

Chapter 2

Principles of Electrical Power Control

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Abstract This chapter contains a review of the scientific works published till date in the field of power theory for systems with periodic non-sinusoidal waveforms. Nowadays, electrical energy belongs to goods indispensable in everyday life. Dynamic increase in the number of installed nonlinear loads, that are the source of higher harmonics in current and voltage waveforms, results in deterioration of electrical energy parameters. Higher harmonics make the electrical energy quality much worse. The number of power theories and papers concerning these issues give evidence about the importance of the problems of working condition optimisation in power systems.

2.1 Power Theory

Power theory is a collection of information about the properties of propagation of energy in electrical circuits. It is the result of research and experience of many generations of scientists and electrical engineers. This concept is often used in phrases such as “Fryze power theory”, “instantaneous pq theory”, etc. In this context it means a way of interpreting the phenomena occurring in the electrical system proposed by the author’s ideas. The definition in this case was accompanied by the necessary formulas that permit the calculation of properties describing

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the electric circuit. Power theories are also used to optimise the operating point of electrical systems. They allow to minimise losses and thereby reduce operating.

Every year, dozens of articles have been published on this subject, in one way or another trying to solve the problem of power quality. Why? The solution is purely economic, electricity is a commodity. In the market the economy which wins is the one with the better quality merchandise at a price comparable to others. The second reason is additional operating costs of the power grid. These costs are caused by:

- Increased losses in resistive elements;
- Increased losses in engines;
- Capacitor failures;
- The need to increase the efficiency of power source;
- Increased current in the neutral wire;
- Resonance phenomena (caused by higher harmonics);
- Production shutdowns caused by improper operation of protection systems.

The methods and ways of describing energy and power-quality properties related to improvement of source and load effectiveness in non-sinusoidal circuits have not been standardised so far. This is proved by the fact that in the past several decades the International Electrotechnical Commission (IEC) has changed reactive power definition several times [1–4].

2.1.1 Critical Review of Classical Power Theory, Power in Sinusoidal-Type Waveforms Circuits

The following sinusoidal waveforms are used for two-terminal networks as shown in Fig. 2.1:

$$v(t) = \sqrt{2}|V| \cos(\omega t + \alpha) \quad (2.1)$$

$$i(t) = \sqrt{2}|I| \cos(\omega t + \beta) \quad (2.2)$$

where $|V|$, $|I|$ —RMS values of voltage $v(t)$ and current $i(t)$, respectively.

The different powers used in discussion of power properties of this circuit are:

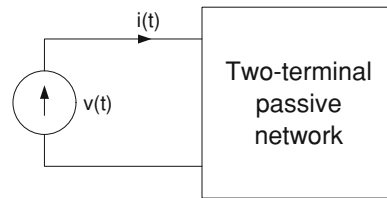
- instantaneous power $p(t)$

$$p(t) = v(t)i(t) = |V||I| \cos \varphi [1 + \cos(2\omega t + 2\alpha)] + |V||I| \sin \varphi \sin(2\omega t + 2\alpha) = p_1(t) + p_2(t) \quad (2.3)$$

It may be expressed as:

$$p(t) = P[1 + \cos(2\omega t + 2\alpha)] + Q \sin(2\omega t + 2\alpha) \quad (2.4)$$

Fig. 2.1 Two-terminal network under consideration



where P —active power, Q —reactive power, φ —argument of impedance Z

$$P = \overline{p(t)} = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T p_1(t) dt = |V||I| \cos \varphi \quad (2.5)$$

$$Q = |V||I| \sin \varphi \quad (2.6)$$

The first component of formula (2.3) describes variable non-negative component of instantaneous power with $2P$ amplitude and average value equal to load's active power P . This component represents one-directional flow of energy from the source to the load.

The second component of instantaneous power (2.3) $p_2(t)$ (alternating component) is characterised by amplitude equal to load's reactive power Q and average value equal to zero. This component characterises the bidirectional flow of energy in source-load system. It is not present if load phase angle is equal to zero. Therefore, in case of resistant load or if the load exhibits phase resonance (circuit scheme as per Fig. 2.1), two-directional oscillations in energy flow between source and load do not take place.

- Apparent power $|S|$

$$|S| = |V||I| \quad (2.7)$$

Apparent power is a purely computational quantity, it does not possess any physical meaning.

- Power factor λ

$$\lambda = \cos \varphi = \frac{P}{|S|} \quad (2.8)$$

In case of sinusoidal waveforms the power properties are described by so-called complex power:

$$S = VI^* = P + jQ \quad (2.9)$$

It must be noted that instantaneous, active, reactive and complex powers may be subjected to power balance, while apparent power may not.

All the specified powers are correctly defined, and in case of linear two-terminal network definition/interpretation is not controversial. The reactive power $Q = |V||I| \sin \varphi$ may be physically interpreted on the basis of formula (2.3) in case of one-phase linear circuits with sinusoidal waveforms. The alternating component $p_2(t)$, with amplitude equal to $Q = |V||I| \sin \varphi$ may be interpreted as the measure of backward flow of energy between circuit's reactance elements and the source. The reactive power may also be related to inductor's magnetic field or condenser's electric field. If sinusoidal current $i(t) = \sqrt{2}|I| \sin \omega t$ flows through induction coil of inductance L , magnetic field exists in the inductor and is equal to:

$$W_L(t) = \frac{1}{2} Li^2(t) = \frac{1}{2} L \left(\sqrt{2}|I| \right)^2 \sin^2 \omega t = W_{L\max} \sin^2 \omega t \quad (2.10)$$

while coil's reactive power

$$Q_L = \omega L |I|^2 = \omega W_{L\max} \quad (2.11)$$

Similarly, in case of condenser with C capacitance and supplied with sinusoidal voltage $v(t) = \sqrt{2}|V| \sin \omega t$, condenser's electric field energy is equal to:

$$W_C(t) = \frac{1}{2} C v^2(t) = \frac{1}{2} C \left(\sqrt{2}|V| \right)^2 \sin^2 \omega t = W_{C\max} \sin^2 \omega t \quad (2.12)$$

and its reactive power

$$Q_C = -\omega C |V|^2 = -\omega W_{C\max} \quad (2.13)$$

Generally, in case of elements which accumulate energy, reactive power may be expressed as:

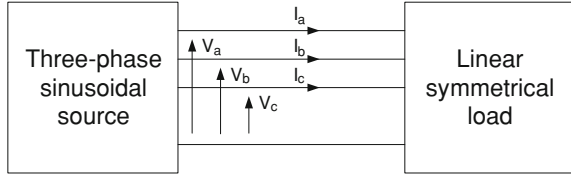
$$Q = Q_L + Q_C = \omega(W_{L\max} - W_{C\max}) \quad (2.14)$$

Compensation (reduction) of reactive power down to zero (circuit as in Fig. 2.1) minimises the RMS value of source current together with apparent power $|S|$, while active power remains unchanged; power factor goes up and attains unity. If one-phase load is non-linear, then it may be proven that reactive power does not relate in any way to energy accumulation and it may be present in purely resistance circuit [5]. Instantaneous power may also be expressed as:

$$P(t) = |V||I| \cos \varphi + |V||I| \cos(2\omega t + 2\alpha - \varphi) = P + p_P(t) \quad (2.15)$$

The first component represents active power, while the second component is alternating with amplitude equal to $|V||I|$ and corresponding to apparent power. If it is generally assumed that apparent power is a computational quantity without any physical meaning, then amplitude of alternating component defined in (2.4) may be assigned to computational quantity only. Formulas (2.4) and (2.15) show that instantaneous power may be expressed by three or two components. The number of components is influenced by mathematical approach and must not be identified with physical interpretation. We may therefore state that while instantaneous power $p(t)$ corresponds to real physical phenomena occurring in source-load

Fig. 2.2 Three-phase system under consideration



networks, the assignment of similar features to different components is, in general, not feasible. Similar interpretation of reactive power based on instantaneous power components is absolutely impossible for three-phase linear circuits in general.

For instance, if we consider symmetrical (balanced) three-phase circuit shown in Fig. 2.2.

where

$$\begin{aligned} v_a(t) &= \sqrt{2}|U| \sin \omega t, \quad v_b(t) = v_a\left(t - \frac{T}{3}\right), \quad v_c(t) = v_a\left(t + \frac{T}{3}\right), \\ Z_a &= Z_b = Z_c = |Z|e^{j\varphi} \end{aligned} \quad (2.16)$$

instantaneous power is equal to:

$$\begin{aligned} p(t) &= v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) \\ &= 2|V_a||I_a| \left[\sin \omega t \sin(\omega t - \varphi) + \sin\left(\omega t - \frac{2\pi}{3}\right) \sin\left(\omega t - \frac{2\pi}{3} - \varphi\right) \right. \\ &\quad \left. + \sin\left(\omega t + \frac{2\pi}{3}\right) \sin\left(\omega t + \frac{2\pi}{3} - \varphi\right) \right] \\ &= 3|V_a||I_a| \cos \varphi = P = \text{const} \end{aligned}$$

So, we cannot discriminate an oscillating component, which might correspond to reactive power expressed by formula:

$$Q = 3|V_a||I_a| \sin \varphi \quad (2.18)$$

On this basis alone (balanced circuit) we are able to say that there is no physical interpretation of reactive power.

