{D} = \frac{1}{2} \rho \bar{V}^2 SC_{D,GE} = 20010 \text{ lbf}$$

$$s_{\text{TOG}} = \frac{V_{G,\text{LOF}}^2/2}{\frac{g}{W} [\bar{T} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]}$$

$$= 293^2/2 / (32.2/735000) / [153715 - 20010 - 0.02(735000 - 285857) - 0]$$

$$= 7856 \text{ ft}$$

Early airborne transition (I):

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} = [(293^2 + 298^2)/2]^{1/2} = 296 \text{ ft/s},$$

$$\bar{T} = 4(46100 - 46.7(296) + 0.0467(296^2)) = 145474 \text{ lbf}$$

$$\bar{C}_L = \frac{W}{\frac{1}{2} \rho \bar{V}^2 S} = 735000 / (0.5 \cdot 0.002378 \cdot 296^2 \cdot 5600) = 1.26$$

$\overline{C_D} = 0.036 + 1.0(0.066)(1.26^2) = 0.141$, noting $\phi = 1.0$ is assumed once airplane is airborne

$$\overline{D} = \frac{1}{2} \rho \overline{V}^2 S \overline{C_D} = 82257 \text{ lbf}$$

$$s_{\text{TOA,I}} = \frac{(V_2^2 - V_{\text{LOF}}^2)/2}{\frac{g}{W}[\overline{T} - \overline{D}]} = (298^2 - 293^2)/2 / (32.2/735000) / [145474 - 82257] = 533 \text{ ft}$$

Late airborne transition (II):

$$V = V_2 = 298 \text{ ft/s}, T = 4(46100 - 46.7(298) + 0.0467(298^2)) = 145322 \text{ lbf},$$

$$C_L = 735000 / (0.5 \times 0.002378 \times 298^2 \times 5600) = 1.243,$$

$$C_D = 0.036 + 0.066(1.243^2) = 0.138$$

$$\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 145322/735000 - 0.138/1.243 = 0.087 \text{ radians or } 5^\circ$$

$$s_{\text{TOA,II}} = \frac{h}{\tan \gamma} = 35/\tan(5^\circ) = 400 \text{ ft}$$

Overall:

$$s_{\text{TO}} = s_{\text{TOG}} + s_{\text{TOA,I}} + s_{\text{TOA,II}} = 7856 + 533 + 400 = 8789 \text{ ft}$$

Note: for a fully loaded Boeing 747, this is a reasonable estimate.

(b) Find the nominal takeoff distance using an approximate method, in this case for sea level altitude ($\rho = 0.002378 \text{ slugs/ft}^3$) and a 30-kt tailwind ($V_w = -51 \text{ ft/s}$) s). $V_{G,\text{LOF}} = V_{\text{LOF}} - V_w = 293 - (-51) = 344 \text{ ft/s}$

Ground roll, using textbook method:

$$\overline{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_w |V_w|}{2}} = \sqrt{\frac{293^2 + (-51)(51)}{2}} = 204 \text{ ft/s},$$

$\overline{T} = 46100 - 46.7 \overline{V} + 0.0467 \overline{V}^2 = 38517 \text{ lbf}$ for **one** engine, 154067 lbf for 4 engines.

$$\phi = 1 - \frac{(1 - 1.32h/b)}{(1.05 + 7.4h/b)} = 1 - (1 - 1.32(20/196)) / (1.05 + 7.4(20/196)) =$$

0.52, from 2.5(a) $C_{D,GE} = C_{D0} + \phi K C_{Lg}^2 = 0.036 + 0.52(0.066)1.0^2 = 0.07$, from 2.5(a)

$$\overline{L} = \frac{1}{2} \rho \overline{V}^2 S C_{Lg} = 277096 \text{ lbf}, \overline{D} = \frac{1}{2} \rho \overline{V}^2 S C_{D,GE} = 19397 \text{ lbf}$$

$$\begin{aligned} s_{\text{TOG}} &= \frac{V_{G,\text{LOF}}^2/2}{\frac{g}{W}[\overline{T} - \overline{D} - \mu(W - \overline{L}) - W \sin \theta_R]} \\ &= 344^2/2 / (32.2/735000) / [154067 - 19397 - 0.02(735000 - 277096) - 0] \\ &= 10761 \text{ ft} \end{aligned}$$

