

Chapter 2

Macroeconomic Analysis and Parametric Control of Equilibrium States in National Economic Markets

Conducting a stabilization policy on the basis of the results of macroeconomic analysis of a functioning market economy is an important economic function of the state.

The *AD-AS*, *IS*, *LM*, *IS-LM*, *IS-LM-BP* models, as well as the Keynesian model of common economic equilibrium for a closed economy and the model of a small country for an open economy [41], are efficient instruments for the macroeconomic analysis of the functioning of a national economy.

In the literature, one can find how these models are used for carrying out a macroeconomic analysis of the conditions of equilibrium in national economic markets. But there are no published results in the context of the estimation of optimal values of the economic instruments on the basis of the Keynesian model of common economic equilibrium and the model of an open economy of a small country in the sense of certain criteria, as well as analysis of the dependence of the optimal criterion value on exogenous parameters.

2.1 Factor Modeling of the Aggregate Demand in a National Economy: *AD-AS* Model

2.1.1 Problem Statement

The problem consists in determining the relative position of the mean (aggregated) curves expressing the values of aggregate demand and aggregate supply for the Republic of Kazakhstan for the period of years from 2000 to 2008 [36]. The level of the gross domestic product in comparable prices calculated by the manufacturing method is used as the index of the aggregate supply. This is the manufacturing method that is mainly used by the statistical services for calculation of the gross

domestic product. The aggregate demand is calculated reasoning from the basic macroeconomic identity $Y_{AD} = C + I + G + NX$. In other words, the level of the gross domestic products calculated by the method of finite use is accepted as the index of the aggregate demand.

2.1.2 Input Data

The official statistics of various state institutes (Statistical Agency of Kazakhstan and National Bank of the Republic of Kazakhstan) are used for carrying out computations. The data are presented in Table 2.1.

The problem consists in determining each term of the basic macroeconomic identity $Y_{AD} = C + I + G + NX$ by the respective regressors.

As might be expected (Table 2.2), most of the macroeconomic parameters closely correlate with the level of the gross domestic product. The basic macroeconomic indexes show considerable correlation with the level of public expenses (sometimes even more considerable than the correlation with the gross domestic product), from which one can draw a conclusion about the significant role of the state in the economy. The correlation of the exchange rate with the level of consumption, investments, public expenses, and taxes is also considerable, from which one can draw a conclusion about the considerable influence of the foreign sector on the economy of Kazakhstan. This is also confirmed by the considerable correlation between the oil price and basic macroeconomic indexes. Thus on the one hand, the economy of Kazakhstan depends to a great extent on state interference, and on the other hand, it depends on the actions of foreign countries.

2.1.3 Model Construction

As mentioned before, the problem consists in constructing the regression equations for each component of $Y_{AD} = C + I + G + NX$. The equations are constructed by reasoning from the theoretically and empirically revealed connections between the variables.

2.1.3.1 Finite Consumption

Let us consider the Keynesian model of consumption as the model for estimation of the consumption level. According to this model, the consumption level depends on the available income (the level of the gross domestic product minus taxes).

Table 2.1 Statistical data of the main indexes of macroeconomic dynamics of the Republic of Kazakhstan for the years 2000–2008

Year	<i>Y</i>	<i>C</i>	<i>I</i>	<i>G</i>	<i>P</i>	<i>NX</i>	<i>E</i>	<i>T</i>	<i>R</i>	<i>PP</i>	<i>PP\$</i>
2000	2,599,901.6	1,589,061.6	519,000	313,984.5	1	195,126.8	142.13	174,530.5	8.68	24,874.62	175.0132
2001	3,055,068.891	1,739,472.838	729,323.3	409,808.647	1.064	−32,354.13534	139.4857	214,944.6	10.25	20,951.37	150.2044
2002	3,329,392.871	1,757,775.184	1,051,820	386,526.03	1.134224	−1,678.151758	140.2352	241,330.4	8.97	20,761.84	148.0502
2003	3,807,298.146	2,009,684.752	1,096,184	428,608.389	1.211351	204,479.0094	147.3835	261,403.6	8.46	24,249.48	164.5332
2004	4,541,648.642	2,362,576.484	1,318,119	527,489.899	1.292512	390,201.2452	114.6345	277,128.8	7.29	23,874.79	208.2689
2005	5,463,019.682	2,652,677.614	1,742,399	614,509.345	1.38945	481,646.4279	108.3157	341,560.2	5.98	31,602.33	291.7614
2006	6,781,287.746	3,019,149.723	1,875,309	690,393.561	1.506164	723,764.696	96.78848	447,985.3	4.61	34,471.47	356.1526
2007	7,181,488.578	3,152,724.009	1,895,757	793,823.402	1.789323	480,370.2316	82.57355	519,542	−4.02	32,179.73	389.7099
2008	8,133,751.486	3,395,129.647	1,957,906	850,104.741	1.959308	1,651,949.086	74.04733	581,246.1	5.77	40,700.53	549.6556

Here, *Y* is the yearly level of the real gross domestic product in millions of tenge (in prices of the year 2000)

C is the consumption level in millions of tenge (in prices of the year 2000)

I is the volume of investment to the capital asset in millions of tenge (in prices of the year 2000)

G is the level of public expenses in millions of tenge (in prices of the year 2000)

NX is the net export in millions of tenge (in prices of the year 2000)

P is the level of prices (calculated for the base of the year 2000)

T is the taxation level in millions of tenge (in prices of the year 2000)

R is the real interest rate calculated by the Fisher equation with use of the current inflation level (the consumer price index is used as the inflation index; the mean credit interest rate is used as the nominal interest rate)

E is the real currency exchange rate of the tenge for the US dollar (corrected for the inflation of both the tenge and the US dollar)

PP is the price of one ton of oil Urals in tenge (in prices of the year 2000)

PP\$ is the price of one ton of oil Urals in US dollars (in prices of the year 2000)

Table 2.2 Correlation matrix of the indexes of macroeconomic dynamics of the Republic of Kazakhstan for the years 2000–2008

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>G</i>	<i>P</i>	<i>NX</i>	<i>E</i>	<i>T</i>	<i>R</i>	<i>PP</i>
<i>Y</i>	1									
<i>C</i>	0.995568	1								
<i>I</i>	0.954742	0.964209	1							
<i>G</i>	0.992147	0.991159	0.946283	1						
<i>P</i>	0.976477	0.963525	0.902736	0.979412	1					
<i>NX</i>	0.852656	0.835273	0.732093	0.823695	0.859411	1				
<i>E</i>	-0.9683	-0.97038	-0.90791	-0.97677	-0.95128	-0.82786	1			
<i>T</i>	0.988341	0.971981	0.917414	0.98303	0.988739	0.843865	-0.95221	1		
<i>R</i>	-0.71187	-0.72126	-0.69824	-0.73769	-0.72103	-0.3307	0.734325	-0.7284	1	
<i>PP</i>	0.928747	0.919677	0.845586	0.901124	0.896504	0.924429	-0.8964	0.912055	-0.55308	1

In addition, let us also include the credit interest rate (taking into account the rather significant correlation that is shown by the consumption level and credit interest rate) and the currency exchange rate (also by virtue of strong correlation) as explanatory variables. Thus, we estimate the following three models:

$$C = a_0 + a_1(Y - T) + u_1,$$

$$C = a_0 + a_1(Y - T) + R + u_2,$$

$$C = a_0 + a_1(Y - T) + R + E + u_3.$$

Estimation of the first model:

Dependent variable: C_{-}				
Method: least squares				
Date: 06/22/09 Time: 03:20				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	t-Statistic	Prob
$Y-T$	0.363289	0.011641	31.20677	0.0000
C	720,078.5	57,818.27	12.45417	0.0000
R-squared	0.992863	Mean dependent var		2,408,695.
Adjusted R-squared	0.991844	S.D. dependent var		676,711.2
S.E. of regression	61,114.51	Akaike info criterion		25.07202
Sum squared resid	2.61E + 10	Schwarz criterion		25.11584
Log likelihood	-110.8241	F-statistic		973.8625
Durbin-Watson stat	1.284489	Prob(F-statistic)		0.000000

Estimation of the second model:

Dependent variable: C_{-}				
Method: least squares				
Date: 06/22/09 Time: 03:21				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	t-Statistic	Prob.
$Y-T$	0.356045	0.017356	20.51473	0.0000
R	-4,468.076	7,597.224	-0.588120	0.5779
C	781,551.9	120,884.5	6.465278	0.0006
R-squared	0.993252	Mean dependent var		2,408,695.
Adjusted R-squared	0.991003	S.D. dependent var		676,711.2
S.E. of regression	64,187.03	Akaike info criterion		25.23819
Sum squared resid	2.47E + 10	Schwarz criterion		25.30393
Log likelihood	-110.5719	F-statistic		441.6029
Durbin-Watson stat	1.160812	Prob(F-statistic)		0.000000

Estimation of the third model:

Dependent variable: C_{-}				
Method: least squares				
Date: 06/22/09 Time: 03:22				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
$Y-T$	0.292251	0.060955	4.794546	0.0049
R	-1,261.369	8,037.910	-0.156928	0.8814
E	-4,067.552	3,730.705	-1.090290	0.3253
C	1,496,067.	666,065.3	2.246127	0.0746
R-squared	0.994548	Mean dependent var		2,408,695.
Adjusted R-squared	0.991278	S.D. dependent var		676,711.2
S.E. of regression	63,200.72	Akaike info criterion		25.24712
Sum squared resid	2.00E + 10	Schwarz criterion		25.33478
Log likelihood	-109.6120	F-statistic		304.0587
Durbin-Watson stat	1.335349	Prob(F-statistic)		0.000004

The first model is the best one, judging by the significance of the model coefficients. Including the interest rate and exchange rate in the number of regressors does not improve the model. Thus, we can assert that the credit market insignificantly influences the consumption level in spite of the revealed correlation between the interest rate and consumption level.

The resulting equation is given by

$$C = 0.3632892184(Y - T) + 720078.5098$$

It should be noted that the level of the limiting propensity to consumption is rather low (0.36). It can be considered as the population uncertainty in the near future, because two-thirds of income is not used for the purposes of current consumption.

2.1.3.2 Investment

The theoretical approach implies that the investment level depends on the interest rate. The high interest rate decreases the investment incentives of the economic agents, since on the one hand, the credit resources rise in price, while on the other hand, investments such as deposits become more attractive (in the view of both profitability and risk). Taking into consideration the high degree of correlation of the investment with the gross domestic product and currency exchange rate, let us include these variables in the analysis. As a result, let us estimate the following model:

$$I = b_0 + b_1 \times R + b_2 \times E + b_3 \times Y + u_2$$

Dependent variable: I

Method: least squares

Date: 06/22/09 Time: 03:26

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
R	-3,464.965	25,562.14	-0.135551	0.8975
E	-1,717.959	11,622.83	-0.147809	0.8883
Y	0.225467	0.176924	1.274371	0.2585
C	435,858.2	2,076,119.	0.209939	0.8420
R-squared	0.912614	Mean dependent var		1,353,980.
Adjusted R-squared	0.860182	S.D. dependent var		539,367.1
S.E. of regression	201,681.3	Akaike info criterion		27.56787
Sum squared resid	2.03E + 11	Schwarz criterion		27.65552
Log likelihood	-120.0554	F-statistic		17.40578
Durbin-Watson stat	0.909211	Prob(F-statistic)		0.004453

In spite of the significance of the model as a whole (F-test), each coefficient turns out to be individually insignificant. Perhaps this is a natural result. Taking into consideration the high cross correlation of the factors included into this model, it turns out to be multicollinear.

Among the one-factor models (the estimation of the investment level depending only on the interest rate, on the currency exchange rate, and on the level of the gross domestic product), the model with the exchange rate as the regressor is the most appropriate (by the results of the main tests), the next is the model with the gross domestic product, and finally, the model with the interest rate. If one model includes the gross domestic product and currency exchange rate, the model parameters deteriorate severely. Therefore, the following model is used for further analysis:

$$I = b_0 + b_1 \times E + u_2$$

Dependent variable: I

Method: least squares

Date: 06/22/09 Time: 02:19

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
E	-15,783.44	2,159.018	-7.310470	0.0002
C	3,053,359.	241,485.3	12.64408	0.0000
R-squared	0.884188	Mean dependent var		1,353,980.
Adjusted R-squared	0.867644	S.D. dependent var		539,367.1
S.E. of regression	196,226.0	Akaike info criterion		27.40505
Sum squared resid	2.70E + 11	Schwarz criterion		27.44888
Log likelihood	-121.3227	F-statistic		53.44297
Durbin-Watson stat	1.256380	Prob(F-statistic)		0.000161

$$I = 3053359 - 15783.44 \times E$$

The constructed model indicates that the “autonomous” volume of the investment, independent of the external factors in the economy, is equal to three billion tenge (with respect to the gross domestic product of the year 2008, it constitutes 37.5%), which is 1.5 times the total volume of investment in comparable prices of the year 2008. Thus, this equation shows an outflow instead of accumulation of the investment because of the features of the external market situation. In this case, the absolute term characterizes the investment potential of the country that theoretically should be realized inside the country under economic conditions more closed and less dependent on external shocks.

Like any time series, the investment volume can show autoregression. Let us examine this hypothesis. For this purpose, let us analyze the correlogram of this series:

Autocorrelation	Partial correlation		AC	PAC	Q-Stat	Prob
. *****	. *****	1	0.686	0.686	5.8172	0.016
. *** .	. ** .	2	0.357	-0.214	7.6186	0.022
. * .	. * .	3	0.086	-0.127	7.7417	0.052
. ** .	. *** .	4	-0.228	-0.346	8.7745	0.067
. *** .	. * .	5	-0.417	-0.107	13.068	0.023
. *** .	. .	6	-0.411	0.040	18.639	0.005
. *** .	. * .	7	-0.356	-0.077	24.929	0.001
. ** .	. .	8	-0.217	0.026	29.576	0.000

The correlogram form shows that the investment is a first-order autoregression AR(1). Constructing this dependence for the investment leads to the following results:

Dependent variable: I

Method: least squares

Date: 10/25/09 Time: 23:14

Sample (adjusted): 2001 2008

Included observations: 8 after adjustments

Variable	Coefficient	Std. error	t -Statistic	Prob.
$I(-1)$	1.105468	0.047283	23.37958	0.0000
R-squared	0.847928	Mean dependent var		1,458,352.
Adjusted R-squared	0.847928	S.D. dependent var		469,497.4
S.E. of regression	183,087.1	Akaike info criterion		27.18978
Sum squared resid	2.35E + 11	Schwarz criterion		27.19971
Log likelihood	-107.7591	Durbin-Watson stat		1.269912

The regression parameters are satisfactory thus allowing representing the investment series as follows:

$$I(n) = 1.105 \times I(n-1) + u$$

Thus, the annual increase of investment in the Republic of Kazakhstan is equal to 10.5%.

However, for the purpose of further analysis, the one-factor model remains the priority. In this model, the level of investment depends on the currency exchange rate.

