

Contents

1	Introduction	1
2	Review of Linear Algebra	3
2.1	Basic Definitions and Notation	3
2.2	Complex Inner Product Spaces	5
2.3	Further Notions from Linear Algebra	7
3	Group Representations	13
3.1	Basic Definitions and First Examples	13
3.2	Maschke's Theorem and Complete Reducibility	20
4	Character Theory and the Orthogonality Relations	27
4.1	Morphisms of Representations	27
4.2	The Orthogonality Relations	31
4.3	Characters and Class Functions	35
4.4	The Regular Representation	41
4.5	Representations of Abelian Groups	47
5	Fourier Analysis on Finite Groups	51
5.1	Periodic Functions on Cyclic Groups	51
5.2	The Convolution Product	52
5.3	Fourier Analysis on Finite Abelian Groups	54
5.4	An Application to Graph Theory	58
5.5	Fourier Analysis on Non-abelian Groups	64
6	Burnside's Theorem	71
6.1	A Little Number Theory	71
6.2	The Dimension Theorem	74
6.3	Burnside's Theorem	78
7	Group Actions and Permutation Representations	83
7.1	Group Actions	83
7.2	Permutation Representations	85
7.3	The Centralizer Algebra and Gelfand Pairs	90

8	Induced Representations	97
8.1	Induced Characters and Frobenius Reciprocity	97
8.2	Induced Representations	100
8.3	Mackey's Irreducibility Criterion	104
9	Another Theorem of Burnside	111
9.1	Conjugate Representations.....	111
10	Representation Theory of the Symmetric Group	117
10.1	Partitions and Tableaux	117
10.2	Constructing the Irreducible Representations.....	122
11	Probability and Random Walks on Groups	131
11.1	Probabilities on Groups	132
11.2	Random Walks on Finite Groups	136
11.3	Card Shuffling	139
11.3.1	The Riffle Shuffle	140
11.4	The Spectrum and the Upper Bound Lemma	144
	References	153
	Index	155

<http://www.springer.com/978-1-4614-0775-1>

Representation Theory of Finite Groups

An Introductory Approach

Steinberg, B.

2012, XIII, 157 p. 4 illus., Softcover

ISBN: 978-1-4614-0775-1