Early airborne transition (I):

$s_{\text{TOA,I}} = s_{\text{TOA,I}}(0) - V_w \Delta t_{\text{TOA,I}}$, where $s_{\text{TOA,I}}(0)$, no wind case, is found in 2.5(a) solution:

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} = [(293^2 + 298^2)/2]^{1/2} = 296 \text{ ft/s},$$

$$\bar{T} = 4(46100 - 46.7(296) + 0.0467(296^2)) = 145474 \text{ lbf}$$

$$\bar{C}_L = \frac{1}{2} \frac{W}{\rho \bar{V}^2 S} = 735000 / (0.5 \cdot 0.002378 \cdot 296^2 \cdot 5600) = 1.26$$

$\bar{C}_D = 0.036 + 1.0(0.066)(1.26^2) = 0.141$, noting $\phi = 1.0$ is assumed once airplane is airborne

$$\bar{D} = \frac{1}{2} \rho \bar{V}^2 S \bar{C}_D = 82257 \text{ lbf} \quad s_{\text{TOA},I}(0) = \frac{(V_2^2 - V_{\text{LOF}}^2)/2}{\frac{g}{W}} [\bar{T} - \bar{D}] = (298^2 - 293^2)/2 /$$

$$(32.2/735000) / [145474 - 82257] = 533 \text{ ft}$$

Given the same $\bar{V}$ as 2.5(a), mean thrust and drag are same as 2.5(a):

$$\Delta t_{\text{TOA},I} = \frac{V_2 - V_{\text{LOF}}}{\frac{g}{W} [\bar{T} - \bar{D}]} = (298 - 293) / (32.2/735000) / [145474 - 82257] = 1.8 \text{ s}$$

$$s_{\text{TOA},I} = 533 - (-51)(1.8) = 625 \text{ ft}$$

Late airborne (II):

Same γ as 2.5(a), since calculated at same V_2 :

$$V = V_2 = 298 \text{ ft/s}, T = 4(46100 - 46.7(298) + 0.0467(298^2)) = 145322 \text{ lbf},$$

$$C_L = 735000 / (0.5 \times 0.002378 \times 298^2 \times 5600) = 1.243,$$

$$C_D = 0.036 + 0.066(1.243^2) = 0.138$$

$$\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 145322/735000 - 0.138/1.243 = 0.087 \text{ radians or } 5^\circ$$

$$s_{\text{TOA},II}(0) = \frac{h}{\tan \gamma} = 35 / \tan(5^\circ) = 400 \text{ ft}$$

$$s_{\text{TOA},II} = s_{\text{TOA},II}(0) - V_w \Delta t_{\text{TOA},II}, \Delta t_{\text{TOA},II} = \frac{s_{\text{TOA},II}(0)}{V_2 \cos \gamma} = 400 / (298 \cdot \cos 5^\circ) =$$

$$1.35 \text{ s}$$

$$s_{\text{TOA},II} = 400 - (-51)(1.35) = 469 \text{ ft}$$

Overall:

$$s_{\text{TO}} = s_{\text{TOG}} + s_{\text{TOA},I} + s_{\text{TOA},II} = 10761 + 625 + 469 = 11855 \text{ ft}$$

(c) Find the nominal takeoff distance using an approximate method, in this case for 5,000 ft altitude ($\rho = 0.002$ slugs/ft³) and no wind ($V_w = 0$ ft/s).