2.1.3.3 State Expenses

The value of the state expenses mainly depends on its preceding values (in part, it can be explained by the budget procedure of planning these expenses). Therefore, let us represent the consumption function as an autoregression function. To determine its order, let us look at the autocorrelation function of the series:

Autocorrelation	Partial correlation		AC	PAC	Q-Stat	Prob
. *****	. *****	1	0.667	0.667	5.5022	0.019
. *** .	. * .	2	0.368	-0.138	7.4169	0.025
. . .	. ** .	3	0.064	-0.226	7.4840	0.058
. ** .	. ** .	4	-0.217	-0.242	8.4193	0.077
. *** .	. * .	5	-0.385	-0.126	12.080	0.034
. *** .	. . .	6	-0.404	-0.010	17.467	0.008
. *** .	. . .	7	-0.347	-0.051	23.430	0.001
. ** .	. * .	8	-0.245	-0.071	29.395	0.000

From the function form we conclude that the consumption function is a first-order autoregression function AR(1):

$$G_t = \alpha \times G_{t-1} + \varepsilon_t.$$

Let us estimate this model:

Dependent variable: G				
Method: least squares				
Date: 06/22/09 Time: 02:32				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	t -Statistic	Prob.
$G-1$	1.000002	1.85E-07	5,398,845.	0.0000
R-squared	1.000000	Mean dependent var		557,249.8
Adjusted R-squared	1.000000	S.D. dependent var		190,471.4
S.E. of regression	0.325331	Akaike info criterion		0.696492
Sum squared resid	0.846722	Schwarz criterion		0.718406
Log likelihood	-2.134213	Durbin-Watson stat		0.151288

In spite of the outside perfection (the determination coefficient equal to one), this model is unsatisfactory, since according to it, the level of public expenses varies almost not at all, which is, of course, contrary to fact. Also, this model shows heteroscedasticity in the residuals. Following the correlation matrix, let us estimate

another two models, whereby we consider the currency exchange rate and the level of gross domestic product as the regressors:

$$G_t = \alpha \times E + c + \varepsilon_t$$

Dependent variable: G				
Method: least squares				
Date: 06/22/09 Time: 02:45				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	t -Statistic	Prob.
E	-5,838.985	385.8181	-15.13404	0.0000
C	1,185,925.	43,153.60	27.48148	0.0000
R-squared	0.970344	Mean dependent var		557,249.8
Adjusted R-squared	0.966107	S.D. dependent var		190,471.4
S.E. of regression	35,065.72	Akaike info criterion		23.96097
Sum squared resid	8.61E + 09	Schwarz criterion		24.00479
Log likelihood	-105.8243	F-statistic		229.0391
Durbin-Watson stat	2.616388	Prob(F-statistic)		0.000001

$$G_t = -5838.985 \times E + 1185925$$

Dependent variable: G				
Method: least squares				
Date: 06/22/09 Time: 03:30				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	t -Statistic	Prob.
Y	0.094588	0.004507	20.98709	0.0000
C	85,435.24	24,030.68	3.555257	0.0093
R-squared	0.984356	Mean dependent var		557,249.8
Adjusted R-squared	0.982121	S.D. dependent var		190,471.4
S.E. of regression	25,468.22	Akaike info criterion		23.32138
Sum squared resid	4.54E + 09	Schwarz criterion		23.36521
Log likelihood	-102.9462	F-statistic		440.4579
Durbin-Watson stat	3.063158	Prob(F-statistic)		0.000000

Let us give preference to the second model:

$$G = 0.0945881296 \times Y + 85435.23751.$$

According to the results of this model, the level of state expenses in Kazakhstan constitutes 9.4% of the gross domestic product of the current year plus the necessary minimum of the autonomous state expenses.

Let us check the residuals on autocorrelation in this model too. The correlogram looks as follows:

Autocorrelation	Partial correlation		AC	PAC	Q-Stat	Prob
.**** .	.**** .	1	−0.567	−0.567	3.9816	0.046
. * .	.**** .	2	−0.147	−0.691	4.2861	0.117
. ** .	.**** .	3	0.328	−0.577	6.0595	0.109
. .	. * .	4	0.038	−0.113	6.0882	0.193
. *** .	. ** .	5	−0.403	−0.282	10.102	0.072
. *** .	. * .	6	0.381	−0.058	14.885	0.021
. * .	. *** .	7	−0.148	−0.388	15.968	0.025
. .	. ** .	8	0.018	−0.273	16.000	0.042

Such a correlogram allows certification of the presence of autoregression in the model residuals. Further analysis results in the clear conclusion that this is a second-order autoregression.

Dependent variable: A

Method: least squares

Date: 07/10/09 Time: 10:27

Sample (adjusted): 2002 2008

Included observations: 7 after adjustments

Variable	Coefficient	Std. error	t-Statistic	Prob.
A(−1)	−0.925561	0.222106	−4.167203	0.0088
A(−2)	−0.796342	0.238031	−3.345539	0.0204
R-squared	0.782105	Mean dependent var		−2,575.656
Adjusted R-squared	0.738526	S.D. dependent var		22,132.43
S.E. of regression	11,317.32	Akaike info criterion		21.74101
Sum squared resid	6.40E + 08	Schwarz criterion		21.72556
Log likelihood	−74.09354	Durbin–Watson stat		3.148337

The situation in which the residuals of the regression model do not correlate with any of the significant factors of the regression is said to be white noise. In this model, there is not only white noise, but *white wind*, i.e., the values of the residuals show autoregression, being in some way dependent on their preceding states. In other words, there is some logic in the values of the derived residuals that cannot be revealed by constructing the autoregression model.

Thus, one may speak of the presence of unrevealed factors affecting the state expenses, whose effects are extended to two subsequent periods. Taking into consideration the considerable institutional transformations that took place in the

Republic of Kazakhstan within the considered period, one may suppose that these transformations affect public expenses.

Let us also estimate the regression of the public expenses depending on the taxation level:

Dependent variable: G				
Method: least squares				
Date: 10/25/09 Time: 23:31				
Sample: 2000 2008				
Included observations: 9				

Variable	Coefficient	Std. error	t -Statistic	Prob.
T	1.303832	0.091964	14.17769	0.0000
C	113,994.6	33,652.48	3.387406	0.0116
R-squared	0.966347	Mean dependent var		557,249.8
Adjusted R-squared	0.961540	S.D. dependent var		190,471.4
S.E. of regression	37,353.89	Akaike info criterion		24.08739
Sum squared resid	9.77E + 09	Schwarz criterion		24.13122
Log likelihood	-106.3933	F-statistic		201.0069
Durbin-Watson stat	1.662048	Prob(F-statistic)		0.000002

$$G = 1.3038 \times T + 113995.$$

If we compare this model with that derived above, $G = 0.0945881296 \cdot Y + 85,435.23751$, we have to admit that it is inferior even in the parameter R^2 (in the latter model, it is equal to 0.98), and the sum of squared errors for the latter model is twice as small (although taking into account the order of this index, this discrepancy can be neglected). However, this regression can be useful in considering the Haavelmo alternative.

For this purpose, let us also estimate the dependence of the investment level on the taxes:

Dependent variable: I				
Method: least squares				
Date: 10/26/09 Time: 00:00				
Sample: 2000 2008				
Included observations: 9				

Variable	Coefficient	Std. error	t -Statistic	Prob.
T	3.445681	0.564900	6.099630	0.0005
C	182,574.0	206,715.3	0.883214	0.4064
R-squared	0.841649	Mean dependent var		1,353,980.
Adjusted R-squared	0.819027	S.D. dependent var		539,367.1
S.E. of regression	229,451.7	Akaike info criterion		27.71790
Sum squared resid	3.69E + 11	Schwarz criterion		27.76173
Log likelihood	-122.7306	F-statistic		37.20548
Durbin-Watson stat	0.696745	Prob(F-statistic)		0.000491

$$I = 3.45 \times T + 182573$$

Though this dependence is not as perfect as the previous, nevertheless it is satisfactory and can be used for the purpose of analysis. From the derived equations it can be seen that increasing the taxes by one unit results in an increment of public expenses by 1.3 units and of investment by 3.45 units. That is, the main part of the investment in Kazakhstan is not private (since it is supposed in this case that raising taxes would result in a decrease of business economic activity and as a result, a decrease in investment), but undertaken by the government. Therefore, in this case, the Haavelmo alternative is out of the question. *At this stage, the state is the sole institution that is able to have an effective influence on the economic situation in the country.*

Let us give preference to the model

$$G = 0.0945881296 \times Y + 85435.23751.$$

Thus, we can assert that the level of public expense in Kazakhstan constitutes 9.4% of the gross domestic product of the current year plus the necessary minimum of the autonomous public expenses.

2.1.3.4 Net Export

Theoretically, the net export must depend on the currency exchange rate; therefore, let us test the following model:

$$NX = e_0 + e_1 \times E + u_4$$

Dependent variable: NX

Method: least squares

Date: 06/22/09 Time: 02:51

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
E	-12,972.00	3,471.498	-3.736716	0.0073
C	1,851,510.	388,285.7	4.768423	0.0020
R-squared	0.666079	Mean dependent var		454,833.9
Adjusted R-squared	0.618376	S.D. dependent var		510,739.4
S.E. of regression	315,512.9	Akaike info criterion		28.35492
Sum squared resid	6.97E + 11	Schwarz criterion		28.39874
Log likelihood	-125.5971	F-statistic		13.96305
Durbin-Watson stat	2.270347	Prob(F-statistic)		0.007294

The coefficient of determination (index R^2) of this model is not as expressive as in the previous models, but it can be accepted as significant (almost 67%). We have to acknowledge that we again observe a picture of confrontation between internal potentials and external factors: when the dollar grows in value (E is greater), the net export of Kazakhstan decreases:

$$NX = 1851510 - 12972 \times E.$$

To complete the analysis, let us test the net export on autoregression. The correlogram for this index is as follows:

Date: 10/26/09 Time: 00:13

Sample: 2000 2008

Included observations: 9

Autocorrelation	Partial correlation		AC	PAC	Q-Stat	Prob
. ** .	. ** .	1	0.250	0.250	0.7752	0.379
. ** .	. ** .	2	0.272	0.224	1.8251	0.402
. .	. * .	3	0.023	-0.097	1.8336	0.608
. * .	. * .	4	-0.097	-0.163	2.0206	0.732
. ** .	. * .	5	-0.215	-0.168	3.1681	0.674
. ** .	. ** .	6	-0.301	-0.192	6.1641	0.405
. ** .	. * .	7	-0.283	-0.127	10.119	0.182
. * .	. .	8	-0.149	0.027	12.316	0.138

The insignificance of the first values of the autocorrelation function gives reason for not analyzing the autocorrelation of this time series. Thus, the hypothesis on the autonomy of the net export and its dependence solely on foreign market opportunities does not prove to be true.

Now let us estimate the dependence of the net export on the currency exchange rate, tax, and oil price (in US dollars).

$$1. NX = 3.001 \times T - 565470$$

Dependent variable: NX

Method: least squares

Date: 10/26/09 Time: 00:16

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
T	3.001217	0.721258	4.161085	0.0042
C	-565,470.2	263,931.9	-2.142485	0.0694
R-squared	0.712107	Mean dependent var		454,833.9
Adjusted R-squared	0.670980	S.D. dependent var		510,739.4
S.E. of regression	292,961.5	Akaike info criterion		28.20660
Sum squared resid	6.01E + 11	Schwarz criterion		28.25043
Log likelihood	-124.9297	F-statistic		17.31463
Durbin-Watson stat	2.269880	Prob(F-statistic)		0.004236

$$2. \quad NX = -15295 \times E + 2231761$$

Dependent variable: NX				
Method: least squares				
Date: 10/26/09 Time: 00:22				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
E	-15,294.99	3,917.023	-3.904749	0.0059
C	2,231,761.	466,379.4	4.785292	0.0020
R-squared	0.685352	Mean dependent var		454,833.9
Adjusted R-squared	0.640402	S.D. dependent var		510,739.4
S.E. of regression	306,272.5	Akaike info criterion		28.29547
Sum squared resid	6.57E + 11	Schwarz criterion		28.33930
Log likelihood	-125.3296	F-statistic		15.24706
Durbin-Watson stat	2.395862	Prob(F-statistic)		0.005863

Note that this regression is somewhat better in the main indexes in comparison to the previous one.

$$3. \quad NX = 3455 \times PP\$ - 479230$$

Dependent variable: NX				
Method: least squares				
Date: 10/26/09 Time: 00:24				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
$PP\$$	3,454.733	485.5019	7.115796	0.0002
C	-479,229.6	145,787.3	-3.287183	0.0134
R-squared	0.878545	Mean dependent var		454,833.9
Adjusted R-squared	0.861194	S.D. dependent var		510,739.4
S.E. of regression	190,284.3	Akaike info criterion		27.34356
Sum squared resid	2.53E + 11	Schwarz criterion		27.38738
Log likelihood	-121.0460	F-statistic		50.63456
Durbin-Watson stat	2.353455	Prob(F-statistic)		0.000191

The regression of net exports depending on oil price is the best in view of parameter R^2 . This fact allows us to observe that net exports do, however, depend to a greater extent on external factors than internal ones, i.e., their volume is determined to a greater extent by external demand, but not the readiness of residents to provide export supply. Moreover, as can be seen from the correlation matrix, the taxation level depends (very considerably) on the world oil price. Thus, preference is given to the last model,

$$NX = 3455 \times PP\$ - 479230.$$

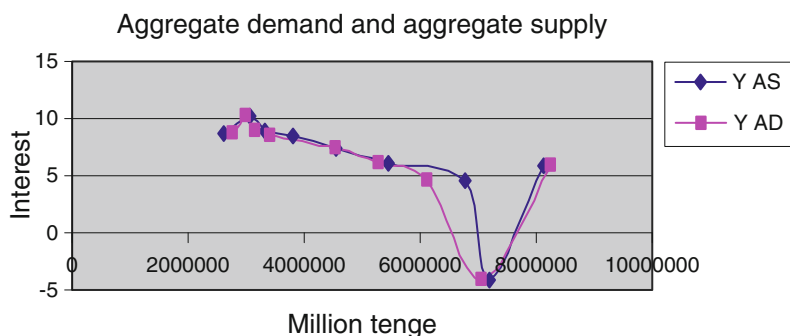


Fig. 2.1 Values of aggregate demand and aggregate supply, million tenge in year-2000 prices and in coordinates of real interest rate

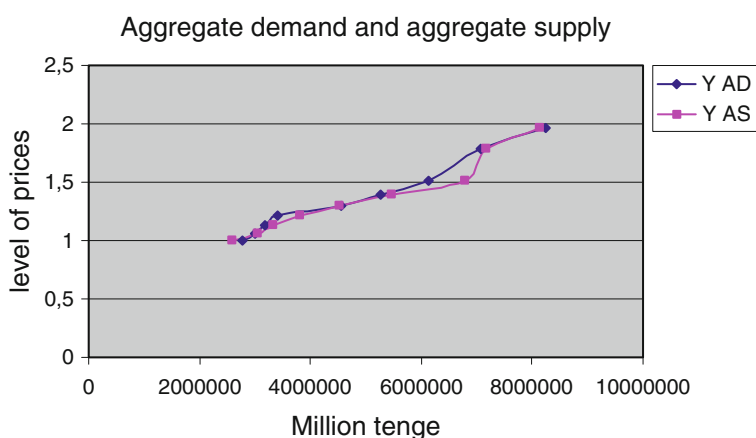


Fig. 2.2 Values of aggregate demand and aggregate supply, million tenge in year-2000 prices and in coordinates of level of prices

Now let us take the model values of the explanatory variables from each of the equations derived above and add them to obtain the calculated value of the aggregate demand. The initially given level of the gross domestic product appears for the values of the aggregate supply. Let us trace their dynamics in Figs. 2.1, 2.2 and Table 2.3.

Based on the plots, one can ascertain that the economy of Kazakhstan is in a state close to equilibrium.

First, our analysis confirms the initial hypotheses on the strong dependence of the economy of the Republic of Kazakhstan on its governmental investment on the one hand, and on the other, its dependence on the foreign sector. Second, public policy appears to be efficient with respect to maintaining macroeconomic equilibrium in the country. When in 2006 there was a tendency toward a

Table 2.3 Values of aggregate demand and aggregate supply, million tenge in year-2000 prices

Years	Y_{AS}	Y_{AD}
2000	2,599,902	2,782,329
2001	3,055,069	3,010,350
2002	3,329,393	3,171,492
2003	3,807,298	3,421,302
2004	4,541,649	4,557,192
2005	5,463,020	5,277,863
2006	6,781,288	6,140,684
2007	7,181,488	7,074,904
2008	8,133,751	8,254,390

recessionary gap (aggregate supply exceeding aggregate demand), the government “bolstered” the demand, and in spite of increasing inflation (18.8% in 2007 against 8.4% in 2006), was able to return the system to a state of equilibrium (it should also be recognized that inflation was successfully restrained to 9.5% in 2008). However, equilibrium by itself cannot be a goal for a developing economy. Growth unavoidably implies instability at certain stages for creating “reserves” for further development. In this connection, the following question is still open: does the maintenance of macroeconomic equilibrium restrain potential growth of the economy? And indeed, the fact of a restraining effect of external economic conditions on the economy of Kazakhstan is unquestionable. The external factor does not allow the realization of domestic investment potential. Thus, one can say that excessive economic openness is not sufficiently strong to counteract external influences.

