Preface

A student's first encounter with abstract algebra is usually in the form of linear algebra. This is typically followed by a course in group theory. Therefore, there is no reason a priori that an undergraduate could not learn group representation theory before taking a more advanced algebra course that covers module theory. In fact, group representation theory arguably could serve as a strong motivation for studying module theory over non-commutative rings. Also group representation theory has applications in diverse areas such as physics, statistics, and engineering, to name a few. One cannot always expect students from these varied disciplines to have studied module theory at the level needed for the "modern" approach to representation theory via the theory of semisimple algebras. Nonetheless, it is difficult, if not impossible, to find an undergraduate text on representation theory assuming little beyond linear algebra and group theory in prerequisites, and also assuming only a modest level of mathematical maturity.

During the Winter term of 2008, I taught a fourth year undergraduate/first year graduate course at Carleton University, which also included some third year students (and even one exceptionally talented second year student). After a bit of searching, I failed to find a text that matched the background and level of mathematical sophistication of my students. Faced with this situation, I decided to provide instead my own course notes, which have since evolved into this text. The goal was to present a gentle and leisurely introduction to group representation theory, at a level that would be accessible to students who have not yet studied module theory and who are unfamiliar with the more sophisticated aspects of linear algebra, such as tensor products. For this reason, I chose to avoid completely the Wedderburn theory of semisimple algebras. Instead, I have opted for a Fourier analytic approach. This sort of approach is normally taken in books with a more analytic flavor; such books, however, invariably contain material on the representation theory of compact groups, something else that I would consider beyond the scope of an undergraduate text. So here I have done my best to blend the analytic and the algebraic viewpoints in order to keep things accessible. For example, Frobenius reciprocity is treated from a character point of view to avoid use of the tensor product.

The only background required for most of this book is a basic knowledge of linear algebra and group theory, as well as familiarity with the definition of a ring. In particular, we assume familiarity with the symmetric group and cycle notation. The proof of Burnside's theorem makes use of a small amount of Galois theory (up to the fundamental theorem) and so should be skipped if used in a course for which Galois theory is not a prerequisite. Many things are proved in more detail than one would normally expect in a textbook; this was done to make things easier on undergraduates trying to learn what is usually considered graduate level material.

The main topics covered in this book include: character theory; the group algebra and Fourier analysis; Burnside's pq-theorem and the dimension theorem; permutation representations; induced representations and Mackey's theorem; and the representation theory of the symmetric group. The book ends with a chapter on applications to probability theory via random walks on groups.

It should be possible to present this material in a one semester course. Chapters 2–5 should be read by everybody; it covers the basic character theory of finite groups. The first two sections of Chap. 6 are also recommended for all readers; the reader who is less comfortable with Galois theory can then skip the last section of this chapter and move on to Chap. 7 on permutation representations, which is needed for Chaps. 8–10. Chapter 10, on the representation theory of the symmetric group, can be read immediately after Chap. 7. The final chapter, Chap. 11, provides an introduction to random walks on finite groups. It is intended to serve as a non-trivial application of representation theory, rather than as part of the core material of the book, and should therefore be taken as optional for those interested in the purely algebraic aspects of the theory. Chapter 11 can be read directly after Chap. 5, as it relies principally on Fourier analysis on abelian groups.

Although this book is envisioned as a text for an advanced undergraduate or introductory graduate level course, it is also intended to be of use for physicists, statisticians, and mathematicians who may not be algebraists, but need group representation theory for their work.

While preparing this book I have relied on a number of classical references on representation theory, including [5–7, 10, 15, 20, 21]. For the representation theory of the symmetric group I have drawn from [7, 12, 13, 16, 17, 19]; the approach is due to James [17]. Good references for applications of representation theory to computing eigenvalues of graphs and random walks are [3, 6, 7]. Chapter 11, in particular, owes much of its presentation to [7] and [3]. Discrete Fourier analysis and its applications can be found in [3, 7, 22].

Thanks are due to the following people for their input and suggestions: Ariane Masuda, Paul Mezo, and Martin Steinberg.

Ottawa Benjamin Steinberg

http://www.springer.com/978-1-4614-0775-1

Representation Theory of Finite Groups
An Introductory Approach
Steinberg, B.
2012, XIII, 157 p. 4 illus., Softcover
ISBN: 978-1-4614-0775-1