General info:

$$\sigma = 0.86 \text{ given } \rho = 0.002 \text{ slugs/ft}^3 \text{ at 5000 ft ISA.}$$

$$V_{S,t/o} = (2W_o / (\rho S C_{L_{\text{max}}}))^{1/2} = (2 \cdot 735000 / (0.002 \cdot 5600 \cdot 1.8))^{1/2} = 270 \text{ ft/s or } 147 \text{ kt}$$

$$V_{\text{LOF}} = 1.18 V_{S,t/o} = 319 \text{ ft/s}, V_2 = 1.20 V_{S,t/o} = 324 \text{ ft/s}$$

Ground roll:

$$T = \sigma^{0.8} T_{S/L}$$

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_w |V_w|}{2}} = \sqrt{\frac{319^2 + (0)(0)}{2}} = 225.6 \text{ ft/s,}$$

$\bar{T} = 40860 - 41.4 \bar{V} + 0.0414 \bar{V}^2 = 33627 \text{ lbf}$ for **one** engine, 134509 lbf for 4 engines.

$$\bar{L} = \frac{1}{2} \rho \bar{V}^2 S C_{Lg} = 285014 \text{ lbf, } \bar{D} = \frac{1}{2} \rho \bar{V}^2 S C_{D,GE} = 19951 \text{ lbf}$$

$$\begin{aligned} s_{\text{TOG}} &= \frac{V_{G,\text{LOF}}^2/2}{\frac{g}{W} [\bar{T} - \bar{D} - \mu(W - \bar{L}) - W \sin \theta_R]} \\ &= 319^2/2 / (32.2/735000) / [134509 - 19951 - 0.02(735000 - 285014) - 0] \\ &= 11002 \text{ ft} \end{aligned}$$

Early airborne transition (I):

$$\bar{V} = \sqrt{\frac{V_{\text{LOF}}^2 + V_2^2}{2}} = [(319^2 + 324^2)/2]^{1/2} = 322 \text{ ft/s,}$$

$$\bar{T} = 4 \cdot 0.86^{0.8} \cdot (46100 - 46.7(322) + 0.0467(322^2)) = 127294 \text{ lbf}$$

$$\bar{C}_L = \frac{W}{\frac{1}{2} \rho \bar{V}^2 S} = 735000 / (0.5 \cdot 0.002 \cdot 322^2 \cdot 5600) = 1.27$$

$\bar{C}_D = 0.036 + 1.0(0.066)(1.27^2) = 0.142$, noting $\phi = 1.0$ is assumed once airplane is airborne

$$\bar{D} = \frac{1}{2} \rho \bar{V}^2 S \bar{C}_D = 0.5(0.002)322^2(5600)0.142 = 82450 \text{ lbf}$$

$$s_{\text{TOA,I}} = \frac{(V_2^2 - V_{\text{LOF}}^2)/2}{\frac{g}{W} [\bar{T} - \bar{D}]} = (324^2 - 319^2)/2 / (32.2/735000) / [127294 - 82450] = 818 \text{ ft}$$

Late airborne transition (II):

$$V = V_2 = 324 \text{ ft/s,}$$

$$T = 4(0.886)(46100 - 46.7(324) + 0.0467(324^2)) = 127129 \text{ lbf,}$$

$$C_L = 735000 / (0.5 \times 0.002 \times 324^2 \times 5600) = 1.25,$$

$$C_D = 0.036 + 0.066(1.25^2) = 0.139$$

$$\gamma = \frac{T}{W} - \frac{C_D}{C_L} = 127129/735000 - 0.139/1.25 = 0.062 \text{ radians or } 3.5^\circ$$

$$s_{\text{TOA,II}} = \frac{h}{\tan \gamma} = 35 / \tan(3.55^\circ) = 564 \text{ ft}$$

Overall:

$$s_{\text{TO}} = s_{\text{TOG}} + s_{\text{TOA,I}} + s_{\text{TOA,II}} = 11002 + 818 + 564 = 12384 \text{ ft}$$

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