Within the current macroeconomic conditions, a strong state is an indispensable condition of the economic stability of the Republic of Kazakhstan.

2.2 Macroeconomic Analysis of the National Economic State Based on IS , LM , $IS-LM$ Models, Keynesian All-Economy Equilibrium. Analysis of the Influence of Instruments on Equilibrium Solution

One of the main economic functions of the state is to carry out a stabilizing policy based on the equilibrium conditions in various markets.

The IS , LM , $IS-LM$ models, as well as the Keynesian model of common economic equilibrium, are efficient instruments for macroeconomic analysis of market states.

This section is devoted to the construction of the IS , LM , $IS-LM$ models, as well as the Keynesian model of common economic equilibrium by the example of the economy of the Republic of Kazakhstan, analysis of the influence of the economic instruments on equilibrium conditions in the respective markets, as well as the estimation of the optimal values of the economic instruments on the basis of the Keynesian mathematical model of common economic equilibrium.

2.2.1 Construction of the IS Model and Analysis of the Influence of Economic Instruments

Let us introduce the notation for the economic indexes used for model construction: T is the tax proceeds (to the state budget, in billions of tenge); S is the net savings, billions of tenge; I is the investment to the capital asset, billions of tenge; G is the public expenses, billions of tenge; Y is the gross national income, billions of tenge; C is household consumption, billions of tenge.

Macro-estimation of the equilibrium conditions in the wealth market can be done on the basis of the *IS* model [41, p. 76] represented as

$$T + S = I + G. \quad (2.1)$$

The tax proceeds T to the state budget represented by the expression $T = T_y Y$ has the following econometric estimation based on statistical information for the years 2000–2008:

$$T = 0.2207 Y. \\ (0.000) \quad (2.2)$$

The statistical characteristics of model (2.2) are as follows: the determination coefficient $R^2 = 0.986$; the standard error $Se = 209.5$; the approximation coefficient $A = 10.47\%$; the Fisher statistics $F = 581.66$. The statistical significance of the coefficient of regression (2.2), as well as the regressions estimated below, is given within parentheses under the respective coefficients of the regressions in the form of p -values.

The net savings S represented by the expression $S = a + S_y Y$ has the following econometric estimation:

$$S = -366.055 + 0.222 Y \\ (0.000) \quad (0.000) \quad (2.3)$$

The statistical characteristics of model (2.3) are as follows: the determination coefficient $R^2 = 0.994$; the standard error $Se = 69.2$; the approximation coefficient $A = 11.47\%$; the Fisher statistics $F = 1,287.2$; the Durbin–Watson statistics $DW = 1.96$.

The investment to capital assets represented by the expression $I = a + I_i i$ after estimation of the parameters of this model using the statistical information becomes the following:

$$I = 1367.9 - 81.3 i + 0.2751 Y_{\text{mean}}. \\ (0.02) \quad (0.03) \quad (0.00) \quad (2.4)$$

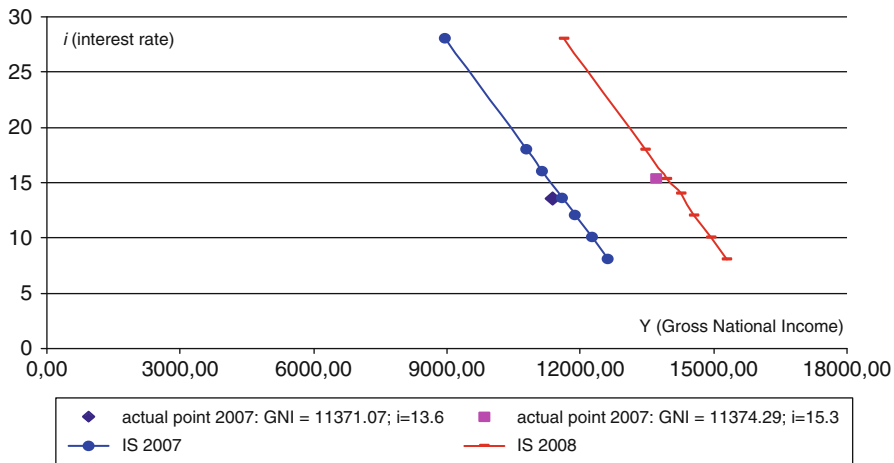


Fig. 2.3 Plots of IS_{2007} and IS_{2008} models

The statistical characteristics of model (2.4) are as follows: the determination coefficient $R^2 = 0.99$; the standard error $Se = 126.8$; the approximation coefficient $A = 4.2\%$; the Fisher statistics $F = 326.48$; the Durbin–Watson statistics $DW = 1.72$. Substituting into (2.3) the value of the mean nominal gross national income for the years 2000–2008 in billions of tenge $Y_m = 6,662.7$ finally yields the following model for the investment:

$$I = 3\,202 - 81.3\,i. \quad (2.5)$$

Substituting expressions (2.2, 2.3), and (2.5) into (2.1), we obtain the IS model representation in the following form:

$$-366.055 + 0.222Y + 0.2207Y = 3202 - 81.3\,i + G_{200X}, \quad (2.6)$$

which allows determining the equilibrium value of i for the given values of Y and G_{200X} . In macroeconomic theory, one has the method [41, p. 77] of plotting the IS curve, which is the set of combinations of the equilibrium values of Y and i (Fig. 2.3).

From the model IS_{2007} (Fig. 2.3) it follows that the equilibrium GNI_{2007} with interest rate 13.6% equals 11,602.75 billion tenge, and that the real GNI_{2007} with interest rate 13.6% equals 11,371 billion tenge, which shows a lack of wealth in the considered market. From the model IS_{2008} (Fig. 2.3), it follows that the equilibrium GNI_{2008} with interest rate 15.3% equals 13,957.91 billion tenge, and that the real GNI_{2008} with interest rate 15.3% equals 13,734 billion tenge, which also shows a lack of wealth in that market.

Table 2.4 Consequences of changing public expenses and taxation

Consequence	Action	
	Public expenses increase by ΔG	Taxes decrease by ΔT
National income increases by	$\frac{1}{T_y + S_y} \Delta G = 2.26 \Delta G$	$\frac{C_{Y^v}}{T_y + S_y} \Delta T = 1.3 \Delta T$
Budgeted deficit increases by	$\left(1 - \frac{T_y}{T_y + S_y}\right) \Delta G = 0.5 \Delta G$	$\left(1 - \frac{T_y C_{Y^v}}{T_y + S_y}\right) \Delta T = 0.7 \Delta T$

To estimate the multiplicative effects [41, p. 78] of the economic instruments T_y and G , let us construct an econometric model of the consumption of households C , which on the basis of statistical information for the years 2000–2008 is given by

$$C = 428.68 + 0.552 Y^v, \\ (0.000) \quad (0.000)$$

where $Y^v = Y - T_y Y$, $C_{Y^v} = 0.552$. The statistical characteristics of this model are as follows: the determination coefficient $R^2 = 0.999$; the standard error $Se = 68.92$; the approximation coefficient $A = 1.78\%$; the Fisher statistics $F = 5,394$; the Durbin–Watson statistics $DW = 1.53$.

Table 2.4 presents the expressions and values of the multipliers [41, p. 83] of instruments T_y and G derived on the basis of the *IS* model (2.6).

Let us estimate the multiplicative effects of the instruments T_y and G based on the data for the year 2008. According to those data, we have $G = 3,859.98$, $Y = 13,734.3$, $T_y = 0.2207$. Now let us change G to $\Delta G = 579$. This change, in accordance with the multiplier of ΔG , results in an increment of *GNI* by the value $\Delta Y = 1,308.54$.

Also, from the data of the year 2008, we have $G = 3,859.98$, $Y = 13,734.3$, $T_y = 0.2207$. Let us change T_y by $\Delta T_y = -0.01$. This change in accordance with the multiplier of ΔT_y results in an increment of *GNI* by the value $\Delta Y = 328.37$. The derived results agree with the macroeconomic theory that considers the influence of the economic instruments on the changes in the domestic national income, which is represented by Table 2.1, “Consequences of changing public expenses and taxation” [41, p. 83].

2.2.2 Macroeconomics of Equilibrium Conditions in the Money Market

The macro-estimation of equilibrium conditions in the money market can be realized on the basis of the *LM* model represented as follows [41, p. 111]:

$$M = l_{pr} + l_{tr}, \quad (2.7)$$

Table 2.5 Value of the money aggregate $M3$ and the velocity of money

Year	GNI	Value of money aggregate $M3$	V , velocity of money
2007	11,371	4,629.8	2.5
2008	13,734	6,266.4	2.2

where M is the money supply, in billions of tenge; l_{pr} is the volume of property (deposits in deposit organizations by sectors and currencies), billions of tenge; l_{pr} is the volume of transaction (the volume of credits given by second-level banks (SLB) taking into account the money velocity), billions of tenge.

To estimate the money velocity, let us use the Fisher equation [41, p. 112]

$$MV = Y,$$

where V is the money velocity, Y is the nominal GNI , and the money aggregate $M3$ is accepted in the Fisher equation as the active money volume M .

Estimation of the money velocity by the expression $V = \frac{Y}{M}$ on the basis of the statistical information for the years 2007–2008 is presented in Table 2.5.

The value of the money supply represented in the Fisher equation by the aggregate $M3$ can be checked again through its estimation determined by yearly values of the money base and the money multiplier μ .

The money multiplier μ is defined by the following relation [41, p. 99]:

$$\mu = \frac{1 + \gamma(1 - \alpha - \beta)}{\alpha + \beta + \gamma(1 - \alpha - \beta)},$$

where $\alpha = RR/D$ is the normative of minimal reserve;

$\beta = ER/D$ is the coefficient of cash remainders of the commercial banks;

$\gamma = CM/K$ is the share of money in cash in the total sum of credits of the commercial banks;

RR the minimal reserves;

D is the check (current) deposits (we used the information about deposits in the deposit organizations by sectors and currencies);

ER is the excess reserves;

K is the credits of the commercial banks accepted in accordance with the expression K_1/V ;

K_1 is the statistical information about the given credits;

CM is the active money in cash.

Estimates of the money supply M by the money bases for the years 2007–2008 and values of μ for the same period are respectively equal to the following: for the year 2007, $M = \mu H = 4,519.9$ billion tenge; for the year 2008, $M = \mu H = 5,343.6$ billion tenge.

Table 2.6 presents the calculated values of the money supply and the values of the money aggregate $M3$ by years. Table 2.7 shows that the calculated values of M

Table 2.6 Values of multipliers

Year	α	β	γ	Values of multipliers		
				Deposit	Credit	Money
2007	0.143	0.043	0.250	2.565	2.087	3.087
2008	0.045	0.069	0.252	2.969	2.632	3.632

Table 2.7 Calculated values of money supply and values of money aggregate

Years	Calculated values of money supply	Values of money aggregate $M3$
2007	4,519.9	4,629.8
2008	5,343.6	6,266.4

and values of the money aggregate $M3$ are of the same order and close to each other. Taking into consideration this fact together with the result on the money velocity derived above, in this specific analysis we accept the calculated values as the money supply, and actual values of credits of the second-level banks are corrected subject to the money velocity.

The property demand represented by the expression $l_{pr} = e^{a+l_i i}$ has the following econometric estimate:

$$l_{pr} = 438883.3 \times 0.66^i. \quad (0.000) \quad (0.01) \quad (2.8)$$

The regression coefficients are statistically significant, although we have the coefficient of determination $R^2 = 0.33$; the standard error $Se = 0.6$; the Fisher statistics $F = 67$. The demand of money for transactions represented by the expression $l_{tr} = a + bY$ describes the following econometric estimation:

$$l_{tr} = -1062.85 + 0.326 Y. \quad (0.0005) \quad (0.0000) \quad (2.9)$$

The statistical characteristics of model (2.9) are as follows: the determination coefficient $R^2 = 0.965$; the standard error $Se = 267$; the Fisher statistics $F = 193.7$.

Substituting expressions (2.8, 2.9) into (2.7), we obtain the representation of the LM model in the following form:

$$M_{200X} = 438883.3 \times 0.66^i - 1062.85 + 0.326 Y, \quad (2.10)$$

which allows the determination of the equilibrium value of i for the given values of Y and M_{200X} . In macroeconomic theory one has a method [41, p. 113] of plotting the LM curve, which is the set of combinations of the equilibrium values of Y and i . Fig. 2.4 presents the plots of the LM models for the years 2007 and 2008.

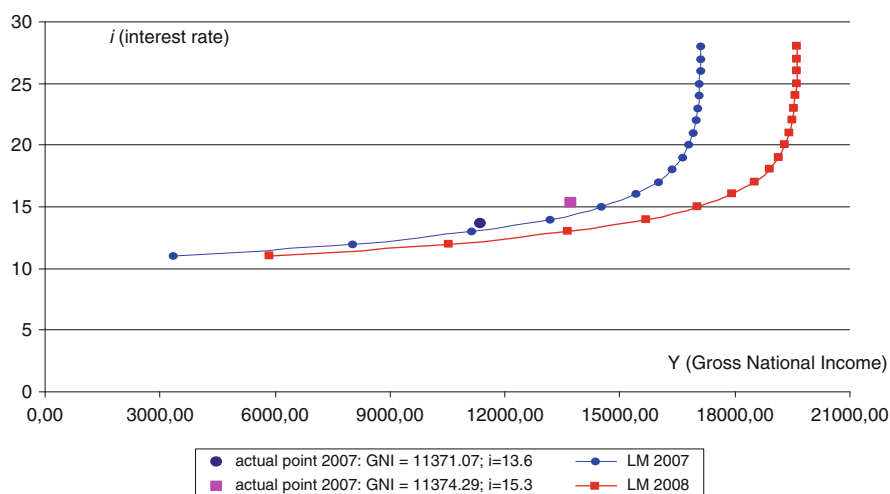


Fig. 2.4 Plots of models LM_{2007} and LM_{2008}

In accordance with the obtained results and plotted LM_{2007} , LM_{2008} , one may conclude that the actual values of Y and i for the years 2007–2008 are situated above the respective curves LM_{2007} , LM_{2008} , which shows the relatively low demand of the monetary assets.

The alarming aspect is that the actual state in which the money market found itself in the year 2008 corresponds to a higher mean market interest rate than in the year 2007, whereas the whole line LM for 2008 is situated below and to the right of the respective line for 2007, i.e., the same volume of GNI corresponds to a lower equilibrium interest rate than that of a year before. This is an indirect indicator that the government has regulated the money market based on the necessity of making money cheaper, but the second-level banks reacted to those signals in the opposite way, raising the commercial rate.

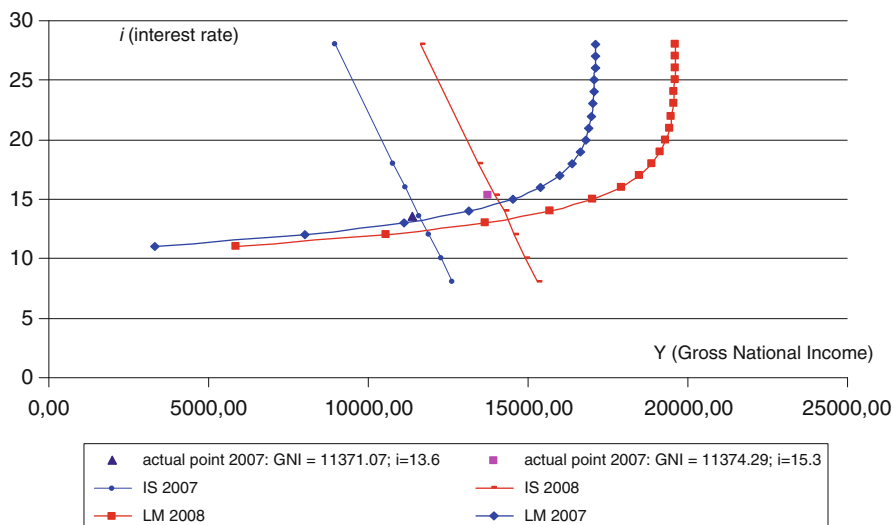
Exactly the same situation occurred in 2008 in most developed countries on the threshold of the economic crisis.