To summarise: in a general case reactive power defined by formula (2.6) must be treated as some computational quantity influencing (loading) the source and decreasing its power factor. Moreover, if apparent power defined with the help of formula (2.7) for a two-terminal network is correct and not controversial, then even in case of sinusoidal three-phase networks, three different definitions of apparent power exist:

- Arithmetic apparent power [6]

$$|S_A| = |V_a||I_a| + |V_b||I_b| + |V_c||I_c| \quad (2.19)$$

- Geometric apparent power [1, 6]

$$|S_G| = \sqrt{P^2 + Q^2} \quad (2.20)$$

- Apparent power according to Buchholz [7, 8]

$$|S_B| = |\mathbf{V}||\mathbf{I}| \quad (2.21)$$

where

$|V_\alpha|, |I_\alpha|$ —RMS values of source phase voltages (currents) (Fig. 2.2) $\alpha \in a, b, c$,
 $P = \text{Re}\{\mathbf{V}^T \mathbf{I}^*\} = \text{Re}\{V_a I_a^* + V_b I_b^* + V_c I_c^*\}$ —active power,
 $Q = \text{Im}\{\mathbf{V}^T \mathbf{I}^*\} = \text{Im}\{V_a I_a^* + V_b I_b^* + V_c I_c^*\}$ —reactive power,
 $|\mathbf{V}| = \sqrt{\mathbf{V}^T \mathbf{V}}$ —RMS value of voltage vector $\mathbf{V}$,
 $|\mathbf{I}| = \sqrt{\mathbf{I}^T \mathbf{I}}$ —RMS value of current vector $\mathbf{I}$,
 $\mathbf{V}^T = [V_a, V_b, V_c]$ —transpose of a matrix of source phase voltage RMS phasors,
 $\mathbf{I}^* = \text{col}[I_a^*, I_b^*, I_c^*]$ —matrix (vector) of complex conjugates of source phasor currents

Values of apparent powers defined by (2.19–2.21) differ from each other and if active power generated by the source is identical in each case, power factors λ are different. Power factor:

$$\lambda = \frac{P}{|S|} \quad (2.22)$$

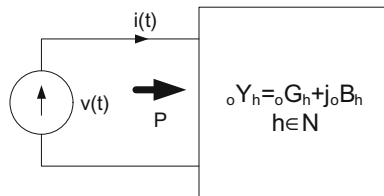
may be considered to be an indicator of source usage. Only in case of symmetrical (balanced) three-phase networks the values of apparent power are identical for all definitions.

This cursory discussion demonstrates that even where linear and sinusoidal networks are concerned, there is no single uniform interpretation of different power quantities. Therefore a universally accepted “power theory” should be based on quantities with unequivocal physical interpretation in one-phase and multi-phase systems both, with sinusoidal and distorted waveforms. In our opinion, such quantities include current, voltage, their RMS values, instantaneous power, active power and—as a computational quantity—apparent power for three-phase circuits in accordance with Buchholz’s formula, since it may be considered to be a natural generalisation of one-phase power concepts.

2.1.2 Budeanu Theory

In 1927 Budeanu presented his ideas of investigating power properties of circuits with non-sinusoidal waveforms. Power theory according to Budeanu [9] is at present the most widely accepted power theory of periodical and distorted waveforms; it has survived in spite of numerous opponents. Budeanu’s theory owes its validity to the fact that reactive power defined thereof complies with the

Fig. 2.3 One-phase linear circuit under consideration



power balance principle. This fact seems to point the scientists to some hidden physical interpretation of this power. Budeanu's theory is set down in every academic textbook's chapters on power phenomena in circuits with periodical and distorted waveforms. That is why we shall pay more attention to this theory here, showing both its merits and drawbacks.

In spite of numerous different approaches to power properties of distorted and periodical waveforms circuits, IEC debating in Stockholm in 1932 did not adopt any of the presented theories, since none offered generalisation features [10].

Let us return to Budeanu theory and consider the one-phase linear circuit shown in Fig. 2.3. Voltage $v(t)$ and current $i(t)$ are given in the form of Fourier series:

$$v(t) = V_0 + \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} V_h \exp(jh\omega t) \quad (2.23)$$

$$i(t) = I_0 + \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} I_h \exp(jh\omega t), \quad \omega = \frac{2\pi}{T} \quad (2.24)$$

where

$V_h = |V_h| \exp(j\alpha_h)$ —voltage $v(t)$ RMS phasors of h th harmonic,

$I_h = |I_h| \exp(j\beta_h)$ —current $i(t)$ RMS phasors of h th harmonic,

ω —pulsation of fundamental harmonic,

$\varphi_h = \beta_h - \alpha_h$ —load impedance phase angle for h th harmonic.

Given this couple of waveforms $v(t)$ and $i(t)$, Budeanu has defined active power P and reactive power Q_B as superposition of active and reactive powers of all $v(t)$ and $i(t)$ harmonics:

- Active power

$$P = \frac{1}{T} \int_0^T v(t)i(t)dt = V_0 I_0 + \sum_{h=1}^{\infty} |V_h| |I_h| \cos \varphi_h = \sum_{h=0}^{\infty} P_h = \operatorname{Re} \sum_{h=0}^{\infty} V_h I_h^* \quad (2.25)$$

- Reactive power

$$Q_B = \sum_{h=1}^{\infty} |V_h| |I_h| \sin \varphi_h = \sum_{h=1}^{\infty} Q_h = \operatorname{Im} \sum_{h=1}^{\infty} V_h I_h^* \quad (2.26)$$

- Apparent power

$$|S| = |V||I| = \sqrt{\sum_{h=0}^{\infty} |V_h|^2 \sum_{h=0}^{\infty} |I_h|^2} \quad (2.27)$$

As opposed to circuits with sinusoidal waveforms, the following inequality is true for defined powers P , Q_B and $|S|$:

$$|S|^2 \geq P^2 + Q_B^2 \quad (2.28)$$

In order to complete inequality (2.28), Budeanu has introduced a new quantity, called distortion power (he has not, however, defined the distortion concept) so that the following equality might be satisfied:

$$|S|^2 = P^2 + Q_B^2 + D_B^2 \quad (2.29)$$

Powers P , Q_B , D_B , $|S|$ may be shown graphically as so-called power rectangular prism (Fig. 2.4), and λ quantity:

$$\lambda = \cos \vartheta = \frac{P}{|S|} \quad (2.30)$$

is called a power factor.

Power theory elaborated by Budeanu has enjoyed a lot of support from the very beginning, but it has also been rejected by numerous opponents.

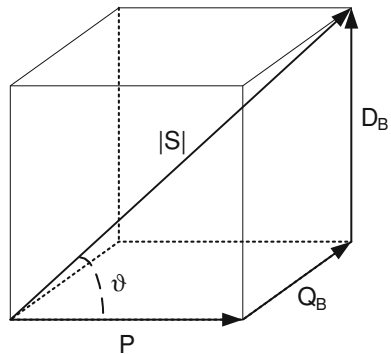
Budeanu's conception was severely criticised by Usatin [10] at IMEKO International Conference in Budapest in 1961. Usatin pointed to lack of physical interpretation of distortion power and unauthorised summing up of amplitudes of oscillating components of different harmonics. Moreover, he criticised the practical significance of this theory drawing attention to the fact that for 34 years no measurement device able to measure Q_B or D_B power has been constructed. Usatin thought it advisable to add squares of different components of reactive power Q_h . He also advocated the use of the forgotten Fryze's theory [5, 11].

Czarnecki discussed the matter further in his publication of 1987 [12]. He criticised Budeanu's theory showing its uselessness on the grounds that:

- apparent power cannot be minimised with the help of this theory, so that power factor cannot be increased;
- reactive power Q_B is not a measure of energy oscillations;
- reactive power does not make it possible to calculate the capacitance, whereat power factor attains highest possible value;
- there is no direct relation between current RMS value and distortion power D_B ;
- independent compensation of powers Q_B and D_B is not possible;
- it implies erroneous interpretation of energy phenomena in non-sinusoidal periodical circuits.

However, Czarnecki's arguments did not convince adherents of Budeanu theory and discussion is still under way (see [13, 14] as well as the latest IEEE recommendations [1]). One of the widely used arguments in favour of applicability of

Fig. 2.4 Power rectangular prism according to Budeanu



reactive power Q_B is the fact that it is subject to energy balance (power is conserved) and—at present—it is relatively simple to design Q_B measurement devices. However, this last argument should not be considered to be substantial, if the present data processing development is taken into account.

2.1.3 Fryze Theory

In 1931 Fryze proposed a novel definition of reactive power of non-sinusoidal and periodical waveforms [5, 11].

