

Preface

Conic optimization is a significant and thriving research area within the optimization community. Conic optimization is the general class of problems concerned with optimizing a linear function over the intersection of an affine space and a closed convex cone. One special case of great interest is the choice of the cone of positive semidefinite matrices for which the resulting optimization problem is called a semidefinite optimization problem.

Semidefinite optimization, or semidefinite programming (SDP), has been studied (under different names) since at least the 1940s. Its importance grew immensely during the 1990s after polynomial-time interior-point methods for linear optimization were extended to solve SDP problems (and more generally, to solve convex optimization problems with efficiently computable self-concordant barrier functions). Some of the earliest applications of SDP that followed this development were the solution of linear matrix inequalities in control theory, and the design of polynomial-time approximation schemes for hard combinatorial problems such as the maximum-cut problem.

This burst of activity in the 1990s led to the publication of the *Handbook of Semidefinite Programming* in the year 2000. That Handbook, edited by Wolkowicz, Saigal, and Vandenberghe, provided an overview of much of the activity in the area.

Research into semidefinite programming has continued unabated, and a new development since the beginning of the twenty-first century has been the fruitful interaction with algebraic geometry through the close connections between semidefinite matrices and polynomial optimization problems. This has brought about several important new results and led to an even higher level of research activity. Much of this activity can be followed on the *Optimization Online* (<http://www.optimization-online.org>) and *ArXiv* (<http://arxiv.org>) websites.

The objective of this *Handbook on Semidefinite, Conic and Polynomial Optimization* is to provide the reader with a snapshot of the state of the art in the growing and mutually enriching areas of semidefinite optimization, conic optimization, and polynomial optimization. Our intention is to provide a compendium of the research activity that has taken place since the publication of the seminal Handbook

mentioned above. It is our hope that this will motivate more researchers, especially doctoral students and young graduates, to become involved in these thrilling areas of optimization.

Overview of the Handbook

The Handbook begins with a chapter presenting the basics of semidefinite, conic, and polynomial optimization. The subsequent 30 chapters are grouped into four parts: *Theory*, *Algorithms*, *Software*, and *Applications*.

Theory

This first part represents approximately one-third of the Handbook. It covers many significant theoretical developments, and several chapters reflect the interactions between conic optimization and polynomial optimization.

Algorithms

This second part documents a number of different directions in which the development of algorithms is taking place. It indicates the breadth of approaches being applied to solve conic optimization problems, including both interior-point methods and more recent approaches.

Software

It is a sign of the maturity of the field that there are now many software packages to solve small- and medium-sized semidefinite optimization problems. The first chapter of this part provides an overview of the state of the art, while the subsequent chapters document the latest developments in three commonly used software packages.

There are also a number of interfaces that facilitate the use of conic optimization software. We have chosen not to include these in the Handbook in order to keep the focus on the theoretical and algorithmic concepts behind the solvers, and thus to help guide the reader to the most appropriate approaches for specific applications.

Like all other aspects of the field, the software offerings are in constant evolution. As a starting point for the interested reader, we provide the URL for the software section of the *Semidefinite Programming* webpage maintained by Christoph Helmberg: <http://www-user.tu-chemnitz.de/~helmberg/sdp-software.html>.

Applications

Finally, the fourth part is concerned with some of the application areas where conic optimization has made a significant impact in recent years. Several of these involve hard combinatorial optimization problems that continue to benefit from the advances in theory, algorithms, and software mentioned in the previous parts.

Acknowledgements This Handbook benefited tremendously from the generous help of all those who kindly agreed to referee one or more chapters, assisting us and the authors to significantly improve the content. They were: Kurt Anstreicher, Michel Baes, Frédéric Bonnans, Valentin Brimkov, Samuel Burer, Brian Borchers, Chek-Beng Chua, Mirjam Dür, Alexander Engau, Anja Fischer, Mituhiro Fukuda, João Gouveia, Luigi Grippo, Stefano Gualandi, Christoph Helmberg, Didier Henrion, Philipp Hungerländer, Michal Kocvara, Nathan Krislock, William Martin, John Mitchell, Baback Moghaddam, Jiawang Nie, Javier Peña, Veronica Piccialli, Daniel Plaumann, Mihai Putinar, Houduo Qi, Grant Schoenebeck, Frank Sottile, David Steurer, Hajime Tanaka, Thorsten Theobald, and Manuel Vieira.

We are also grateful for the support of GERAD in the compilation of this Handbook, and particularly the hard work and dedication of Ms. Marilyne Lavoie.

Finally, financial support from the Alexander von Humboldt Foundation and the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged by the first editor.

Montréal and Toulouse

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Handbook on Semidefinite, Conic and Polynomial
Optimization

Anjos, M.F.; Lasserre, J.B. (Eds.)

2012, XI, 960 p., Hardcover

ISBN: 978-1-4614-0768-3