2.2.3 Macro-Estimation of the Mutual Equilibrium State in Wealth and Money Markets. Analysis of the Influence of Economic Instruments

On the basis of the derived IS and LM models, the model for macro-estimation of the joint equilibrium state in the wealth and money markets can be represented by the following system:

Table 2.8 Joint equilibrium and actual values of Y and i

	Actual values		Joint equilibrium conditions	
	i , Interest rate of SLB, %	Y , gross domestic income, billion tenge	i^*	Y^* , Keynesian effective demand
2007	13.6	11,371.1	13.23	11,670.89
2008	15.3	13,734.3	13.29	14,327.31

**Fig. 2.5** Plots of models IS_{2007} , LM_{2008} , LM_{2007} , and LM_{2008}

$$\begin{cases} -366.055 + 0.222Y + 0.2207Y = 3202 - 81.3i + G_{200x}, \\ M_{200x} = 438833.3 \times 0.66^i - 1062.85 + 0.326Y. \end{cases} \quad (2.11)$$

The results of solving system (2.11) to estimate the joint equilibrium state in the wealth and money markets for the years 2007 and 2008 are presented in Table 2.8. The plots of the IS and LM models in the same period are shown in Fig. 2.5.

From Fig. 2.5 it follows that the coordinates of the effective demand point for years 2007 and 2008 are respectively represented by $Y^*_{2007} = 11,670.89$; $i^*_{2007} = 13.23$ and $Y^*_{2008} = 14,327.31$; $i^*_{2008} = 13.29$. The points of the actual state of the economy of the Republic of Kazakhstan in 2007 and 2008 are respectively situated to the left of the corresponding IS_{2007} and IS_{2008} plots and above the respective LM_{2007} and LM_{2008} plots. Such location of the points of the actual economic state means a respective lack of wealth in the wealth market and excess of money in the money market in 2007 and 2008.

Let us estimate the influence of the instruments G and M on the joint equilibrium conditions using the data for the year 2008.

Based on 2008 data, solution of (2.1.11) result in $G = 3,859.98$ and $M = 5,343.6$. Let us now increase G by $\Delta G = 579$. With unchanged M , this fluctuation results in an increase of the Keynesian effective demand GNI up to 15,522 billion tenge and an increase of the interest rate up to 13.9% due to the shift of IS to the right as a result of the multiplicative effect from increasing the public expenses.

Let us now increase M_{2008} by $\Delta M = 534$. With unchanged G_{2008} , this fluctuation results in an increase of GNI up to 15,438.6 billion tenge and a decrease of the interest rate to 12.7% due to the shift of IS to the right as a result of the multiplicative effect from increasing the money supply.

The obtained results also agree with the macroeconomic theory on the influence of economic instruments in the wealth and money markets [41, p. 78; 114].

2.2.4 Macro-Estimation of the Equilibrium State on the Basis of the Keynesian Model of Common Economic Equilibrium. Analysis of the Influence of Economic Instruments

The Keynesian mathematical model of common economic equilibrium on the basis of the IS , LM models, as well as the econometric function of the labor supply price and the econometric expression of the production function is given by the following [41, p. 223]:

$$\begin{cases} T(Y) + S(Y) = I(i) + G, & (2.12) \\ M = l(Y, i), & (2.13) \\ W^s(N, P) = PY_N, & (2.14) \\ Y = Y(N), & (2.15) \end{cases}$$

where $W^s(N, P)$ is the function representing the labor supply price; Y_N is the derivative of the production function; $Y(N)$ is the production function.

Equations (2.12, 2.13) of the common economic equilibrium model are given by the respective IS and LM equations (2.11).

The econometric representation of the labor supply price using the statistical data for the years 2000–2008 is given by

$$W^s(N, P) = 60.12 P - 0.007 N, \\ (0.000) \quad (0.000) \quad (2.16)$$

where P is the level of prices for the year 2000; N is the busy population in thousand. The respective p -values (of t -statistics) in the equation in W^s are presented in parentheses below the regression coefficients. The results of the

Table 2.9 Comparative analysis of actual and equilibrium values of GNI, interest rate, level of prices, employed population

		Y	i	P	N
2007	Actual	11,371.1	13.6	1.789	7,631.1
	Equilibrium	11,670.89	13.23	1.05	7,751.6
	Deviation	2.64%	-0.37	-0.74	1.58%
2008	Actual	13,734.3	15.3	1.959	7,857.2
	Equilibrium	14,327.3	13.3	1.103	8,048.8
	Deviation	4.32%	-2	-0.9	2.44%

analysis of the statistical significance of the model for W^s are as follows: the determination coefficient $R^2 = 0.99$; the standard error $Se = 3.37$; the Fisher statistics $F = 522.6$; the approximation coefficient $A = 7.4\%$.

The econometric representation of the production function $Y(N)$ using the statistical data for the years 2000–2008 is given by

$$Y = -5.654N + 0.0009N^2. \quad (0.000) \quad (0.000) \quad (2.17)$$

The results of analysis of statistical significance of the model for W^s are as follows: the determination coefficient $R^2 = 0.98$; the standard error $Se = 122$; the Fisher statistics $F = 172$.

The Keynesian model of common economic equilibrium on the basis of relations (2.11, 2.16), and (2.17) is given by

$$\begin{cases} -366.055 + S_y Y + T_y Y = 3202 - 81.30i + G_{200X}, \\ M_{200X} = 438883.3 \times 0.66^i - 1062.85 + 0.326 Y, \\ 60.12 P - 0.00698N = -5.65 P + 0.0018N P, \\ Y = -5.65 N + 0.0009 N^2. \end{cases} \quad (2.18)$$

In this system describing the behavior of the macroeconomic subjects, the exogenously given parameters include the value of public expenses G and the nominal values of the money in cash M . The values of five endogenous parameters, Y^* , i^* , P^* , N^* , W^* , that result in attaining equilibrium simultaneously in all three given markets are determined from the solution of this system of equations.

Substituting the actual values of G_{200X} and M_{200X} of the respective year and solving system (2.18), we obtain the values of variables that are in equilibrium simultaneously in all three markets.

Table 2.9 presents the equilibrium values of the endogenous parameters using the solution of system (2.18) on the basis of the data for the years 2007 and 2008.

Let us estimate the influence of instruments G and M on the Keynesian common economic equilibrium from the data from 2008.

Increasing G by $\Delta G = 579$ while keeping the values of M results in an increase of the GNI to 15,522.6 billion tenge and a decrease of the interest rate to 13.9%, while at the same time, the unemployment drops by 1.6%, and the level of prices increases to 1.12.

Increasing M_{2008} by $\Delta M = 534.4$ while keeping the values of G results in an increase of the GNI to 14,438.56 billion tenge and a decrease of the interest rate to 12.68%, while unemployment is reduced by 0.15%, and the level of prices increases insignificantly to 1.105.

Increasing G by $\Delta G = 579$ and increasing M_{2008} by $\Delta M = 534.4$ results in an increase of GNI to 15,658.85 billion tenge and a decrease of the interest rate to 13.15%, while unemployment is reduced by 1.77%, and the level of prices increases to 1.13.

2.2.5 Parametric Control of the Open Economy State Based on the Keynesian Model

Let us estimate the optimal values of the instruments M and G for the given external exogenous parameters S_y , T_y on the basis of model (2.18) for the year 2008 in the sense of the GNI criterion

$$Y \rightarrow \max. \quad (2.19)$$

Such an estimate can be obtained by solving the following mathematical programming problem:

Problem

1. On the base of mathematical model (2.18), find the values of (M, G) maximizing criterion (2.19) under the constraints

$$\begin{cases} |M - M^*| \leq 0.1M^*, \\ |G - G^*| \leq 0.1G^*, \\ |N - N^*| \leq 0.1N^*, \\ |P - P^*| \leq 0.1P^*, \\ |i - i^*| \leq 0.1i^*, \\ |Y - Y^*| \leq 0.1Y^*. \end{cases} \quad (2.20)$$

Here M^* и G^* are the respective actual values of the money and public expenses supplies in 2008. The symbol (*) for the unknown variables of system (2.20) corresponds to the equilibrium values of these variables with fixed values of M^* and G^* .

For Problem 1, the optimal values of the parameters are $M = 5,877.96$, $G = 4,245.98$, which ensure attaining the maximum value of the criterion

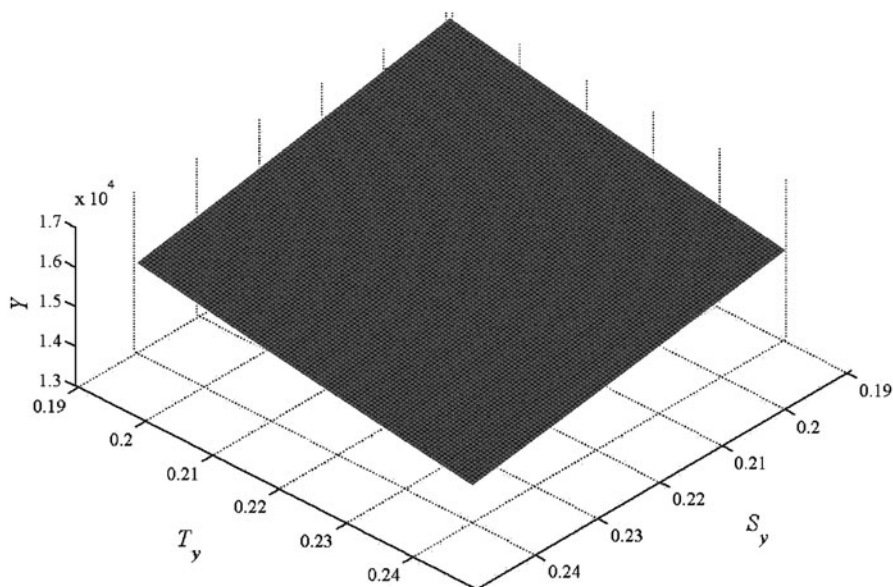


Fig. 2.6 Plot of dependence of optimal values of criterion Y on parameters T_y , S_y

$Y = 15,255.9$. The value of this criterion without control is equal to 14,327.3. For the optimal values of the instruments M and G that were obtained, the equilibrium values of the other endogenous variables turn out to be $N = 8,148.539$; $P = 1.1210$; $i = 12.986$. Here we should also note that solving this optimization problem results in an increase of the working segment of the population by approximately 100,000 people.

On the basis of Problem 1, we carry out the analysis of the dependence of the optimal values of criterion Y on the pair of the exogenous parameters $\{T_y, S_y\}$ given in their respective regions. The obtained plot of the optimal values of criterion (2.19) is presented in Fig. 2.6.

2.3 Long-Term *IS-LM* Model and Mundell–Flemming Model

2.3.1 Problem Statement and Data Preparation

The problem consists in the construction of the long-term *IS-LM-BP* model for the economy of Kazakhstan. The modeling is underlain by the construction of the regression equations for each of the curves included in the model, namely, investment–savings (*IS*), liquidity–money (*LM*), and the balance of payments (*BP*). The derived equation allows plotting the model in the traditional coordinates “income–interest rate.”

The statistical basis for constructing the model is the data of the official statistics of the Republic of Kazakhstan, namely, the following indexes: the gross domestic product (*GDP*), the interest rate, money aggregates *M2* and *M3*, the investment level, the volume of public expenses, the exchange rate of tenge to US dollars, and the net export, as well as the price of Urals oil (the data are presented in Table 2.10).

Let us choose correlation analysis (Table 2.11) as the instrument of preliminary analysis needed in further modeling.

2.3.2 Model Construction

The model is constructed under the assumption that the curves of *IS*, *LM*, and *BP* remain immovable over the considered period, an assumption that, strictly speaking, does not correspond to reality. Therefore, as a result of computations, we obtain the “averaged” Mundell–Fleming long-term model.

2.3.2.1 “Investment–Savings” (*IS*) Curve

The *IS* curve is the formalized reflection of all possible equilibrium states in the market of goods and services. This equilibrium assumes equality between the volume of national savings and the level of gross investment. The former depends immediately on the national income (*GDP*), while the latter depends on the interest rate. So let us accept the regression dependence of *GDP* on the interest rate as the initial dependence including the investment level or public expenses as the explanatory variables:

$$Y_{IS} = Y(R; I), \quad Y_{IS} = Y(R; G).$$

Correlation analysis reflects the presence of a connection between the variables entering the regression equation at which the level is significant (the correlation coefficient between *GDP* and the interest rate is equal to -0.71 , between *GDP* and the investment is equal to 0.95 , between *GDP* and the level of public expenses is equal to 0.99). However, the correlation between the explanatory variables of the model, namely the interest rate and investment as well as the interest rate and public expenses, is also considerable (-0.698 and -0.737 , respectively). This fact entails the problem of multicollinearity in the model. Let us analyze this problem by the basic econometric indexes:

$$Y_{IS} = c + a_1 R + a_2 G + \varepsilon,$$

$$Y = 20699.17234 \times R + 10.74660517 \times G - 1129241.822$$

Table 2.10 Statistical data on basic indexes of macroeconomic dynamics of the Republic of Kazakhstan for the years 2000–2008

Year	<i>Y</i>	<i>R</i>	<i>I</i>	<i>G</i>	<i>NX</i>	<i>E</i>	<i>M2</i>	<i>M3</i>	<i>PP</i>
2000	2,599,901.6	8,679417	519,000	313,984.5	195,126.8	142.13	249,142.9	322,312.7	24,874.62
2001	3,055,068.891	10,25376	729,323.3	409,808.6	−32,354.1	139.4857	304,050.5	443,335.1	20,951.37
2002	3,329,392.871	8,968105	1,051,820	386,526	−1,678.15	140.2352	338,320	554,087	20,761.84
2003	3,807,298.146	8,464419	1,096,184	428,608.4	204,479	147.3835	497,687.5	729,412.4	24,249.48
2004	4,541,648.642	7,291471	1,318,119	527,489.9	390,201.2	114.6345	702,109.3	960,779.1	23,874.79
2005	5,463,019.682	5,981395	1,742,399	614,509.3	481,646.4	108.3157	968,457.6	1,327,192	31,602.33
2006	6,781,287.746	4,612546	1,875,309	690,393.6	723,764.7	96.78848	1,394,648	1,835,797	34,471.47
2007	7,181,487.945	−4,02357	1,895,757	793,823.4	480,370.2	82.57355	1,901,954	2,384,855	32,179.73
2008	8,133,751.486	5,771689	1,957,906	850,104.7	1,651,949	74.04733	2,017,338	2,738,204	40,700.53

Here *Y* is the annual level of real *GDP* in millions of tenge (in prices of the year 2000)

I is the investment volume in million tenge (in prices of the year 2000)

G is the level of public expenses in millions of tenge (in prices of the year 2000)

NX is the net export in millions of tenge (in prices of the year 2000)

R is the real interest rate calculated by the Fisher equation with use of the current inflation level (the consumer price index is used as the inflation index; the mean credit interest rate is used as the nominal interest rate)

E is the real interest rate calculated by the Fisher equation with use of the current inflation level (the consumer price index is used as the inflation index; the mean credit interest rate is used as the nominal interest rate)

M2, *M3* are the respective money aggregates in millions of tenge (in prices of the year 2000)*PP* is the price of one ton of Urals oil in tenge (in prices of the year 2000)