The ruling concept was:

- first of all, for any periodical current and voltage waveform measurement of $|V|$, $|I|$, active power and power factor should be made simple; power factor was defined as:

$$\lambda = \frac{P}{|S|} = \frac{\frac{1}{T} \int_0^T v(t)i(t)dt}{\sqrt{\frac{1}{T} \int_0^T v^2(t)dt} \sqrt{\frac{1}{T} \int_0^T i^2(t)dt}} \quad (2.31)$$

- next, to generalise the description of energy properties true for sinusoidal waveforms in such a way that they should also hold for any periodical waveforms.

We know that sinusoidal current may be decomposed into the sum of two reciprocally orthogonal components, i.e.:

$$i(t) = i_a(t) + i_b(t) \quad (2.32)$$

where

$i_a(t)$ —current's active component,

$i_b(t)$ —current's reactive component.

The following relation is also true for current components (it proves their orthogonality):

$$\int_0^T i_a(t) i_b(t) dt = 0 \quad (2.33)$$

and:

$$\frac{1}{T} \int_0^T v(t) i(t) dt = \frac{1}{T} \int_0^T v(t) i_a(t) dt = P \quad (2.34)$$

Active power P may also be calculated from the expression:

$$P = |V| |I_a| \quad (2.35)$$

and reactive power Q_F :

$$Q_F = |V| |I_b| \quad (2.36)$$

Identical algorithm has been applied to periodical and non-sinusoidal waveforms. The following steps in reasoning can be distinguished here:

1. Axiomatic determination of active source current in accordance with equation

$$i_a(t) = {}_eG v(t) \quad (2.37)$$

where ${}_eG$ —overall (equivalent) source conductance defined by the following relation:

$${}_eG = \frac{P}{\|v\|_{L^2}^2} = \frac{\frac{1}{T} \int_0^T v(t) i(t) dt}{\frac{1}{T} \int_0^T v^2(t) dt} = \frac{\frac{1}{T} \int_0^T v(t) i_a(t) dt}{\frac{1}{T} \int_0^T v^2(t) dt} \quad (2.38)$$

The active current defined here is characterised by minimum RMS value, while it ensures required power flow into the load.

2. Representation of source current as superposition of active and reactive current:

$$i(t) = i_a(t) + i_b(t) \quad (2.39)$$

but currents' orthogonality must be maintained:

$$\int_0^T i_a(t)i_b(t)dt = 0 \quad (2.40)$$

3. Power definition; in case of Fryze's theory power is secondary to current decomposition:

$$\|i\|_{L^2}^2 = \|i_a\|_{L^2}^2 + \|i_b\|_{L^2}^2 \quad (2.41)$$

If we multiply both sides of Eq. 2.41 by $\|v\|^2$, the following power equation is obtained:

$$|S|^2 = P^2 + Q_F^2 \quad (2.42)$$

This reactive power $Q_F = \|v\|\|i_b\|$ has been called Fryze's reactive power. However, it must be pointed out that it is not subject to conservation.

Greater functionality of Fryze's ideas as compared to Budeanu's theory is based on the fact that decomposition into active and reactive components is carried out with primary source quantities (voltage, current), and Fourier series need not be applied here, while Budeanu's theory is based on apparent power.

Fryze was deeply opposed to the idea of elaborating power theory on the basis of Fourier series; he pointed out that taking into account Gibbs phenomenon at discontinuity points (jumps), it is not possible to minimise error produced by approximating a given function with Fourier series. Fryze's concept makes it easy to account for reactive current component, both analytically and by measurement [15, 16]. However, it does not demonstrate its physical sense, apart from the fact of excessive loading of the source. It does not provide any information about how to compensate this component with the help of two-terminal reactance networks. This current can be compensated in linear circuits by applying a controlled source with current value $i_k = -i_b$ (Fig. 2.5). This source is called active power filter.

These filters are expensive and therefore other methods of arriving at optimum system working point are used. *LC* compensators and hybrid compensators are applied [17–20].

2.1.4 Shepherd and Zakikhani Theory

Attention must be paid to the Shepherd and Zakikhani conception [21], even though its authors restricted its application to one-phase circuits. The current source has been decomposed into two components:

$$i(t) = i_R(t) + i_r(t) \quad (2.43)$$

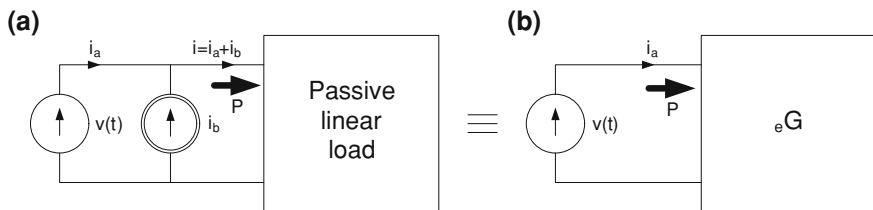


Fig. 2.5 Idea of compensation according to Fryze's theory

where

$$i_R(t) = \sqrt{2} \sum_{h=1}^{\infty} |I_h| \cos \varphi_h \cos(h\omega t + \alpha_h) \text{—current resistance component,}$$

$$i_r(t) = \sqrt{2} \sum_{h=1}^{\infty} |I_h| \sin \varphi_h \sin(h\omega t + \alpha_h) \text{—current reactance component,}$$

$\alpha_h - \arg V_h$, $\varphi_h - \angle(V_h, I_h)$, and these currents are reciprocally orthogonal.

$$\int_0^T i_R(t) i_r(t) dt = 0 \quad (2.44)$$

Apparent power equation may be derived from (2.43) to (2.44):

$$|S|^2 = \|v\|^2 \|i\|^2 = \|v\|^2 \|i_R\|^2 + \|v\|^2 \|i_r\|^2 = S_R^2 + Q_r^2 \quad (2.45)$$

The active power P is not present in this equation. This was one of reasons why decomposition as per (2.43) has been severely criticised. Moreover, powers S_R and Q_r are not subject to power balance.

If we adopt decomposition into reciprocally orthogonal components (which is not always true [22]) and work out new power theories in accordance with these decompositions, then it seems that reactive power Q_r (see 2.45) as proposed by Shepherd and Zakikhani is most appropriate. Sharon [23] has modified (2.45), introducing active power as supplementary power. Apparent power equation may then be expressed as:

$$|S|^2 = P^2 + Q_r^2 + S_c^2, \text{ where } S_c^2 = S_r^2 - P^2 \text{—supplementary power.}$$

The authors of the above concepts did not propose any physical interpretation of S_R or S_C powers.

Interpretation of currents defined with (2.43) is easy for linear, stationary, lumped elements loads described with admittances ${}_0Y_h = {}_0G_h + j{}_0B_h$ (Fig. 2.3). The discussed currents may then be defined with the following relations:

$$i_R(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} {}_0G_h V_h \exp(jh\omega t) \quad (2.46)$$

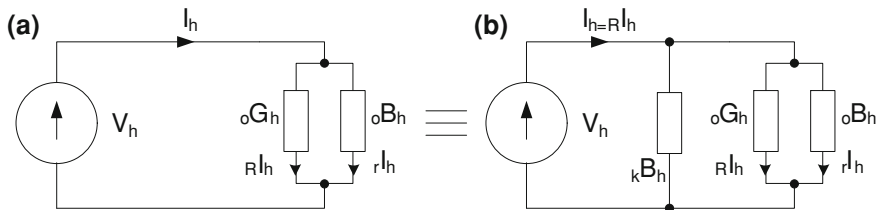


Fig. 2.6 Illustration of Shepherd's and Zakikhani's decomposition of source current

$$i_r(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} j_0 B_h V_h \exp(jh\omega t) \quad (2.47)$$

Current defined by (2.47) is called reactance (reactive) current and may be physically interpreted as current related to backward flow of energy (between source and load), and its measure is the reactive power Q_r . Current $i_r(t)$ may be compensated for a finite number of harmonics with the help of a two-terminal network connected to the load in parallel (Fig. 2.6b) [24]; as each considered harmonic susceptance is equal to ${}_k B_h = -{}_o B_h$. This property has been originally observed by Emanuel [24]. Basing on Shepherd and Zakikhani theory we may determine compensating capacitor's capacity, the so-called optimum capacity whereat the source factor is maximum:

$$C_{\text{opt}} = \frac{\sum_{h=1}^{\infty} h |V_h| |I_h| \sin \varphi_h}{\omega \sum_{h=1}^{\infty} h^2 |V_h|^2} \quad (2.48)$$

Among the merits of this concept we can count the following:

- Definition of current $i_r(t)$, which may be compensated for a finite number of harmonics by a reactance two-terminal network;
- Determination of so-called optimum capacity value

Among its faults are:

- Active power is not present in apparent power equation;
- S_R , Q_R powers do not fit into the balance of energy;
- This theory does not cover more complex circuits than one-phase systems, even though with Sharon-added modifications active power is displayed in apparent power equation.