Table 2.11 Correlation matrix of indexes of the macroeconomic dynamics of the Republic of Kazakhstan for 2000–2008

<i>Y</i>	<i>R</i>	<i>I</i>	<i>G</i>	<i>NX</i>	<i>E</i>	<i>M2</i>	<i>M3</i>	<i>PP</i>
<i>Y</i>	1							
<i>R</i>	-0.711698533	1						
<i>I</i>	0.954742036	-0.69807	1					
<i>G</i>	0.992147203	-0.73752	0.946283	1				
<i>NX</i>	0.852656102	-0.33054	0.732093	0.823695	1			
<i>E</i>	-0.968299283	0.734224	-0.90791	-0.82786	1			
<i>M2</i>	0.984931584	-0.78136	0.908705	0.827694	-0.96861	1		
<i>M3</i>	0.98991353	-0.74259	0.916602	0.853583	-0.96661	0.997587	1	
<i>PP</i>	0.928746735	-0.55291	0.845586	0.924429	-0.8964	0.911159	0.92066	1

Dependent variable: Y
 Method: least squares
 Date: 11/27/09 Time: 21:35
 Sample: 2000 2008
 Included observations: 9

Variable	Coefficient	Std. error	t -Statistic	Prob.
R	20,699.17	34,610.14	0.598067	0.5717
G	10.74661	0.770466	13.94819	0.0000
C	-1,129,242.	613,042.2	-1.842030	0.1151
R-squared	0.985236	Mean dependent var		4,988,095.
Adjusted R-squared	0.980315	S.D. dependent var		1,997,879.
S.E. of regression	280,309.4	Akaike info criterion		28.18638
Sum squared resid	4.71E + 11	Schwarz criterion		28.25212
Log likelihood	-123.8387	F-statistic		200.1997
Durbin-Watson stat	2.869894	Prob(F-statistic)		0.000003

In spite of the high value of the index R^2 , the value of the coefficient of the interest rate turns out to be insignificant (high p -level). This can be partially explained by the sign of this coefficient. It is positive, though the theoretical derivation of the model shows the presence of a negative connection between GDP and the interest rate, and this connection is confirmed by the sign of the correlation coefficient between them:

$$Y_{IS} = c + a_1 \times R + a_2 \times I + \varepsilon,$$

$$Y = -41559.38539 \times R + 3.308409085 \times I + 767164.6307$$

Dependent variable: Y
 Method: least squares
 Date: 11/27/09 Time: 21:40
 Sample: 2000 2008
 Included observations: 9

Variable	Coefficient	Std. error	t -Statistic	Prob.
R	-41,559.39	78,083.25	-0.532245	0.6137
I	3.308409	0.613837	5.389718	0.0017
C	767,164.6	1,241,178.	0.618094	0.5592
R-squared	0.915521	Mean dependent var		4,988,095.
Adjusted R-squared	0.887361	S.D. dependent var		1,997,879.
S.E. of regression	670,522.1	Akaike info criterion		29.93070
Sum squared resid	2.70E + 12	Schwarz criterion		29.99644
Log likelihood	-131.6882	F-statistic		32.51176
Durbin-Watson stat	1.164057	Prob(F-statistic)		0.000603

In this case, we are faced with the same problem of statistical insignificance of the coefficient of the interest rate, although its sign completely agrees with the theory.

Let us also estimate the “lite” version of the equation:

$$Y_{IS} = c + a_1 \times R + \varepsilon,$$

$$Y = -335340.1444 \times R + 7074627.596$$

Dependent variable: <i>Y</i>				
Method: least squares				
Date: 11/27/09 Time: 21:44				
Sample: 2000 2008				
Included observations: 9				
Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
<i>R</i>	−335,340.1	125,105.8	−2.680452	0.0315
<i>C</i>	7,074,628.	925,242.8	7.646239	0.0001
R-squared	0.506515	Mean dependent var		4,988,095.
Adjusted R-squared	0.436017	S.D. dependent var		1,997,879.
S.E. of regression	1,500,383.	Akaike info criterion		31.47347
Sum squared resid	1.58E + 13	Schwarz criterion		31.51730
Log likelihood	−139.6306	F-statistic		7.184823
Durbin–Watson stat	1.672475	Prob(F-statistic)		0.031519

Now, the coefficient of the interest rate is significant, but the common reliability of the model suffers. It is expressed in the decrease of the coefficient R^2 . This model is not worth considering for the characterization of the curve *IS*, since the connection “*GDP*–interest rate” is present in all three curves of the model *IS-LM-BP*. Moreover, it would lead to an unacceptable situation when different dependencies will be described by the same question.

In this situation, when none of the derived regression equations are fully satisfactory, let us consider the other possible dependencies describing the curve *IS*. The insignificant coefficients of the interest rate in the regressions with two explanatory variables lead to the idea of some “lag” of the reaction of the *GDP* to the variation of the interest rate. From a logical point of view, this hypothesis is not groundless. The macroeconomic indexes are rather resilient with respect to the reactions, and the time lag with such reactions is a regular phenomenon. Therefore, let us construct a model in which the *GDP* depends only on the value of the interest rate in the preceding year. Also, let us keep the investment level in the model:

$$Y_{IS} = c + a_1 \times R_{-1} + a_2 \times I + \varepsilon,$$

$$Y(n) = -152650.914 \times R(n-1) + 2.855488855 \times I(n) + 2080721.113$$

Dependent variable: Y

Method: least squares

Date: 11/27/09 Time: 22:40

Sample (adjusted): 2001 2008

Included observations: 8 after adjustments

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
$R(-I)$	-152,650.9	38,816.77	-3.932602	0.0110
I	2.855489	0.374470	7.625414	0.0006
C	2,080,721.	746,195.3	2.788441	0.0385
R-squared	0.978132	Mean dependent var		5,286,619.
Adjusted R-squared	0.969384	S.D. dependent var		1,909,217.
S.E. of regression	334,061.3	Akaike info criterion		28.55603
Sum squared resid	5.58E + 11	Schwarz criterion		28.58582
Log likelihood	-111.2241	F-statistic		111.8211
Durbin-Watson stat	2.273508	Prob(F-statistic)		0.000071

This model seems to be perfect with respect to all the indexes (the significance of the coefficients, the significance of the model as a whole, R^2). The sign of the coefficient of the interest rate also meets the theoretical requirements. Thus, we will use just this regression equation as the curve describing the dependence “investment–savings.”

While carrying out the analysis, the following regressions are also constructed: $Y_{IS} = c + a_1 \times R_{-1} + a_2 \times G + \varepsilon$, $Y_{IS} = c + a_1 \times R_{-1} + \varepsilon$. However, the chosen model turns out to be the best one for all the parameters.

2.3.2.2 “Liquidity–Money” (LM) Curve

This curve describes the money market equilibrium, as well as the stock market equilibrium. According to the theory, the money demand depends on the income level and the real interest rate, $M^s = M(Y; R)$. To preserve the model’s logic, let us consider just this dependence, but not the implicit function $Y = Y(M; R)$:

$$M^s = b_1 \times Y + b_2 \times R + \varepsilon,$$

$$M2 = 0.2490654962 \times Y - 48136.28677 \times R$$

Dependent variable: M_2

Method: least squares

Date: 11/28/09 Time: 01:38

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
Y	0.249065	0.010246	24.30966	0.0000
R	-48,136.29	7,386.492	-6.516799	0.0003

(continued)

R-squared	0.971418	Mean dependent var	930,412.0
Adjusted R-squared	0.967335	S.D. dependent var	688,173.9
S.E. of regression	124,377.4	Akaike info criterion	26.49316
Sum squared resid	1.08E + 11	Schwarz criterion	26.53699
Log likelihood	−117.2192	Durbin–Watson stat	1.247213

$$M3 = 0.3236175168 \times Y - 54199.73886 \times R$$

Dependent variable: M_3

Method: least squares

Date: 11/28/09 Time: 01:40

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	<i>t</i> -Statistic	Prob.
<i>Y</i>	0.323618	0.015388	21.03010	0.0000
<i>R</i>	−54,199.74	11,094.16	−4.885432	0.0018
R-squared	0.960611	Mean dependent var		1255108.
Adjusted R-squared	0.954984	S.D. dependent var		880,468.3
S.E. of regression	186,808.9	Akaike info criterion		27.30669
Sum squared resid	2.44E + 11	Schwarz criterion		27.35052
Log likelihood	−120.8801	Durbin–Watson stat		1.412212

The regression equation with the aggregate $M2$ as the index of the money supply gives some better results in comparison with $M3$. This can be explained by the fact that certificates of deposit, public bonds, exchequer savings stock, commercial securities, by the value of which $M3$ differs from $M2$, are not so marketable to represent the money demand as such. Therefore, for the further representation of curve LM we use the first regression equation.

2.3.2.3 “Balance of Payment” (BP) Curve

The balance of payment curve characterizes the foreign market equilibrium (the equilibrium in operations with the “foreign” sector). Therefore, the levels of the real currency exchange rate, net export, as well as the oil price (besides the interest rate) can appear for the regressors. Since from the correlation analysis it can be seen that these additional regressors demonstrate a close connection, let us estimate the regression equations with respect to each of them individually (of course, including the interest rate):

$$Y_{BP} = c + d_1 \times R + d_2 \times E + \varepsilon,$$

$$Y = -767.0093069 \times R - 69893.29419 \times E + 13112868.16$$

Dependent variable: Y
 Method: least squares
 Date: 11/28/09 Time: 01:54
 Sample: 2000 2008
 Included observations: 9

Variable	Coefficient	Std. error	t -Statistic	Prob.
R	-767.0093	70,774.86	-0.010837	0.9917
E	-69,893.29	10,855.55	-6.438485	0.0007
C	13,112,868	1,002,904.	13.07490	0.0000
R-squared	0.937605	Mean dependent var		4,988,095.
Adjusted R-squared	0.916806	S.D. dependent var		1,997,879.
S.E. of regression	576,254.6	Akaike info criterion		29.62769
Sum squared resid	1.99E + 12	Schwarz criterion		29.69343
Log likelihood	-130.3246	F-statistic		45.08056
Durbin-Watson stat	2.277324	Prob(F-statistic)		0.000243

Here, the statistical insignificance of the interest rate coefficient can be explained by multicollinearity, since the interest rate and the exchange rate are highly correlated. Unfortunately, the interest rate cannot be removed from this model, since by doing so, the required connection between the GDP and interest rate would be corrupted:

$$Y_{BP} = c + d_1 \times R + d_2 \times NX + \varepsilon,$$

$$Y = -227386.5054 \times R + 2.711387294 \times NX + 5169694.357$$

Dependent variable: Y
 Method: least squares
 Date: 11/28/09 Time: 01:56
 Sample: 2000 2008
 Included observations: 9

Variable	Coefficient	Std. error	t -Statistic	Prob.
R	-227,386.5	52,175.57	-4.358103	0.0048
NX	2.711387	0.433159	6.259562	0.0008
C	5,169,694.	474,598.0	10.89279	0.0000
R-squared	0.934467	Mean dependent var		4,988,095.
Adjusted R-squared	0.912623	S.D. dependent var		1,997,879.
S.E. of regression	590,565.3	Akaike info criterion		29.67675
Sum squared resid	2.09E + 12	Schwarz criterion		29.74249
Log likelihood	-130.5454	F-statistic		42.77860
Durbin-Watson stat	1.517209	Prob(F-statistic)		0.000281

$$Y_{BP} = c + d_1 \times R + d_2 \times PP + \varepsilon,$$

$$Y = -134497.9641 \times R + 224.5877659 \times PP - 505075.5724$$

Dependent variable: Y

Method: least squares

Date: 11/28/09 Time: 01:57

Sample: 2000 2008

Included observations: 9

Variable	Coefficient	Std. error	t -Statistic	Prob.
R	−134,498.0	65,645.92	−2.048840	0.0864
PP	224.5878	40.58780	5.533381	0.0015
C	−505,075.6	1,428,299.	−0.353620	0.7357
R-squared	0.919141	Mean dependent var		4,988,095.
Adjusted R-squared	0.892188	S.D. dependent var		1,997,879.
S.E. of regression	655,997.4	Akaike info criterion		29.88690
Sum squared resid	2.58E + 12	Schwarz criterion		29.95264
Log likelihood	−131.4911	F-statistic		34.10173
Durbin–Watson stat	1.704565	Prob(F-statistic)		0.000529

As the p -level shows, in the third model, the confidence level with respect to the value of the free term is low. Thus, the character of the dependence between Y and R is revealed, but the precise position of the respective curve BP still remains unclear.

It is obvious that the second model most precisely reflects reality. In this model, the interest rate and the net export appear for the regressors.

Let us draw our attention to the fact that in all three models of the curve BP , the dependence between Y and the market average interest rate becomes negative, and moreover, the curve BP has a steeper slope than the curve IS situated similarly. This is evidence of the relatively closed economic system of Kazakhstan for short-term capital mobility and the presence of investment barriers to entering local markets.

2.3.3 Final *IS-LM-BP* Model

Now let us introduce the complete model, reasoning from the derived regression equations:

$$IS : Y(n) = -152650.914 \times R(n-1) + 2.855488855 \times I(n) + 2080721.113,$$

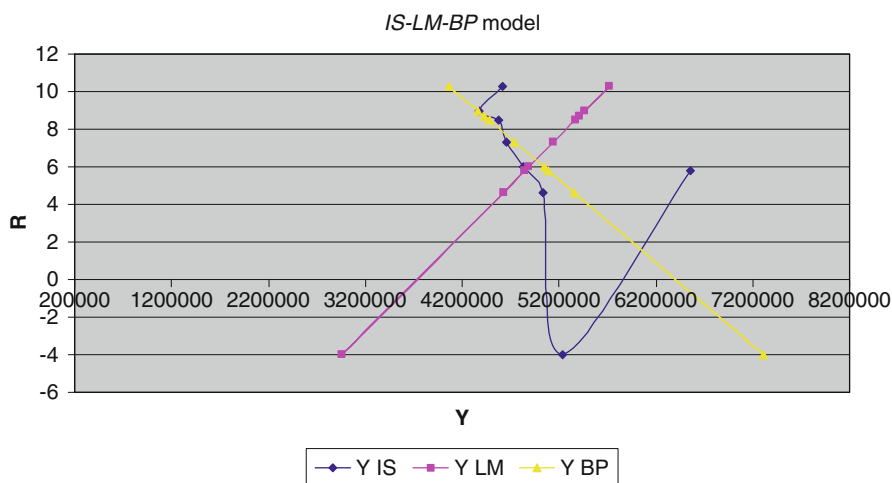
$$LM : M2(n) = 0.2490654962 \times Y(n) - 48136.28677 \times R(n),$$

$$BP : Y(n) = -227386.5054 \times R(n) + 2.711387294 \times NX(n) + 5169694.357.$$

Averaging the values of the “floating” variables I , $M2$, and NX for the considered period, let us derive three equations expressing the dependence between the current yearly GDP and the interest rate. The combination of these three equations gives the *Mundell-Flemming long-term model* for the present-day Republic of Kazakhstan.

Table 2.12 Model values of Y

Y IS	Y LM	Y BP
	5,413,062	4,429,343
4,622,074	5,717,331	4,071,359
4,381,749	5,468,856	4,363,699
4,578,006	5,371,510	4,478,230
4,654,894	5,144,817	4,744,943
4,833,945	4,891,622	5,042,837
5,033,930	4,627,068	5,354,095
5,242,886	2,957,986	7,317,831
6,561,197	4,851,092	5,090,521

**Fig. 2.7** *IS-LM-BP* model in coordinates “income–interest rate”

Using these equations, let us find the respective model values for Y (Table 2.12).

The obtained data allow us to plot Fig. 2.7.