2.1.5 Kusters and Moore Theory

The next noteworthy theory is the one worked out by Kusters and Moore. In 1980 they published a paper presenting the main points of this concept [25]. They have decomposed source current (in case of RL -type load) into active current (Fryze's current), capacitive reactive current i_{qC} and residual reactive current i_{qCr} :

$$i(t) = i_a(t) + i_{qC}(t) + i_{qCr}(t) \quad (2.49)$$

where

$$i_a(t) = \frac{P}{\|v\|^2} v(t) - \text{Fryze's activecurrent}, \quad (2.50)$$

$$i_{qC}(t) = \frac{\frac{1}{T} \int_0^T \frac{dv}{dt} i(t) dt}{\left\| \frac{dv}{dt} \right\|^2} \frac{dv}{dt} \quad (2.51)$$

$$i_{qCr}(t) = i(t) - (i_a(t) + i_{qC}(t)) \quad (2.52)$$

According to their conception these currents are reciprocally orthogonal, so that RMS values fulfil the following relation:

$$\|i\|^2 = \|i_a\|^2 + \|i_{qC}\|^2 + \|i_{qCr}\|^2 \quad (2.53)$$

Power equation is expressed as:

$$|S|^2 = P^2 + Q_C^2 + Q_{Cr}^2 \quad (2.54)$$

The authors have shown that Q_C power may be fully compensated with the help of capacitor connected to the load, capacity is equal to C_{opt} :

$$C_{opt} = - \frac{Q_c}{\left\| \frac{dv}{dt} \right\| \|v\|} \quad (2.55)$$

Similar reasoning has been proposed for RC -type load [25]. This concept has been quickly supported by IEC and gained a lot of popularity [4]. However, it has also been criticised and Willams [25] in particular has proved that not all Kusters-Moore statements are true.

2.1.6 Czarnecki Theory

Czarnecki has also shown in his works (1983) that Kusters and Moore theory does not satisfy all its presumed properties. Czarnecki has enriched both Fryze and Shepherd-Zakikhani concepts. His own theory is also based upon the source current decomposition into reciprocally orthogonal components. Czarnecki has exchanged Fryze's reactive current $i_b(t)$ for reactance current and scatter current:

$$i(t) = i_a(t) + i_b(t) = i_a(t) + (i_s(t) + i_r(t)) \quad (2.56)$$

Developing Shepherd-Zakikhani conception he exchanged resistance current for active current and scatter current:

$$i(t) = i_R(t) + i_r(t) = (i_a(t) + i_s(t)) + i_r(t) \quad (2.57)$$

obtaining the following decomposition:

$$i(t) = i_a(t) + i_s(t) + i_r(t) \quad (2.58)$$

If we assume that $v(t)$ may be expressed as:

$$v(t) = V_0 + \sqrt{2}\text{Re} \sum_{h=1}^{\infty} V_h \exp(jh\omega t) \quad (2.59)$$

then the following relations are obtained (Fig. 2.7):

$$i_a(t) = {}_eGv(t) = {}_eGV_o + \sqrt{2}\text{Re} \sum_{h=1}^{\infty} {}_eGV_h \exp(jh\omega t) \quad (2.60)$$

$$i_r(t) = \sqrt{2}\text{Re} \sum_{h=1}^{\infty} j_0B_hV_h \exp(jh\omega t) \quad (2.61)$$

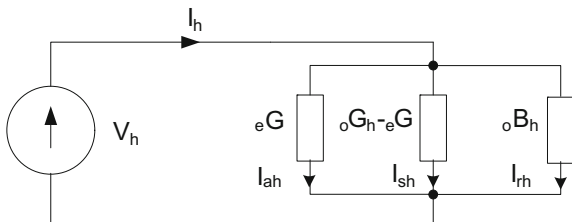
$$i_s(t) = \sqrt{2}\text{Re} \sum_{h=1}^{\infty} j_0B_hV_h \exp(jh\omega t) \quad (2.62)$$

Since currents defined in (2.58) are reciprocally orthogonal, the following relation is also true:

$$\|i\|_{L^2}^2 = \|i_a\|_{L^2}^2 + \|i_r\|_{L^2}^2 + \|i_s\|_{L^2}^2 \quad (2.63)$$

According to Czarnecki, Eq. 2.63 explains, for the first time, why RMS current value $\|i\|$ in linear loads with periodical and non-sinusoidal voltage is greater than active current RMS value $\|i_a\|$. It is greater, when $\Lambda_h B_h \neq 0$ as well when load conductance ${}_oG_h$ varies with frequency [26]. For a finite number of harmonics current $i_r(t)$ is fully compensated with a two-terminal reactance network, but current $i_s(t)$ cannot be compensated with passive LC two-terminal network.

Fig. 2.7 Illustration of Czarnecki's decomposition of source



In 1991 Czarnecki stated in [27] that it is possible to compensate current with reactance-type compensator. This statement appears to be controversial. On the basis of the discussion presented in Ref. [28] we can only say that circuit shown in Fig. 2.8 may be transformed into Fig. 2.8b circuit with the help of LC four-terminal networks. However, optimum conditions for circuit shown in Fig. 2.8a are demonstrated in Fig. 2.8c. Figure 2.8b helps to show that source current after being compensated by the reactance-type compensator may be expressed as:

$$i_{a1} = G_a v(t) \quad (2.64)$$

and from Fig. 2.8c we can see that optimum i_a current may be expressed as:

$$i_a = eGv(t) \quad (2.65)$$

In both cases source generates active power P only and source current waveform replicates voltage waveform's shape; however, minimum value of equivalent conductance G_a is dictated by oG_h maximum value. Since in general the following inequality is true:

$$G_a \geq (oG_h)_{\max} > eG \quad (2.66)$$

then it is obvious that we cannot attain optimum state by this method, since $\|i_{a1}\| > \|i_a\|$.

Moreover, it has been demonstrated in Ref. [28], that $\|i_{a1}\|$ current RMS value after compensation may be greater than $\|i\|$ source current RMS value before compensation and $P_I > P$.

Power equation must be treated as secondary product in accordance with Czarnecki conception; it may be expressed as:

$$|S|^2 = P^2 + Q_r^2 + Q_s^2 = P^2 + Q_F^2 \quad (2.67)$$

This equation will eventually lead to power rectangular prism (Fig. 2.9) different from the prism shown in Budeanu's theory; however, the sides corresponding to reactive powers Q_r and Q_s are not subject to energy balance.

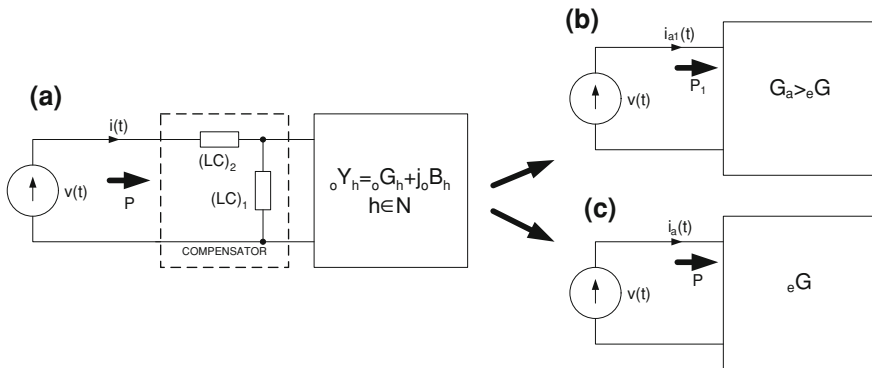
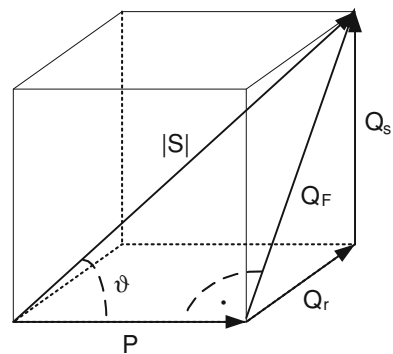


Fig. 2.8 Conception of compensating reactive and scattered currents with LC system [26, 28]

Fig. 2.9 Power rectangular prism in accordance with Czarnecki's conception



2.1.7 Optimization Theory

The idea of correlating energy and power-quality properties of a given system to solution of optimisation problem, where the input data applied uses universally accepted quantities, emerged in Institute of Electrical Circuit Theory and Engineering in 1985.