The curve *IS* is not a straight line, since it depends on the preceding values of the interest rate. Furthermore, the “incorrect” slope of the line *BP* is very noticeable. In theory, with an increase in the domestic interest rate, capital inflow to the country must take place, i.e., the plot must be ascending. But the equilibrium of the balance of payments (the line *BP* reflects just this) depends not so much on the value of the domestic interest rate as on its deviation from the world average rate. Thus, we can assume that during the considered period, investment abroad promised greater profitability for the residents of Kazakhstan, that is, the difference between the domestic and world interest rates was negative, which becomes apparent from the sign of the coefficient of the variable R . Hence even the growth of the domestic interest rate was accompanied by the outflow of capital from the country. Also, the same may be said about the level of risk. As a rule, investment abroad is less risky than that inside countries suffering from problems of reform and modernization.

Let us draw attention to the fact that the current equilibrium of the money market is achieved with little deficit of the balance of payments, and the current equilibrium of the real market is attained with an excess and deficit of the money supply (in the points situated below curve LM , the money demand exceeds the money supply). This means that the monetization level of the economy of Kazakhstan is insufficient, and as long as this drawback is not overcome, application of Keynesian arrangements for stimulating economic growth is rather dangerous, because the economy runs the risk of choking over the deficit of means of payment. However, this does not reflect world problems concerned with a liquidity deficit. This is the result of the rather tough monetary policy of the government of Kazakhstan.

As a whole, it should be noted that the year 2005 was closest to equilibrium. However, in spite of two later years, the points on the curves are also rather close one to another. It is evidence of the state's effort to carry out the policy of maintaining equilibrium in the real and money markets.

2.4 Macroeconomic Analysis and Parametric Control of the National Economic State Based on the Model of a Small Open Country

Ensuring a double equilibrium, that is, a common economic equilibrium in conditions of full employment with a planned (assumed zero) balance of payments, is an urgent problem in the conditions of an open economy when the country is engaged in the free exchange of goods and capital with the outside world.

All remaining states of the national economy differing from double equilibrium represent various kinds of nonequilibrium states. Hence unemployment remains the same in spite of an excess in the balance of payments. Unemployment can be accompanied by an excess in the balance of payments. The excess of employment can be accompanied by both the excess and deficiency of the balance of payments. Therefore, public economic policy aims at attainment of a double equilibrium. The estimation of the equilibrium conditions for an open economy can be partially considered on the basis of the model of a small country [41, p. 433].

This section is devoted to the construction of a mathematical model of an open economy of a small country using the example of the Republic of Kazakhstan, to the analysis of the influence of economic instruments on the conditions of common economic equilibrium and state of the balance of payments, and to the estimation of the optimal values of the economic instruments on the basis of the model of an open economy of a small country, as well as an analysis of the dependencies of the optimal values of the criteria on the values of one, two, and three parameters from the set of the external economic parameters given in the respective regions.

2.4.1 Construction of the Model of an Open Economy of a Small Country and the Estimation of Equilibrium Conditions

Let us introduce the following notation for the economic indexes used for the model construction: Y is the gross national income (GNI); C is household consumption; I is the investment in capital assets; G is public expenses; NE is the net export of wealth; P is the level of prices of RK ; l is the real cash remainder; i is the interest rate of second-level banks; N is the number of employed; dY/dN is the derivative of the gross national income as a function of the number of employed; W^S is the level of wages; NKE is the net capital export; e is the rate of exchange of the national currency; e^e is the expected rate of exchange of the national currency; $\hat{e}^e$ is the expected rate of increase of the exchange rate of the national currency [41, p. 121]; M is the money supply determined from [41, p. 412] by the formula $M = \mu H$, where H is the money base of each year; μ is the money multiplier calculated from the balance equations of the banking system and defined by the formula

$$\mu = \frac{(1 + \gamma(1 - \alpha - \beta))}{(\alpha + \beta + \gamma(1 - \alpha - \beta))} \quad (2.21)$$

where $\alpha = RR/D$ is the norm of the minimal reserve; $\beta = ER/D$ is the coefficient of the cash remainder of the second-level banks; $\gamma = CM/K$ is the share of cash in the whole sum of the credits of second-level banks; RR is the minimal reserve; ER is the excessive reserve; D is the check deposits; CM is the active money in cash; K is the credits of second-level banks corrected subject to the velocity of money.

Let us begin to construct a mathematical model of an open economy of a small country by estimating the money multiplier, real cash remainders, and economic functions characterizing the national economic state.

The estimations of values of the money multiplier calculated by formula (2.21) using the statistical data for the period of years 2,006–2008 are presented below:

Year	2006	2007	2008
μ	2.372	3.087	3.632

The real cash remainder l is determined by the formula

$$l = l_{pr} + l_{tr}, \quad (2.22)$$

where l_{pr} is the property volume (deposits in the deposit organizations, by sectors and kinds of currency), billions of tenge; l_{tr} is the volume of the transaction (the volume of the credits given by second-level banks subject to the money velocity), billions of tenge.

The estimation of the money velocity is calculated by the Fisher equation [43]:

Table 2.13 Values of *GNI* (billions of tenge), money aggregate *M3* (billions of tenge), and money velocity *V*

Year	<i>GNI</i>	<i>M3</i>	<i>V</i>
2007	11,371	4,629.8	2.5
2008	13,734	6,266.4	2.2

$$MV = Y,$$

where *V* is the money velocity; *M* is the quantity of the active money usually represented by the money aggregate *M3* in the Fisher equation.

From the latter formula, the estimates of the money velocity calculated by the formula $V = Y/M$ on the basis of the statistical information for 2006–2008 [40] are presented in Table 2.13.

In the macroeconomic theory, the behavior of the national economy is characterized by the following functions constructed by econometric methods [1] on the basis of official statistical information.

The consumption *C* represented by the expression $C = a + C_Y Y$ has the following econometric estimation derived on the basis of the statistical information of the Republic of Kazakhstan for the period 2000–2008:

$$C = 474.2 + 0.4531 Y. \\ (0.00) + (0.00) \quad (2.23)$$

The statistical characteristics of the constructed model of the consumption *C* are as follows: the determination coefficient $R^2 = 0.999$, the approximation coefficient $A = 1.9\%$. The statistical significance of the coefficients of regression (2.23), as well as the regressions estimated below, are presented in parentheses under the respective regression coefficients as the *p*-values.

The consumption of the imported wealth Q_{im} is represented by the regression equation

$$Q_{im} = a_1 Y + b_1 e P^Z / P$$

or, in estimated form,

$$Q_{im} = 0.3946 Y - 2.6125 e P^Z / P \\ (0.00) \quad (0.03) \quad (2.24)$$

with the determination coefficient $R^2 = 0.91$ and the approximation coefficient $A = 11\%$.

The model of the demand of the real cash remainder is given by $l = a_2 + b_2 Y + b_3 i + b_4 e$ or, after estimating the parameters of this model using the statistical information,

$$l = -6758.3 + 0.9973 Y - 175.5 i + 38.4 e. \quad (2.25)$$

(0.3) (0.04) (0.7) (0.5)

In constructing model (2.25), the values of l calculated in accordance with formula (2.22) are accepted as the data for the left-hand side. The determination coefficient is given by $R^2 = 0.995$, and the approximation coefficient is $A = 6\%$. The statistical insignificance of the latter model concerns the fact that in the model there are correlated factors. Thus, the gross national income has a strong correlation with the exchange rate ($R = 0.92$) and a direct connection with the interest rate ($R = 0.65$).

The model of the labor supply price is given by $W^s = b_5 N + b_6 P_{\text{mean}}$, where $P_{\text{mean}} = (1-\alpha)P + \alpha eP^z/e^0$ has the following econometric estimation derived on the basis of the statistical information:

$$W^s = -0.025 N + 175.5 P_{\text{mean}} \quad (2.26)$$

(0.00) (0.00)

where $P_{\text{mean}} = 0.6 P + 0.4 eP^z/e^0$, e^0 is the currency exchange rate within the base period (year 2000); α is the share of the imported goods in their entire volume accepted at the level of 0.4. We also have the determination coefficient $R^2 = 0.98$ and the approximation coefficient $A = 0.07\%$.

The model of the net capital export is given by $NKE = b_7 e(i^z + \bar{e}^e - i)$ or, after estimating the parameters of this model using the statistical information,

$$NKE = -0.47 e(i^z + \bar{e}^e - i) \quad (2.27)$$

(0.02)

with the determination coefficient $R^2 = 0.62$ and the approximation coefficient $A = 3.2\%$.

The production function is represented in the regression pair $Y = a_3 + b_8 N$ or, in the estimated form,

$$Y = -44477.9 + 7.5 N \quad (2.28)$$

(0.00) (0.00)

with the determination coefficient $R^2 = 0.88$ and the approximation coefficient $A = 12\%$.

The model of investment in capital assets is given by

$$I_t = a_4 + b_9 Y_{t-1} + b_{10} i_t,$$

where I_t and i_t are the values of the investments in the current period; Y_{t-1} is the value of the gross national income in the preceding period.

After estimating the latter model parameters using the statistical data, the following expression is derived:

$$I_t = 1367.9 + 0.2753 Y_{t-1} - 81.3 i_t \quad (0.02) \quad (0.03) \quad (0.00) \quad (2.29)$$

We have the determination coefficient $R^2 = 0.98$ and the approximation coefficient $A = 5\%$.

Substituting the value $Y_{t-1} = Y_{2007}$ into (2.29), finally we obtain the following model of investment in the year 2008:

$$I_{2008} = 5148.9 - 81.3i. \quad (2.30)$$

Similarly, substituting the value $Y_{t-1} = Y_{2006}$ into (2.29) for investment in 2007, we obtain the following model:

$$I_{2007} = 3857.6 - 81.3i \quad (2.31)$$

The wealth export model is a regression of the form $Q_{ex} = b_{11} eP^Z/P$. After estimating the parameters, this model becomes

$$Q_{ex} = \frac{25.68 eP^Z}{P}. \quad (0.02) \quad (2.32)$$

The determination coefficient is $R^2 = 0.50$.

On the basis of derived econometric estimates (2.23–2.32) characterizing the state of the national economy, let us proceed to the construction of a model of an open economy of a small country for the year 2008.

Within the framework of the *IS* curve, we constructed the function $Y = C + I + G + Q_{ex} - Q_{im}$, which subject to (2.23, 2.24, 2.29–2.32), becomes

$$\begin{aligned} Y &= 474.2 + 0.4531 Y + 5148.9 - 81.3i + G + 28.29 eP^Z/P - 0.3946 Y \text{ or} \\ Y &= 5985.2 - 86.54 i + 30.11 eP^Z/P + 1.064 G \end{aligned} \quad (2.33)$$

The equation of the *LM* line $M/P = l$ subject to the econometric model (2.25) becomes

$$\frac{M}{P} = -6758.3 + 0.9973 Y - 175.5 i + 38.4 e,$$

from which one can derive the following relation:

$$i = -38.51 + 0.2190 e + 0.0057 Y - 0.0057 M/P. \quad (2.34)$$

Substituting (2.34) into (2.33), we obtain the value of the aggregate demand Y^D :

$$Y^D = 6246.1 - 12.70 e + 20.18 eP^Z/P + 0.7135G + 0.3305M/P. \quad (2.35)$$

Let us substitute (2.33) into (2.34) and determine the function of the domestic commercial interest rate:

$$i = -3.0147 + 0.1468 e - 0.0038 M/P + 0.1147 eP^Z/P + 0.0041 G. \quad (2.36)$$

The condition of equilibrium in the labor market is given by $P dY/dN = W^S$ [41, p. 435], which subject to the econometric functions (2.26) and (2.28) can be represented by the expression

$$7.5P = -0.025 N + 175.5(0.6P + 0.4 eP^Z/e^0). \quad (2.37)$$

From (2.37) we obtain the following relation for N :

$$N = 3915.9 P + 19.7758 eP^Z. \quad (2.38)$$

Substituting expression (2.38) into the production function (2.28), we obtain the function of the aggregate supply:

$$Y^S = -44477.9 + 29368.9 P + 148.3 eP^Z. \quad (2.39)$$

The balance of payments has a zero balance if the net wealth export equals the net capital export, i.e., the following holds: $NE = NKE$. The econometric representation of the latter equality on the basis of (2.24, 2.27, 2.32) is given by

$$25.68 eP^Z/P - (0.3946 Y - 2.6125 eP^Z/P) = -0.47e(i^z + \hat{e}^e - i)$$

Substituting the value of domestic interest rate (2.36) into the latter equality, after some transformation we obtain the following equation of the curve of the zero balance of payments:

$$Y^{ZBO} = 72.0543 eP^Z/P - 1.1971 e i^Z/P - 1.1971 e^e/P - 2.412 e/P + 0.1757 e^2/P - 0.0046 eM/P^2 + 0.1373 e^2P^Z/P^2 + 0.0049 eG/P. \quad (2.40)$$

Thus, the model of an open economy of a small country in the year 2008 is given by the following system of equations:

$$\left\{ \begin{array}{l} Y^D = 6246,1 - 12.7e + 2018 \frac{eP^Z}{P} + 0,7135G + 0,3305 \frac{M}{P}, \\ Y^S = -44477.9 + 29368.9P + 148.3eP^Z, \\ Y^{ZBO} = 72.05 \frac{eP^Z}{P} - 1.1971 \frac{e^Z}{P} - 1.1971 \frac{e^e}{P} - 2.412 \frac{e}{P} + 0.1757 \frac{e^2}{P} \\ \quad - 0.0046 \frac{eM}{P^2} + 0.1373 \frac{e^2P^Z}{P^2} + 0.0049 \frac{eG}{P}, \\ Y^D = Y^S = Y^{ZBO}. \end{array} \right. \quad (2.41)$$

A model of an open economy of a small country in 2007 can be constructed similar to (2.41).

Solving system (2.41) with prescribed values of the external economic indexes P^Z , i^Z , e^e and the economic instruments M and G , let us determine the equilibrium conditions of the gross national income $Y^* = Y^D = Y^S = Y^{ZBO}$, level of prices P^* , and exchange rate of the national currency e^* . The equilibrium values of the credit interest rate of the second-level banks i^* and the number of employed are calculated by formulas (2.36) and (2.38), respectively.

The following equilibrium values of the endogenous variables are obtained by solving system (2.41) for the given external uncontrolled economic indexes P^Z , i^Z , e^e and the controlled economic instruments M and G :

- In the year 2007: $Y^* = 9,398.1$; $P^* = 1.1699$; $e^* = 109.0$; $i^* = 16.8$; $N^* = 7,183.5$
- In the year 2008: $Y^* = 11,383.0$; $P^* = 1.1924$; $e^* = 116.3$; $i^* = 26.1$; $N^* = 7,448.1$

Figure 2.8 presents the double equilibrium state, where the point of intersection of the $IS-LM-ZBO$ curves ($i^* = 16.8\%$, $Y^* = 9,398.1$) corresponds to a simultaneous equilibrium in the wealth, money, and labor markets with full employment and zero balance of payments in the year 2007. All combinations of the values of the national income and interest rate besides this point represent various kinds of nonequilibrium states. According to the plotted curves, Kazakhstan has cyclical unemployment [41, p. 206] and a deficit in the balance of payments, which is confirmed by the official statistics. In Fig. 2.8, such a situation is represented by the point A ($Y_{2007} = 11,371.1$; $i_{2007} = 13.6\%$).

Figure 2.9 presents the double equilibrium state, and the intersection point of the $IS-LM-ZBO$ curves corresponds to the simultaneous equilibrium in the wealth, money, and labor markets with full employment and zero balance of payments in the year 2008. All combinations of the values of the national income and interest rate besides this point represent various kinds of nonequilibrium states. According to the plotted curves, Kazakhstan also has cyclical unemployment and a deficit in the balance of payments, which is confirmed by the official statistics. In Fig. 2.8, such a situation is represented by the point B ($Y_{2008} = 13734$, $i_{2008} = 15.3\%$).

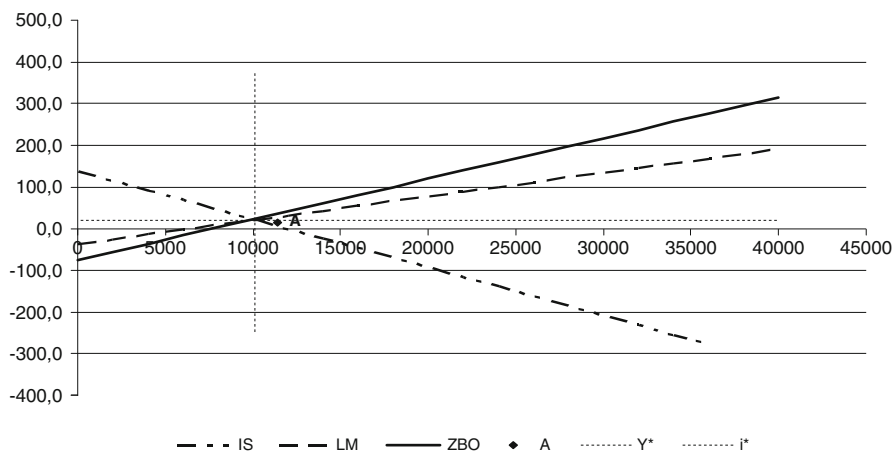


Fig. 2.8 Double balance in the year 2007

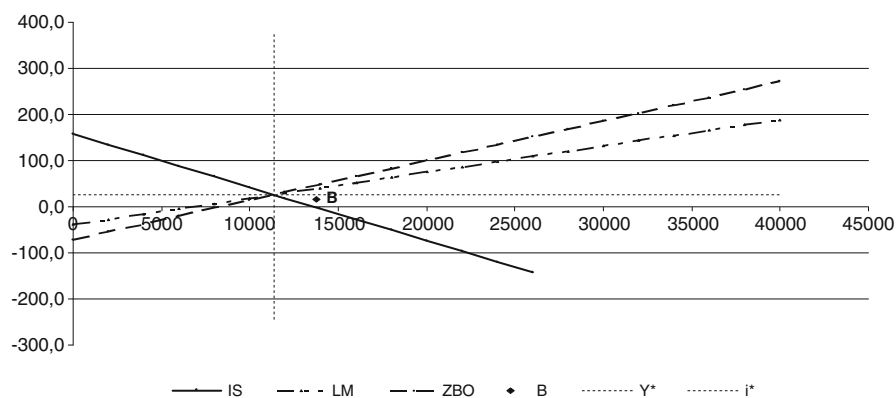


Fig. 2.9 Double equilibrium in the year 2008

However, it can be noted that in accordance with the official statistics, Kazakhstan has an excessive balance of payments.

Taking into account the obtained equilibrium values, the equilibrium values of the economic indexes C , I , and others calculated by the econometric models constructed above, in Table 2.14 we present the results of comparison of the equilibrium indexes with actual values of these indexes in 2007. Table 2.15 shows similar results for 2008.

Table 2.14 Equilibrium and actual values of indexes in 2007

Indexes	2007		Deviation $Y_{\text{actual}} - Y^*$	
	Equilibrium value of Y^*	Actual value of Y_{actual}		
			Absolute	%
Level of prices P	1.1699	1.7893	0.6194	34.6
Currency exchange rate e	109.0	122.6	13.6	11.1
Interest rate of SLB i	16.8	13.6	-3.2	-23.5
National income Y	9,398.1	11,371.1	1973	17.4
Consumption C	4,732.5	5,641.2	908.7	16.1
Import Q_{im}	3,395.5	5,481.8	2,086.3	38.1
Investment I	2,495.2	3,392.1	896.9	26.4
Export Q_{ex}	2,891.1	6,360.5	3,469.4	54.5

Table 2.15 Equilibrium and actual values of indexes in 2008

Indexes	2008		Deviation $Y_{\text{actual}} - Y^*$	
	Equilibrium value of Y^*	Equilibrium value of Y_{actual}		
			Absolute	%
Level of prices P	1.1924	1.96	0.76	38.8
Currency exchange rate e	116.3	120.3	4	3.3
Interest rate of SLB i	26.1	15.3	-10.8	-70.0
National income Y	11,383.0	13,734.3	2,351.0	17.1
Consumption C	5,641.9	6,652.0	1,010.1	15.1
Import Q_{im}	4,161.1	4,558.0	396.9	8.7
Investment I	3,026.2	3,836.0	809.8	21.0
Export Q_{ex}	3,026.1	8,563.4	5,618.4	65.6

2.4.2 Influence of Economic Instruments on Equilibrium Solutions and State of the Balance of Payments

Below, let us estimate the influence of economic instruments, namely the money supply and public expenses, on the conditions of common economic equilibrium and the state of the balance of payments using the following algorithm:

1. Changing the value M_{2007} by $\Delta M = 0.01 M_{2007}$ while keeping the values G_{2007} and i^Z_{2007} , P^Z_{2007} , e^e_{2007} unchanged, define the values $(M\Delta Y^*)/(Y^*\Delta M)$, $(M\Delta P^*)/(P^*\Delta M)$, $(M\Delta e^*)/(e^*\Delta M)$, and $(M\Delta i^*)/(i^*\Delta M)$ that show the percentage by which the equilibrium values of the indexes Y^* , P^* , e^* , i^* change with variation of M_{2007} by 1%.
2. Changing the value G_{2007} by $\Delta G = 0.01 G_{2007}$ while keeping the values M_{2007} and i^Z_{2007} , P^Z_{2007} , e^e_{2007} unchanged, define the values $(G\Delta Y^*)/(Y^*\Delta G)$, $(G\Delta P^*)/(P^*\Delta G)$, $(G\Delta e^*)/(e^*\Delta G)$, and $(G\Delta i^*)/(i^*\Delta G)$ that show the percentage by which

Table 2.16 Influence of the money supply instrument on the equilibrium state of the national economy in 2007 for $\Delta M = 0.01 M_{2007}$ (%)

$(M\Delta Y^*)/(Y^*\Delta M)$	$(M\Delta P^*)/(P^*\Delta M)$	$(M\Delta e^*)/(e^*\Delta M)$	$(M\Delta i^*)/(i^*\Delta M)$
0.1829	-0.0709	0.2130	-0.5216

Table 2.17 Influence of the public expenses instrument on the equilibrium state of the national economy in 2007 for $\Delta G = 0.01 G_{2007}$ (%)

$(G\Delta Y^*)/(G^*\Delta M)$	$(G\Delta P^*)/(P^*\Delta G)$	$(G\Delta e^*)/(e^*\Delta G)$	$(G\Delta i^*)/(i^*\Delta G)$
0.2031%	0.0174	0.0672	0.7658

Table 2.18 Influence of money supply and public expenses instruments on the equilibrium state of the national economy in 2007 for $\Delta M = 0.01 M_{2007}$ and $\Delta G = 0.01 G_{2007}$ (%)

$100\Delta Y^*/Y^*$	$100\Delta P^*/P^*$	$100\Delta e^*/e^*$	$100\Delta i^*/i^*$
0.3859	-0.0534	0.2799	0.2439

the equilibrium values of the indexes Y^* , P^* , e^* , i^* change with variation of G_{2007} by 1%.

3. Changing the value M_{2007} by $\Delta M = 0.01 M_{2007}$ and the value G_{2007} by $\Delta G = 0.01 G_{2007}$ while keeping the values i_{2007}^Z , P_{2007}^Z , e_{2007}^e unchanged, define the values $100\Delta Y^*/Y^*$, $100\Delta P^*/P^*$, $100\Delta e^*/e^*$, and $100\Delta i^*/i^*$ that show the percentage by which the equilibrium values of the indexes Y^* , P^* , e^* , i^* change with simultaneous variation of M_{2007} and G_{2007} by 1%.

The results of computations carried out by the above algorithm are given in Tables 2.16–2.18.

According to the proposed algorithm, first we estimate the influence of the economic instruments, namely, the money supply and public expenses, on the conditions of the common economic equilibrium and the state of the balance of payments individually. From Tables 2.16 and 2.17 it follows that increasing G_{2007} by ΔG while keeping the value M_{2007} results in growth of the national income and an increase in the interest rate, whereas increasing M_{2007} by ΔM while keeping the value G_{2007} also results in growth of the common economic equilibrium of the GNI , but also in a decrease in the interest rate. Moreover, from the tables it follows that the growth in public expenses shows a stronger influence on the national income growth, whereas the money supply growth affects the currency exchange rate more strongly.

Here Y^* , P^* , e^* , i^* are the equilibrium solutions for the year 2007, $\Delta Y^* = Y_M^* - Y^*$, $\Delta P^* = P_M^* - P^*$, $\Delta e^* = e_M^* - e^*$, $\Delta i^* = i_M^* - i^*$, where Y_M^* , P_M^* , e_M^* , i_M^* are the equilibrium solutions corresponding to $M = M_{2007} + \Delta M$.

According to the macroeconomic theory, the money supply growth shows the following influence on the equilibrium solutions of system (2.41): The national income, level of prices, and national currency exchange must increase, whereas the interest rate must decrease. The results of the influence of the money supply instrument on the equilibrium state of the national economy in 2007 presented in Table 2.16 coincide with the theoretical assumptions, except the price level index, which in this case decreases.

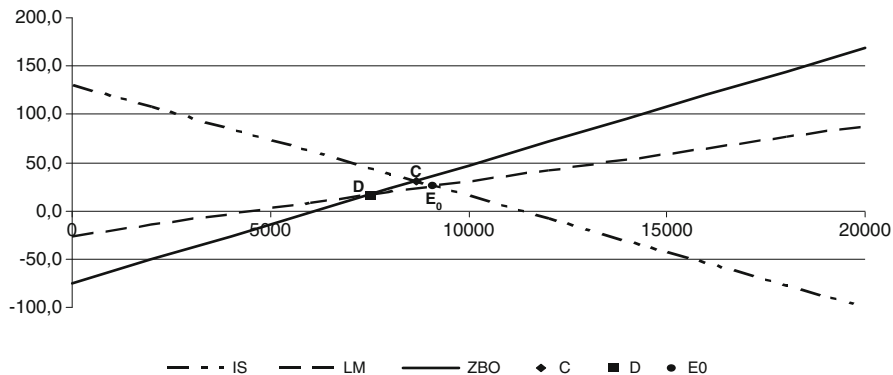


Fig. 2.10 Plots *IS-LM-ZBO* by actual values of P, e for 2007

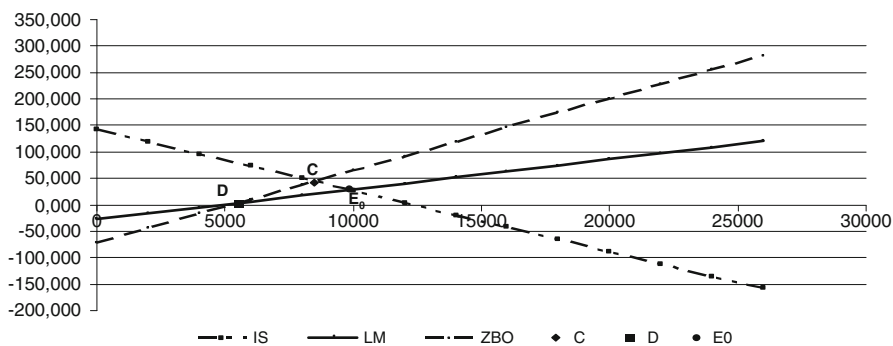


Fig. 2.11 Plots *IS-LM-ZBO* by actual values of P, e for 2008

Here $\Delta Y^* = Y_G^* - Y^*$, $\Delta P^* = P_G^* - P^*$, $\Delta e^* = e_G^* - e^*$, $\Delta i^* = i_G^* - i^*$, where Y_G^* , P_G^* , e_G^* , i_G^* are the equilibrium solutions corresponding to $G = G_{2007} + \Delta G$.

According to macroeconomic theory, the public expenses growth exerts the following influence on the equilibrium solutions of system (2.41): The national income, level of prices, national currency exchange rate, and interest rate must grow. The results of the money supply instrument influence on the equilibrium state of the national economy in 2007 presented in Table 2.16 completely coincide with these theoretical assumptions.

Here $\Delta Y^* = Y_{MG}^* - Y^*$, $\Delta P^* = P_{MG}^* - P^*$, $\Delta e^* = e_{MG}^* - e^*$, $\Delta i^* = i_{MG}^* - i^*$, where Y_{MG}^* , P_{MG}^* , e_{MG}^* , i_{MG}^* are the equilibrium solutions corresponding to $M = M_{2007} + \Delta M$ and $G = G_{2007} + \Delta G$.

Figures 2.10 and 2.11 present the plots of the *IS*, *LM*, and *ZBO* curves from the derived econometric models for the actual statistical information for 2007 and 2008.

As stated above (Figs. 2.8 and 2.9), the country has cyclical unemployment and a deficit in the balance of payments from the constructed models. In Figs. 2.10 and 2.11, such a situation is represented by point E_0 . According to macroeconomic theory, the balance of payments deficit can be eliminated by applying a restrictive monetary policy, namely by shifting the LM curve to the left up to its intersection with the IS curve at the point C , or the counteractive fiscal policy by means of the IS curve to the left up to its intersection with the LM curve at the point D .

2.4.3 Parametric Control of an Open Economy State Based on a Small Country Model

Let us estimate the optimal values of the instruments M and G given the external exogenous parameters e^e, i^Z, P^Z on the basis of model (2.41) for the year 2008 in the sense of the criteria

$$Q_{ex} = \frac{aeP^Z}{P} \rightarrow \max \quad (2.44)$$

and

$$Q_{imp} = bY^S + ceP^Z/P \rightarrow \min. \quad (2.45)$$

Such an estimate can be obtained by solving the following problems of mathematical programming:

Problem

1. On the basis of mathematical model (2.41), find the values (M, G) maximizing criterion (2.44) under the constraints

$$\begin{cases} |M - M^*| \leq 0.1M^*, \\ |G - G^*| \leq 0.1G^*, \\ |P - P^*| \leq 0.1P^*, \\ |e - e^*| \leq 0.1e^*, \\ |i - i^*| \leq 0.1i^*, \\ |Y - Y^*| \leq 0.1Y^*. \end{cases} \quad (2.46)$$

Here M^* and G^* are the actual values of the money supply and public expenses in the year 2008.

2. On the basis of mathematical model (2.41), find the values (M, G) minimizing criterion (2.44) under constraints (2.46).

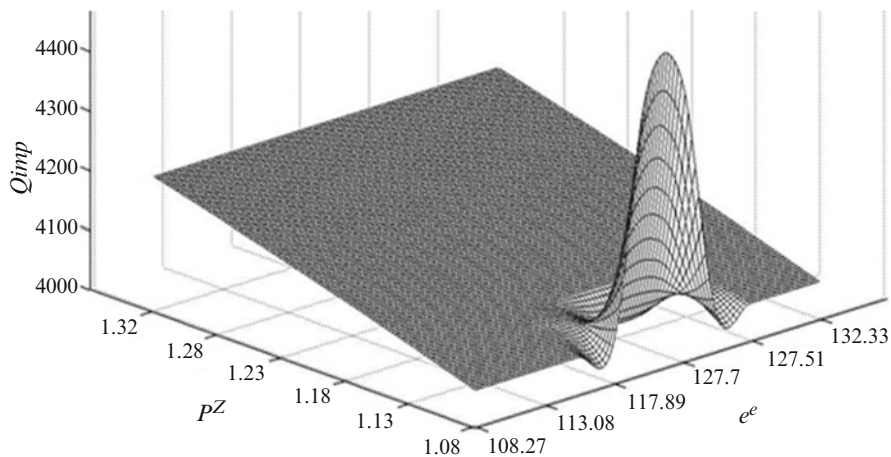


Fig. 2.12 Plot of the dependence of optimal values of criterion Q_{imp} on pair P^Z, e^e

Solving Problems 1 and 2 by the iterative technique [66] given the values $e^e = 120.3$, $i^Z = 1.32$, $P^Z = 1.2002$, the following results are obtained:

For Problem 1, the optimal values of the parameters are $M = 5,877.96$, $G = 4,246$, providing the attainment of the maximum value $Q_{ex} = 3,122.74$. The value of this criterion without control is 3,023.01.