The idea may be presented as a series of claims as follows:

1. In order to characterise energy properties of non-sinusoidal circuits the following quantities are exclusively used: currents, voltages, their RMS values, instantaneous power and active power P
2. Optimum circuit current is defined as current calculated by solving optimisation problem with imposed side constraints
3. The optimisation quality indicator defined for a given circuit should make it possible to assess:

- (a) energy properties of waveforms—on the basis of RMS values and active power losses

- (b) waveform distortion (in relation to requisite sinusoidal waveform)
4. Separate set of optimum currents defines optimum circuit condition in a given sense (by defined criteria)
 5. Optimum circuit operating conditions are accomplished with the help of modifying circuits (compensators)

I. One-phase circuits supplied from ideal periodical non-sinusoidal voltage sources

Let us discuss the circuit shown in Fig. 2.3 with passive stationary, linear, lumped elements load. Load consumes active power P at a given voltage $e(t)$:

$$v(t) = \sqrt{2}\text{Re} \sum_{h=1}^n |E_h| \exp(jh\omega t) \quad (2.68)$$

Source current is calculated as:

$$i(t) = \sqrt{2}\text{Re} \sum_{h=1}^n ({}_0G_h + j_0\beta_h) V_h \exp(jh\omega t) \quad (2.69)$$

Optimisation problem for source operation conditions are formulated in the following way:

$$\min \|i\|_{L^2}^2 = \min \frac{1}{T} \int_0^T i(t)^2 dt \quad (2.70)$$

with imposed side constraint:

$$P = (v|i)_{L^2} = \frac{1}{T} \int_0^T v(t) i(t) dt \quad (2.71)$$

Lagrange functional was used to solve this problem. Minimisation of Lagrange functional expressed as:

$$\Phi(i, \lambda) = \|i\|_{L^2}^2 + \lambda(P - (v|i)_{L^2}) \quad (2.72)$$

results in discrimination of optimum current:

$$i^{\text{opt}}(t) = i_a(t) = G_e e(t) = \sqrt{2}\text{Re} \sum_{h=1}^n G_e V_h \exp(jh\omega t) \quad (2.73)$$

where

λ —Lagrange's multiplier

$$G_e = \frac{P}{\|v\|^2} \quad (2.74)$$

The form of optimum current coincides with Fryze's active current. Current difference:

$$i_b(t) = i(t) - i_a(t) \quad (2.75)$$

may be decomposed into reciprocally orthogonal components and may compensate different components or else current $i_b(t)$ may be compensated with the help of active filters. Complete compensation of $i_b(t)$ current helps to minimise source current RMS value, but it does not minimise current's distortion.

II. *One-phase circuits supplied from periodical non-sinusoidal voltage sources with non-zero internal impedance*

Let us discuss the circuit shown in Fig. 2.10 (for a specific harmonic), consisting of non-sinusoidal periodical voltage source with non-zero internal impedance and one-phase load. Let us assume that:

Let us assume that:

- source voltage may be expressed as:

$$e(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^n E_h \exp(jh\omega t) \quad (2.76)$$

- source's internal impedance belongs to stationary, linear, lumped elements class systems:

$$Z_{Zh} = R_{Zh} + jX_{Zh} \quad (2.77)$$

- load admittance belongs to stationary, linear, lumped elements class systems:

$$Y_h = G_h + jB_h \quad (2.78)$$

minimisation of Lagrange functional expressed as:

$$\Phi(i, \lambda) = \|i\|_{W^{2,\delta}(0,T)}^2 + \lambda \left(P - (v|i)_{L^2(0,T)} \right) \quad (2.79)$$

results in discrimination of optimum (active) current ${}_a i(t)$:

$${}_a i(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} {}_e G_h E_h \exp(jh\omega t) \quad (2.80)$$

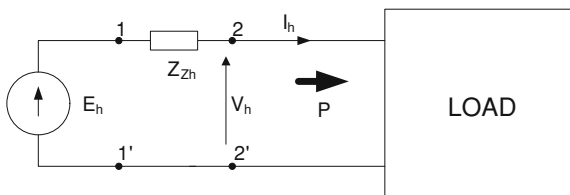
where:

$${}_e G_h = \frac{\lambda_*}{2(1 + \lambda_* R_{Zh})} \quad (2.81)$$

λ_* —positive solution of equation derived from active power balance

$$P = \sum_{h=0}^n \left[\frac{\lambda}{2(1 + \lambda R_{Zh})} - R_h \frac{\lambda^2}{4(1 + \lambda R_{Zh})^2} \right] |E_h|^2 \quad (2.82)$$

Fig. 2.10 One-phase system model for a specific (given) harmonic; source characterised by non-zero internal impedance



Taking the above into account, the remaining components may be expressed as:

- reactance component

$${}_r i(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} j B_{weh} E_h \exp(jh\omega t) \quad (2.83)$$

- scatter component

$${}_s i(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} (G_{weh} - e G_h) E_h \exp(jh\omega t) \quad (2.84)$$

The author of paper [29] has proved that different current components ${}_a i(t)_{(W)}, {}_r i(t)_{(W)}, {}_s i(t)_{(W)}$ and ${}_a i(t), {}_r i(t), {}_s i(t)$ are not reciprocally orthogonal any longer, while scatter component takes part in active energy (active power) transfer. This means that elimination of one component causes changes in the remaining ones. That is why the literature of the subject [17, 30–32] proposed a different approach for sources with non-zero internal impedance. The following series of steps has to be carried out:

- discrimination of optimum current for required optimising criterion;
- calculation of compensator's current on the basis of source current (before compensation) and optimum current;
- calculation of compensator's terminal voltage on the basis of optimum current and load constants equations;
- calculation of compensator admittance on the basis of ordered pairs compensator voltage-compensator current values for specific investigated harmonics.

III. *Three-phase circuits supplied from periodical non-sinusoidal voltage sources with non-zero internal impedance*

In this section we will show how to formulate and solve exemplary optimisation problems for a selected class of three-phase circuit with a frequency approach. Three-phase system shown in Fig. 2.11 is described with the help of following data.

$$e_x(t) = \sqrt{2} \operatorname{Re} \sum_{h=1}^{\infty} E_{xh} \exp(jh\omega t) \quad (2.85a)$$

$$e_b(t) = e_a\left(t - \frac{T}{3}\right), e_c(t) = e_a\left(t + \frac{T}{3}\right) \quad (2.85b)$$

internal impedance matrix of three-phase source:

$$\mathbf{Z}_h = \mathbf{R}_h + j\mathbf{X}_h; \mathbf{Z}_h = \mathbf{Z}_h^T, h \in N_0 \quad (2.86)$$

where superscript T denotes transpose of a load admittance matrix:

$${}_o\mathbf{Y}_h = {}_o\mathbf{G}_h + j{}_o\mathbf{B}_h; {}_o\mathbf{Y}_h = {}_o\mathbf{Y}_h^T, h \in N_0 \quad (2.87)$$

Internal impedances of both source and load are of linear, stationary, lumped elements class. Now, we will formulate the problem: **select appropriate compensators for a circuit depicted in Fig. 2.11**. Compensators admittances are defined as:

$${}_k\mathbf{Y}_{\alpha 0h} = {}_k\mathbf{G}_{\alpha 0h} + j{}_k\mathbf{B}_{\alpha 0h}, \alpha \in \{a, b, c\}, h \in N_0 \quad (2.88)$$

Compensators should be connected between a given phase and neutral conductor (it is assumed that neutral conductor's impedance is equal to zero). This modification of the circuit should result in obtaining optimum currents as in, for instance, optimisation problem **P1**.

Problem P1. Carry out minimisation of active power losses in circuit represented by $\mathbf{R}_h$:

$$\min \sum_{h=1}^n \mathbf{I}_h^T \mathbf{R}_h \mathbf{I}_h^* \quad (2.89)$$

Optimisation problem should be solved with three different sets of constraints related to active power P .

Variant A

$$P = \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{E}_h^T \mathbf{I}_h^* - \sum_{h=1}^n \mathbf{I}_h^T \mathbf{Z}_h^T \mathbf{I}_h^* \right\} = \operatorname{const} \quad (2.90)$$

This variant is widely illustrated in references [17, 30, 32, 33], it ensures that active power remains constant in a given cross-section (see Fig. 2.11) before and after compensation. It does not ensure specific active power delivered to the load after compensation. Usually load active power after compensation is greater than before.

Variant B

$$P = \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{V}_{ho}^T \mathbf{Y}_h^* \mathbf{V}_h^* \right\} = \operatorname{const} \quad (2.91)$$

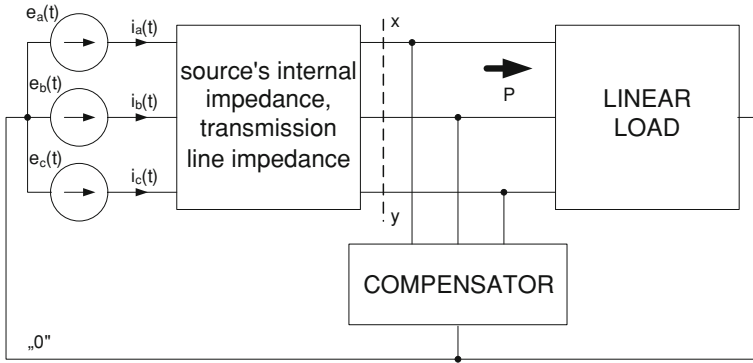


Fig. 2.11 Three-phase system under consideration

It ensures that load active power remains constant before and after compensation.

Variant C

$$\operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{E}_h^T \mathbf{I}_h^* - \sum_{h=1}^n \mathbf{I}_h^T \mathbf{Z}_h^T \mathbf{I}_h^* \right\} = \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{V}_{h0}^T \mathbf{Y}_h^* \mathbf{V}_h^* \right\} \quad (2.92)$$

Compensator does not consume active power ($P_{\text{komp}} = 0$). The following designations are used in (2.90–2.92):

$\mathbf{E}_h^T = [E_a, E_b, E_c]_h$ —transpose of matrix of RMS phasors of source voltage's h th harmonic,

$\mathbf{I}_h^T = [I_a, I_b, I_c]_h$ —transpose of matrix of RMS phasors of source current's h th harmonic,

$\mathbf{V}_h = \mathbf{E}_h - \mathbf{Z}_h \mathbf{I}_h$ —matrix of load RMS phasors for h th harmonic,

$\mathbf{I}_h^*$ —adjugate matrix of $\mathbf{I}_h$ matrix.

$$P = \sum_{\alpha=a,b,c} \frac{1}{T} \int_0^T v_\alpha(t) i_\alpha(t) dt = \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{V}_h^T \mathbf{I}_h^* \right\}, \quad \alpha \in \{a, b, c\} \quad (2.93)$$

Solving of different problems is carried out with Lagrange multipliers. For a given model (Fig. 2.11) the formulated problem **P1** and its variants should result in finding optimum currents of compensator currents with waveform in one phase (e.g. phase a).

Lagrange functional, e.g. for *variant A* of **P1** problem is expressed as:

$$\Phi(\lambda, (\mathbf{I}_h)) = \sum_{h=1}^n \mathbf{I}_h^T \mathbf{R}_h \mathbf{I}_h^* + \lambda \left(P - \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{E}_h^T \mathbf{I}_h^* - \sum_{h=1}^n \mathbf{I}_h^T \mathbf{Z}_h^T \mathbf{I}_h^* \right\} \right) \quad (2.94)$$

Arbitrary increment $\Delta \mathbf{I}_h$ is assigned to current vector $\mathbf{I}_h$, therefore $\mathbf{I}_{1h} = \mathbf{I}_h + \Delta \mathbf{I}_h$, and:

$$\Phi_1(\lambda, (\mathbf{I}_{1h})) = \sum_{h=1}^n \mathbf{I}_{1h}^T \mathbf{R}_h \mathbf{I}_{1h}^* + \lambda \left(P - \operatorname{Re} \left\{ \sum_{h=1}^n \mathbf{E}_h^T \mathbf{I}_{1h}^* - \sum_{h=1}^n \mathbf{I}_{1h}^T \mathbf{Z}_h^T \mathbf{I}_{1h}^* \right\} \right) \quad (2.95)$$

Lagrange functional increment may be expressed as:

$$\Delta \Phi = \Phi_1 - \Phi = \operatorname{Re} \sum_{h=1}^n [2\mathbf{I}_h^T (\mathbf{1} + \lambda \mathbf{R}_h) - \lambda \mathbf{E}_h^T] \Delta \mathbf{I}_h^* + \sum_{h=1}^n \Delta \mathbf{I}_h^T (\mathbf{1} + \lambda \mathbf{R}_h) \Delta \mathbf{I}_h^* \quad (2.96)$$

Sufficient condition for existence of a minimum is that:

$$\bigwedge_{h \in N_0} \Delta \Phi > 0 \quad (2.97)$$

Necessary condition for existence of a minimum is satisfying following vector equation:

$$\bigwedge_{h \in N_0} \{2\mathbf{I}_h^T (\mathbf{1} + \lambda \mathbf{R}_h) - \lambda \mathbf{E}_h^T\} = 0 \quad (2.98)$$

The sufficient condition is met for any arbitrary increment $\Delta \mathbf{I}_h$, when quadratic form denoted with (2.75) is positive, i.e.:

$$\bigwedge_{h \in N_0} \{\Delta \mathbf{I}_h^T (\mathbf{1} + \lambda \mathbf{R}_h) \Delta \mathbf{I}_h^*\} > 0 \quad (2.99)$$

When assumptions are considered, it is seen that matrix $\mathbf{R}_h$ is symmetrical and non-singular, and therefore for every $\lambda \geq 0$ the quadratic form defined by (2.99) is positive. Coefficient $\lambda \geq 0$ is derived from power condition (2.96), where $\mathbf{I}_h(\lambda)$ current is defined as follows:

$$\mathbf{I}_h(\lambda) = \frac{\lambda}{2} (\mathbf{1} + \lambda \mathbf{R}_h)^{-1} \mathbf{E}_h \quad (2.100)$$

If we continue to act in the manner shown in [17, 29–31, 33], we obtain optimum source current for a given λ_* , and this current ensures minimisation of active power losses in the circuit:

$$\bigwedge_h \left(\mathbf{I}_h^{\text{opt}} = \frac{\lambda_*}{2} (\mathbf{1} + \lambda_* \mathbf{R}_h)^{-1} \mathbf{E}_h \right) \quad (2.101)$$

Load voltage at optimum current (2.101) is equal to:

$$\mathbf{V}_h^{\text{opt}} = \mathbf{E}_h - \mathbf{Z}_h \mathbf{I}_h^{\text{opt}} = \mathbf{E}_h - \frac{\lambda_*}{2} \mathbf{Z}_h (\mathbf{1} + \lambda_* \mathbf{R}_h)^{-1} \mathbf{E}_h \quad (2.102)$$

When optimum current $\mathbf{I}_h^{\text{opt}}$ and optimum voltage $\mathbf{V}_h^{\text{opt}}$, $h \in N_0$ are known, the compensator's current vector ${}_k \mathbf{I}_h$ is calculated. The obtained matrix ${}_k \mathbf{I}_h$ is a column

matrix, same as in the case of compensator's voltage matrix $\mathbf{V}_h^{\text{opt}} = {}_k\mathbf{V}_h$. Compensator's admittance for each harmonic is calculated from the following relationship (compensator is connected between a given phase and neutral conductor):

$${}_kY_{\alpha h} = \frac{{}_kI_{\alpha h}}{{}_kV_{\alpha h}}, \alpha \in \{a, b, c\} \quad (2.103)$$

Note! Theory of orthogonal distributions is true for ideal sources only.

2.2 Instantaneous Power Theories

Instantaneous power theory is based on time domain. This enables it to analyse systems in both steady-state and transient conditions. Therefore, such theories are often used in control devices to control and improve the power quality.

2.2.1 *pq Theory*

The most currently used “power theory” (in the area of improving of power quality) was presented in Japan in 1983. Instantaneous power theory proposed by Akagi and Nabae, can only be applied to the analysis of three-phase systems [34], and therefore does not have the characteristics of the general theory of power [35, 36] explain many physical phenomena), but is characterised by many advantages, of which the most important is the ability to calculate optimal current (in a given sense) with simple mathematical operations.

This theory is based on a scalar transformation of three-phase phase voltage $[v_a, v_b, v_c]^T$ and load currents $[i_{oa}, i_{ob}, i_{oc}]^T$ for the rectangular coordinate system $\alpha - \beta - 0$ (Fig. 2.12). This transformation is performed converting the instantaneous values using the formula (2.104):

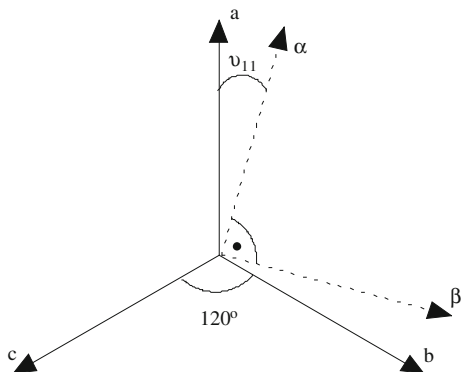
$$\begin{bmatrix} F_\alpha \\ F_\beta \\ F_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos v_{11} & \cos v_{12} & \cos v_{13} \\ -\sin v_{11} & -\sin v_{12} & -\sin v_{13} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \quad (2.104)$$

where: v_{Ix} —angle between the axis $\mathbf{x}$ ($\mathbf{x} = \mathbf{a}, \mathbf{b}, \mathbf{c}$) of the natural three-phase system and the α axis of the rectangular coordinate system (Fig. 2.12).

When axes $\mathbf{a}$ and α overlap (i.e. if $v_{11} = 0$) the transformation matrix assumes the following form:

$$\begin{bmatrix} F_\alpha \\ F_\beta \\ F_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos 0 & \cos \frac{4}{3}\pi & \cos \frac{2}{3}\pi \\ -\sin 0 & -\sin \frac{4}{3}\pi & -\sin \frac{2}{3}\pi \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \quad (2.105)$$

Fig. 2.12 Illustration of the transformation from $a - b - c$ to $\alpha - \beta$ coordinate system



which leads to:

$$\begin{bmatrix} F_\alpha \\ F_\beta \\ F_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \quad (2.106)$$

For three-phase system of phase voltages and currents the instantaneous power expressed by the instantaneous phase currents and voltages in a $a-b-c$ system may be determined by the formula:

$$p = v_a i_{oa} + v_b i_{ob} + v_c i_{oc} \quad (2.107)$$

After transform voltage and current phase of a three-phase system for the rectangular coordinate system $\alpha - \beta - 0$ using the formula (2.106), the instantaneous power (due to the orthogonality of transformation) preserves the form (in the new coordinate system), i.e.:

$$p = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \quad (2.108)$$

In most cases, the transfer of energy in the medium voltage network is realised via three-wire line (with a symmetrical sinusoidal voltage source), then the transformation matrix can be omitted of elements v_0 , i_0 , and the zero component of instantaneous power.

If the p_α and p_β denote the instantaneous axis powers in α and β , then the instantaneous power can be represented as:

$$\begin{aligned} p &= p_\alpha + p_\beta = v_\alpha i_{\alpha p} + v_\alpha i_{\alpha q} + v_\beta i_{\beta p} + v_\beta i_{\beta q} \\ &= v_\alpha \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} p - v_\alpha \frac{v_\beta}{v_\alpha^2 + v_\beta^2} q + v_\beta \frac{v_\beta}{v_\alpha^2 + v_\beta^2} p + v_\beta \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} q \\ &= p_{\alpha p} + p_{\alpha q} + p_{\beta p} + p_{\beta q} \end{aligned} \quad (2.109)$$

where:

$i_{\alpha p}$ —instantaneous active current in the α axis,
 $i_{\beta p}$ —instantaneous active current in the β axis,
 $i_{\alpha q}$ —instantaneous reactive current in the α axis,
 $i_{\beta q}$ —instantaneous reactive current in the β axis,
 $p_{\alpha p}$ —instantaneous active power in the α axis,
 $p_{\alpha q}$ —instantaneous reactive power in the α axis,
 $p_{\beta p}$ —instantaneous active power in the β axis,
 $p_{\beta q}$ —instantaneous reactive power in the β axis.

In this decomposition the sum of power components:

$$p_{\alpha q} + p_{\beta q} = 0 \quad (2.110)$$

These components (called instantaneous reactive powers) cancel each other and do not participate in the transfer of energy from source to load. The sum of the other two components (called instantaneous active powers):

$$p = p_{\alpha p} + p_{\beta p} \quad (2.111)$$

is identical to the standard interpretation of the instantaneous power used in the three-phase circuits, and its average value is the active power P .

Conventional passive power [9] is defined in the frequency domain and cannot in any way be compared with the instantaneous values defined in time domain (one can compare the average values for the period of the instantaneous power course with P and with Q).

The authors of the instantaneous power theory, introduced an entirely new concept—the instantaneous imaginary power (this power unit with an analogy to the *var* is marked as *vai*, or volt-ampere-imaginary). Instantaneous imaginary power is computed using the formula:

$$q = v_{\alpha} i_{\beta} - v_{\beta} i_{\alpha} \quad (2.112)$$

This component is treated as an undesirable element (it must be eliminated from the system), but it has no physical interpretation.

The main advantage of the use of rectangular coordinates α – β is the ability to write simple equations for the phase currents. Source current of three-phase three-wire system, transformed into α – β system can be written as:

$$\begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} = \frac{1}{v_{\alpha}^2 + v_{\beta}^2} \begin{bmatrix} v_{\alpha} & -v_{\beta} \\ v_{\beta} & v_{\alpha} \end{bmatrix} \begin{bmatrix} \bar{p} + \tilde{p} \\ \bar{q} + \tilde{q} \end{bmatrix} \quad (2.113)$$

where $\bar{x}$ —DC component of the instantaneous power, $\tilde{x}$ —AC component of the instantaneous power.

For a symmetrical voltage source:

$$v_{\alpha}^2 + v_{\beta}^2 = 3|E_a|^2 = \text{const} \quad (2.114)$$

One of the tasks of optimisation may be elimination of unwanted components of the current source so that the current source was active. This task can be achieved using an active power filter. Using formula (2.115) can be calculated compensator currents (active power filter) to elimination of unnecessary parts (all or selected), leaving only the DC component of the instantaneous active power (desired), according to the formula:

$$\begin{bmatrix} i_{\alpha k} \\ i_{\beta k} \end{bmatrix} = \frac{1}{v_{\alpha}^2 + v_{\beta}^2} \begin{bmatrix} v_{\alpha} & -v_{\beta} \\ v_{\beta} & v_{\alpha} \end{bmatrix} \begin{bmatrix} p_k \\ q_k \end{bmatrix} \quad (2.115)$$

Depending on which component you want to eliminate, instead of p_k and q_k insert value from Table 2.1.

Instantaneous power theory should not be used in three-phase systems with asymmetric or distorted supply voltage pq theory has disadvantages as a correct power theory, as has been demonstrated in several publications [35, 36]. However, it is now often used as a useful control algorithm for active power filters.

In case where non-linear load is powered from distorted voltage source even after compensation source current will contain components related to the higher voltage harmonics. These deformations are caused by incorrect calculation of optimal currents.

2.2.2 Extensions pq Theory

A simple and effective method of analysis based on the pq theory proposed by Akagi [34] does not work in systems with unbalanced supply voltage. Since systems with small power asymmetries occur quite often we need a different approach to the analysis of such a system. In 1995, Komatsu and Kawabata [37–39] presented instantaneous power theory called the “extension pq ”. This theory uses a broader approach to ensure proper analysis of both the asymmetry and low distortion voltage.

In this theory, the instantaneous powers: active and passive are defined:

$$p = v_a i_a + v_b i_b + v_c i_c \quad (2.116)$$

$$q = v'_a i_a + v'_b i_b + v'_c i_c \quad (2.117)$$

where the transverse voltages v'_a, v'_b, v'_c are determined by the transfer phase voltages v_a, v_b, v_c separately for each phase of constant angle $\pi/2$.

In addition to the three-phase three-wire systems (such systems are the most common) can be written (according to Kirchhoff's first law):

$$i_a + i_b + i_c = 0 \quad (2.118)$$

Taking into account formula (2.118), we can simplify the formulas (2.116, 2.117) to form:

Table 2.1 Types of components to be eliminated from the instantaneous power

Component of the current source	p_k	q_k
Component associated with the instantaneous imaginary power	0	q
Component of the opposite order and higher harmonics	$\tilde{p}$	$\tilde{q}$
Component associated with the fixed component of the instantaneous imaginary power	0	$\tilde{q}$
Optimum variant—full compensation	$\tilde{p}$	q
components associated with the instantaneous imaginary power and higher harmonics		
Component of the opposite order	$p_{2\omega}$	$q_{2\omega}$
Components associated with higher harmonics	p_h	q_h
Component associated with the variable component of the instantaneous active power	$\tilde{p}$	0

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_a - v_c & v_b - v_c \\ v'_a - v'_c & v'_b - v'_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \quad (2.119)$$

This formula allows you to specify the energy state of the system (calculation of instantaneous power p and q). Transforming this formula allows to calculate the value of source currents.

$$\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} v'_b - v'_c & v_c - v_b \\ v'_c - v'_a & v_a - v_c \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \quad (2.120)$$

$$i_c = -i_a - i_b$$

where

$$\Delta = (v_a - v_c)(v'_b - v'_c) - (v'_a - v'_c)(v_b - v_c) \quad (2.121)$$

Analysing the form of formula (2.120) can be seen similar to the corresponding formula in the pq theory (2.122)

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{\Delta'} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \quad (2.122)$$

where

$$\Delta' = v_\alpha^2 + v_\beta^2 \quad (2.123)$$

Currents obtained from formula (2.122) should be transformed to the **a-b-c** system. At the same time, you can prove that this theory is equivalent to pq theory in the field of symmetrical voltage power system, i.e. if:

$$v_a + v_b + v_c = 0 \quad (2.124)$$

Then the transverse voltages can be written as:

$$v'_a = \frac{v_c - v_b}{\sqrt{3}} \quad (2.125)$$

$$v'_b = \frac{v_a - v_c}{\sqrt{3}} \quad (2.126)$$

$$v'_c = \frac{v_b - v_a}{\sqrt{3}} \quad (2.127)$$

and phase current in phase a can be written as:

$$i_a = \frac{2v_b p + v_a(-p + \sqrt{3}q) - v_c(p + \sqrt{3}q)}{2(v_a^2 + v_b^2 - v_b v_c + v_c^2 - v_a(v_b + v_c))} \quad (2.128)$$

By contrast, given that in pq theory:

$$v_\alpha = \sqrt{\frac{2}{3}} \left(v_a - \frac{1}{2}v_b - \frac{1}{2}v_c \right) \quad (2.129)$$

$$v_\beta = \sqrt{\frac{2}{3}} \left(\frac{\sqrt{3}}{2}v_b - \frac{\sqrt{3}}{2}v_c \right) \quad (2.130)$$

this current phase a (in theory pq) can be written as:

$$i_a = \frac{2v_b p + v_a(-p + \sqrt{3}q) - v_c(p + \sqrt{3}q)}{2(v_a^2 + v_b^2 - v_b v_c + v_c^2 - v_a(v_b + v_c))} \quad (2.131)$$

Comparing formulas (2.128) and (2.131) can be seen that they are identical. Proceeding similarly with the currents in other phases we receive the same results. This demonstrates the broader approach in the theory of “extension pq ”, and the equivalence of these theories in the case of symmetric voltages source (because the formulas (2.125–2.127) are true in this case).

2.2.3 Synchronous Reference Frame Theory

This generalisation of the pq theory should be applied especially in cases of deformed voltage systems. This approach uses the transformation [40] vectors of the input signal from the natural three-phase system to a rotating $d-q$ coordinate system as shown in Fig. 2.13.

Transformation is carried out in two stages, the first stage transforms the vectors of the three-phase system to rectangular coordinate system $\alpha - \beta$ similarly as in pq theory [34], i.e. according to dependence:

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \quad (2.132)$$

In the next step vectors are transformed into $\mathbf{d} - \mathbf{q}$ rotating system. Values in the new coordinate system are derived from the dependence:

$$\vec{F}_{dq}^> = \vec{F}_{\alpha\beta}^> e^{-j\theta} \quad (2.133)$$

leads to:

$$\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} \quad (2.134)$$

This transformation in the literature is also called Park transformation.

Similarly, the inverse transformation in the first place requires a transformation of vectors from the $\mathbf{d} - \mathbf{q}$ rotating to rectangular $\alpha - \beta$ coordinate system:

$$\begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix} \quad (2.135)$$

Then the transformation to the natural three-phase system:

$$\begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} F_\alpha \\ F_\beta \end{bmatrix}. \quad (2.136)$$

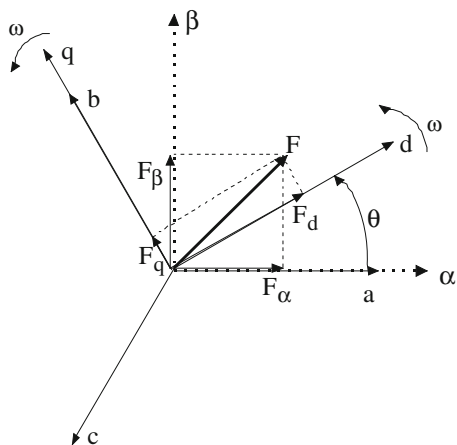
The advantage of this transformation is that the system does not require calculation of instantaneous active and reactive power. If the course function $\cos\theta$ coincides with the course of the basic harmonic voltage phase a , then the system dq rotates synchronously with the basic harmonic voltage course. In this reference system components that are in phase with the fundamental harmonic voltage (e.g., active current), are represented by constant values. In this case, the mean value of current component of $\mathbf{d}$ -axis (Fig. 2.13) corresponds to the active component of source current in the rotating coordinate system [41]. Therefore, the optimal values of instantaneous currents can be calculated directly from knowledge of the components of phase currents in the new coordinate system ($\mathbf{d}$ and $\mathbf{q}$). This follows from the fact that the only desired component is a constant (DC) component of i_d :

$$\begin{bmatrix} i_{ka} \\ i_{kb} \\ i_{kc} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \tilde{i}_d \\ i_q \end{bmatrix} \quad (2.137)$$

where $\tilde{i}_d$ —AC component of i_d current component.

Eliminate the calculation of instantaneous active and reactive power values, causing significant reduction in the number of required mathematical operations in each calculation cycle. This leads to improved dynamic properties of the system. Use PLL to determine θ angle ensures correct calculation of the optimal currents and compensation currents, even in the case of distorted periodic voltage. In the case of a symmetric voltage, this method is equivalent to the method proposed by Akagi [34].

Fig. 2.13 Illustration of transformation from a three-phase $a - b - c$ system to rotating $d - q$ system



2.3 Power Theory on Basis of Orthogonal Components

The CPC theory [42] proposed by L. Czarnecki, uses a frequency domain for description of system working point. In this theory source current orthogonal decomposition has been used. These components are referred to physical interpretation for the phenomena in electrical circuits. Basic to this theory for single-phase systems have been presented already in 1984, a complete theory for the different supply conditions and different loads was published in 1994. It combines the ideas proposed by Fryze (time domain) and Shepherd and Zakikhani (frequency domain). Power theory developed by Czarnecki select from these theories this elements that allow to explain energy phenomenas in electrical circuits and establish the theoretical basis of their compensation. To describe current and voltages waveforms Fourier series are used, for example phase voltage is expressed in the form of a column vector $\mathbf{u}$:

$$\mathbf{V} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2} \text{Re} \left[\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} e^{j\omega t} \right] = \sqrt{2} \text{Re} \mathbf{V} e^{j\omega t} \quad (2.138)$$

Phase currents are presented, analogous to voltage, using a column vector $\mathbf{i}$:

$$\mathbf{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{2} \text{Re} \left[\begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} e^{j\omega t} \right] = \sqrt{2} \text{Re} \mathbf{I} e^{j\omega t} \quad (2.139)$$

In the example system shown in Fig. 2.14 (with sinusoidal voltages and currents) current source can be presented as:

$$\mathbf{i} = \sqrt{2}\text{Re}\{[G_e + jB_e)\mathbf{V} + A\mathbf{V}^\#]e^{j\omega t}\} \quad (2.140)$$

where

$$G_e + jB_e = Y_{ab} + Y_{bc} + Y_{ac} \equiv Y_e \quad (2.141)$$

Y_e —equivalent admittance.

$$A \equiv -(Y_{bc} + \alpha Y_{ac} + \alpha^* Y_{ab}) \quad (2.142)$$

In this case, the author of CPC power theory proposed a source decomposition into three orthogonal components:

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_r + \mathbf{i}_u \quad (2.143)$$

Each of them is related to another energy phenomenon, and so:

- Component $\mathbf{i}_a$ called “active current”

$$\mathbf{i}_a = \sqrt{2}\text{Re}\{G_e \mathbf{V} e^{j\omega t}\} \quad (2.144)$$

is related to permanent energy conversion.

- Component $\mathbf{i}_r$ called “reactive current”

$$\mathbf{i}_r = \sqrt{2}\text{Re}\{B_e \mathbf{V} e^{j\omega t}\} \quad (2.145)$$

is related to phase-shift.

- Component $\mathbf{i}_u$ called “unbalanced current”

$$\mathbf{i}_u = \sqrt{2}\text{Re}\{A\mathbf{V}^\# e^{j\omega t}\} \quad (2.146)$$

is related to load imbalance.

If a system contains non-linear loads and waveforms of currents and voltages are distorted, then the supply voltage is described by the following equation:

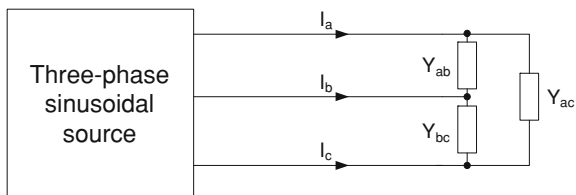
$$\mathbf{V} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}\text{Re} \sum_{n \in N} \begin{bmatrix} \frac{V_a}{V_b} \\ \frac{V_b}{V_c} \\ \frac{V_c}{V_a} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\text{Re} \sum_{n \in N} \underline{\mathbf{V}}_n e^{jn\omega_1 t} \quad (2.147)$$

Thus, according to the theory of current physical components, source current was in this case, distributed into four components:

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_s + \mathbf{i}_r + \mathbf{i}_u \quad (2.148)$$

These components are mutually orthogonal, i.e.:

Fig. 2.14 Example system with three-phase load



$$\|\mathbf{i}\|^2 = \|\mathbf{i}_a\|^2 + \|\mathbf{i}_s\|^2 + \|\mathbf{i}_r\|^2 + \|\mathbf{i}_u\|^2 \quad (2.149)$$

And each of them is related to another energy phenomenon:

$$\mathbf{i}_a = \sqrt{2}\text{Re}\left\{\sum_{n \in N} G_e \mathbf{V}_n e^{jn\omega_1 t}\right\} \quad (2.150)$$

Active current is responsible for the flow of energy from a power source to the receiver. This occurs when the receiver has non-zero active power:

$$\mathbf{i}_s = \sqrt{2}\text{Re}\left\{\sum_{n \in N} (G_{en} - G_e) \mathbf{V}_n e^{jn\omega_1 t}\right\} \quad (2.151)$$

This component (scattered current) does not participate in energy flow from a source to the load. It is due to change in load conductance G_{en} with a row of harmonic n .

$$\mathbf{i}_r = \sqrt{2}\text{Re}\left\{\sum_{n \in N} B_{en} \mathbf{V}_n e^{jn\omega_1 t}\right\} \quad (2.152)$$

Reactive current occurs when there is a phase shift between voltages and current harmonics (related to load susceptance), this current component does not participate in energy transfer from a source to the load.

$$\mathbf{i}_u = \sqrt{2}\text{Re}\left\{\sum_{n \in N} A_n \mathbf{V}_n^\# e^{jn\omega_1 t}\right\} \quad (2.153)$$

Imbalance of the receiver causes the appearance of unbalanced current, it occurs only in three-phase systems, like in the previous component which also does not participate in energy transfer from a source to the load.

The theory proposed by Czarnecki was introduced to allocate the constituent phenomena in electrical systems, so that they can be correctly interpreted. However, its practical use in systems to improve power quality is difficult to achieve. Already the author in one of his publications shows that the compensator using this theory would be very complicated. Also applied is the theory of assumption of the orthogonality components. This theory used in other theories, was in the past often criticised by many scientists.

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Power Theories for Improved Power Quality

Benysek, G.; Pasko, M. (Eds.)

2012, X, 214 p., Hardcover

ISBN: 978-1-4471-2785-7