For Problem 2, the optimal values of the parameters are $M = 4,809.234$, $G = 3,474$, providing the attainment of the minimum value $Q_{imp} = 4,010.64$. The value of this criterion without control is 4,183.73.

On the basis of Problems 1 and 2, we carried out the analysis of the dependencies of the optimal values of the criteria Q_{ex} and Q_{imp} on the one pair and one set of three of the parameters from the set of the external parameters $\{e^e, i^Z, P^Z\}$ given within the respective regions. The plots of the dependencies of the optimal values of criteria (2.44) and (2.45) for the single cases including that on the pair of the parameters (P^Z, e^e) and (i^Z, e^e) are shown in Figs. 2.12–2.14.

From the plots in Figs. 2.12–2.14, one can see the general growth of Q_{imp} and Q_{ex} with increasing values of combinations P^Z, e^e and i^Z, e^e , in which the spikes of values Q_{imp} and Q_{ex} are observed for the pair P^Z, e^e .

2.5 Modeling of Inflationary Processes by Means of Regression Analysis: Rational and Adaptive Expectations

The goal of this section is to construct models of present-day inflation in the Republic of Kazakhstan, reasoning from the concept of rational expectations (factor regression models) and the concept of adaptive expectations (autoregression models).

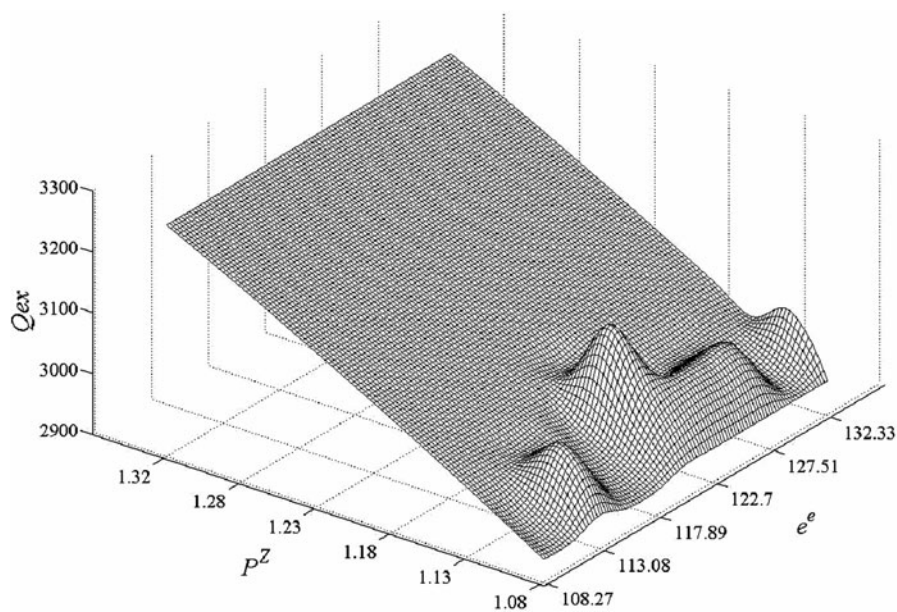


Fig. 2.13 Plot of the dependence of optimal values of criterion Q_{ex} on pair P^Z, e^e

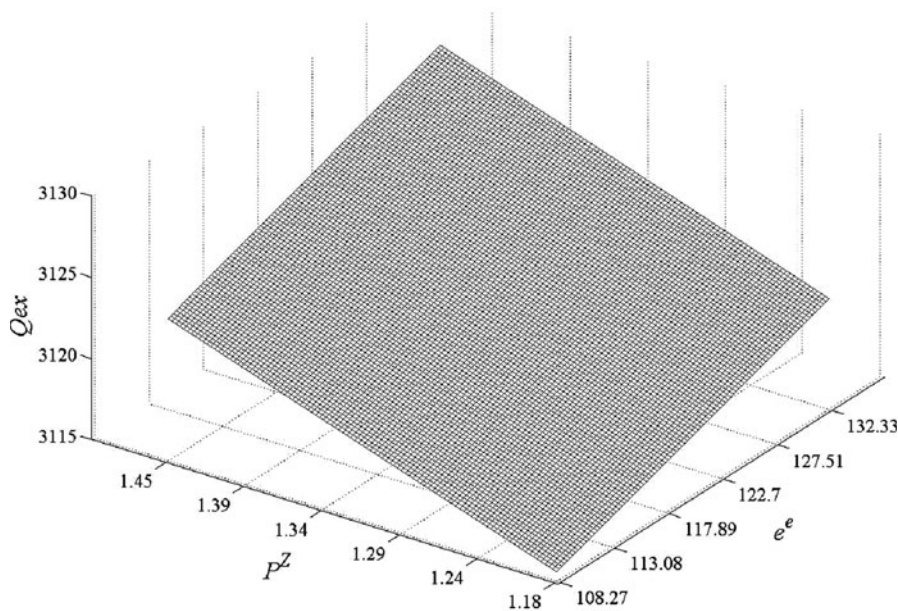


Fig. 2.14 Plot of the dependence of optimal values of criterion Q_{ex} on pair i^Z, e^e

2.5.1 *Preparation of the Data for Factor Regression Models of Inflation*

First of all, let us distinguish a number of factors obviously wielding strong influence on the rate of inflation in the Republic of Kazakhstan. These factors are quite standard, and the level of effect for each of them on inflation processes is subject to further analysis.

The initial data for the analysis are as follows (see Table 2.19):

The data expressed in value units are necessarily deflated to be put into comparable prices. Then, to compare values (and put them in the same order), they are converted from absolute values to relative ones (fractions of a unit). For data expressed in percentages, the fraction of the yearly sum is calculated. As a result, we obtain processed information suitable for carrying out regression analysis (Table 2.20).

The matrix of the partial correlations of the derived dynamic series is presented in Table 2.21.

In the regression formulas below, the following notation is used:

Infl	Is the inflation rate
Nx	Is the net export
Rd	Is the wear and tear of capital assets
Rlr	Is the ratio of the withdrawal coefficient to the renewal coefficient
Cipc	Is the change in prices of consumer goods and services
Innov	Is the cost of research and development (R&D) and innovation
Mh	Is the income of the households used for consumption, mean yearly value per capita
Mm	Is the monetary aggregate
Fv	Is the index of the physical volume of industrial production
Invst	Is the investment in capital assets
Cr	Is the currency exchange rate (tenge per US dollar)

2.5.2 *Construction of One-Factor Regression Models of Inflation*

Now let us construct one-factor regression models of inflation, each time with one of the distinguished factors.

2.5.2.1 *Dependence of the Inflation Rate on the Net Export Volume*

$$d \text{ Infl} = 0.00056 \times d \text{ Nx} + 0.1156$$

Table 2.19 Initial data for regression analysis

Initial statistics	2000	2001	2002	2003	2004	2005	2006	2007	2008
Deflator of <i>GDP</i> of USA (%)	4.10	1.10	1.80	2.50	3.60	3.10	2.70	2.10	4.08
Monetary aggregate <i>M3</i> (millions of tenge)	397,015.00	576,023.00	764,954.00	971,213.00	1,650,115.00	2,065,348.00	3,677,561.00	4,629,829.00	6,266,395.00
Export of wealth and services (in% of <i>GDP</i>)	56.60	45.90	46.99	48.42	52.23	53.54	51.14	49.43	60.71
<i>GDP</i> (in current prices, US dollars)	18,291,990,619	22,152,689,130	24,636,598,581	30,833,692,831	43,151,647,003	57,123,671,734	81,003,864,916	104,853,480,212	132,228,697,116
Import of wealth and services (in% of <i>GDP</i>)	49.10	46.95	47.04	43.05	43.49	44.73	40.43	42.60	40.66
Inflation, deflator of <i>GDP</i> (yearly, in%)	17.43	10.16	5.80	11.74	16.13	17.87	21.55	15.53	19.95
Exchange rate (tenge/US dollar)	142.13	147.93	153.28	168.79	131.40	132.88	126.09	122.55	120.30
Change of prices of consumer goods and services (% increment to the preceding year)	5.40	5.70	6.00	6.40	6.90	7.60	8.60	10.80	17.00
Costs of R&D and innovations (\$)	188,740,000	147,880,000	188,240,000	269,970,000	267,710,000	222,420,000	171,760,000	202,470,000	347,610,001
Coefficient of wear of capital asset, %	29.70	33.10	30.10	32.20	35.20	37.40	40.60	37.80	

Index of physical volume of industrial production, in% w.r.t. preceding year	115.50	113.80	110.50	109.10	110.40	104.80	107.20	105.00	102.10
Income of households used for consumption, mean monthly value per capita, tenge	4,731.00	5,398.00	6,478.00	7,569.00	8,387.00	9,751.00	13,723.00	16,935.00	20,037.08
Investment in capital asset, in% w.r.t. preceding year	149.00	145.00	111.00	117.00	123.00	134.00	111.00	114.00	105.00
Ratio of withdrawal coefficient to renewal coefficient	14.49	12.88	8.73	6.34	8.33	9.74	10.32	12.00	13.23

Table 2.20 Data prepared for carrying out regression analysis

Relative increments	2001–2000 (%)	2002–2001 (%)	2003–2002 (%)	2004–2003 (%)	2005–2004 (%)	2006–2005 (%)	2007–2006 (%)	2008–2007 (%)
Net export	–116.90	–94.80	–13,110.28	119.74	29.57	67.72	–19.10	255.42
Monetary aggregate	37.88	25.90	12.48	110.66	20.05	82.72	26.87	32.47
Costs of R&D and innovations	–22.50	25.04	39.92	–4.28	–19.42	–24.81	15.46	64.95
Income of households used for consumption, mean yearly value per capita	8.43	13.77	3.52	37.39	11.51	44.41	24.36	15.81
Change of prices of consumer goods and services	7.26	7.66	8.06	8.60	9.27	10.22	11.56	14.52
Wear and tear of capital assets	10.76	11.99	10.90	11.66	12.75	13.55	14.70	13.69
Index of physical volume of industrial production	11.80	11.63	11.29	11.15	11.28	10.71	10.96	10.73
Investment to capital assets	13.44	13.07	10.01	10.55	11.09	12.08	10.01	10.28
Ratio of withdrawal coefficient to renewal coefficient	–11.11	–32.22	–27.38	31.39	16.93	5.95	16.28	10.30
Currency exchange rate	4.08	3.62	10.12	–22.15	1.13	–5.11	–2.81	–1.84
Inflation rate	12.80	7.46	4.26	8.62	11.85	13.13	15.83	11.40

Table 2.21 Partial correlation matrix

	Net export	Monetary aggregate	Costs of R&D and innovation	Income of households used for consumption, mean yearly value per capita	Change of prices of consumer goods and services	Wear and tear of capital assets	Index of physical volume of industrial production	Investment in capital assets	Ratio of withdrawal coefficient to renewal coefficient	Currency exchange rate	Inflation rate
Net export	1										
Monetary aggregate	0.3731	1									
Costs of R&D and innovation	-0.3660	-0.4233	1								
Income of households used for consumption, mean yearly value per capita	0.4675	0.8598	-0.3838	1							
Change of prices of consumer goods and services	0.2827	-0.0466	0.5125	0.2165	1						
Wear and tear of capital assets	0.4661	-0.0074	0.1445	0.4433	0.7967	1					
Index of physical volume of industrial production	-0.1210	-0.3065	-0.2458	-0.5998	-0.8160	-0.7611	1				

(continued)

Table 2.21 (continued)

	Net export	Monetary aggregate	Costs of R&D and innovation	Income of households used for consumption, mean yearly value per capita	Change of prices of consumer goods and services	Wear and tear of capital assets	Index of physical volume of industrial production	Investment in capital assets	Ratio of withdrawal coefficient to renewal coefficient	Currency exchange rate	Inflation rate
Investment in capital assets	0.3665	0.0444	-0.4898	-0.0574	-0.5439	-0.3836	0.6220	1			
Ratio of withdrawal coefficient to renewal coefficient	0.5255	0.5575	-0.2768	0.5955	0.4682	0.5003	-0.5607	-0.4570	1		
Currency exchange rate	-0.5067	-0.8876	0.2556	-0.7903	-0.1694	-0.2115	0.3844	0.2184	-0.7887	1	
Inflation rate	0.7026	0.0909	-0.3935	0.3636	0.4485	0.6869	-0.3252	0.0825	0.5448	-0.2009	1

Regression statistics					
Multiple <i>R</i>					0.702646212
Squared <i>R</i>					0.493711699
Normalized squared <i>R</i>					0.409330315
Standard error					0.028263362
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	1	0.004673846	0.004673846	5.85095525	0.051942589
Residual	6	0.004792906	0.000798818		
Total	7	0.009466752			
		Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection		0.115623264	0.010654327	10.85223578	3.6266E-05
Net export		0.000555822	0.000229785	2.418874789	0.05194259

The dependence of the inflation rate on a single factor with the rather high determination coefficient (almost 50%) and low flexibility with respect to the selected factor (0.00056) suggests that the inflation rate dynamics may weakly depend on some dynamic factors and to a greater extent is determined by the dynamics of its own preceding states. To test this hypothesis, one can apply autoregression methods (see Sect. 2.5.4 below).

2.5.2.2 Dependence of the Inflation Rate on Monetary Aggregate Volume

$$d\text{ Infl} = 0.0097 \times d\text{ Mm} + 0.1024$$

Regression statistics					
Multiple <i>R</i>					0.090896264
Squared <i>R</i>					0.008262131
Normalized squared <i>R</i>					−0.157027514
Standard error					0.039556997
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	1	7.82155E-05	7.82155E-05	0.04998577	0.830505924
Residual	6	0.009388536	0.001564756		
Total	7	0.009466752			
		Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection		0.102447184	0.023546608	4.350825545	0.00481825
Monetary aggregate		0.009707317	0.043418618	0.223574986	0.83050592

It can easily be seen that this model is not itself significant in any statistical criteria. This unambiguously proves that there is no significant connection between the monetary aggregate volume and the inflation rate in the Republic of Kazakhstan. The absence of this connection is not a characteristic feature of Kazakhstan; it is confirmed by theoretical reasoning [30, 31] and typical for a large number of countries, in particular, for countries with transitional economies during the last decade of the twentieth century [32].

2.5.2.3 Dependence of the Inflation Rate on the Costs of R&D and Innovation

$$d \text{ Infl} = -0.0442 \times d \text{ Innov} + 0.1108$$

Regression statistics					
Multiple R					0.393505471
Squared R					0.154846556
Normalized squared R					0.013987648
Standard error					0.036516795
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	1	0.001465894	0.001465894	1.09930255	0.334805228
Residual	6	0.008000858	0.001333476		
Total	7	0.009466752			
	Coefficients		Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.110790788		0.013492163	8.211491911	0.00017599
Costs of R&D and innovation	-0.044197082		0.04215363	-1.048476298	0.33480523

2.5.2.4 Dependence of the Inflation Rate on the Volume of Incomes of Households Used for Consumption per Capita

$$d \text{ Infl} = 0.0929 \times d \text{ Mh} + 0.0882$$

Regression statistics	
Multiple R	0.363609741
Squared R	0.132212044
Normalized squared R	-0.012419282
Standard error	0.037002552
Observations	8

(continued)

Variance analysis

variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	1	0.001251619	0.001251619	0.91413145	0.375940254
Residual	6	0.008215133	0.001369189		
Total	7	0.009466752			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.088202262	0.023339782	3.779052507	0.00919056
Yearly incomes of households used for consumption per capita	0.092864177	0.097127875	0.956102218	0.37594025

2.5.2.5 Dependence of the Inflation Rate on Changes in the Prices of Consumer Goods and Services

$$d \text{ Infl} = 0.6808 \times d \text{ Cipc} + 0.041$$

Regression statistics

Multiple R	0.448455855
Squared R	0.201112654
Normalized squared R	0.067964763
Standard error	0.035503211
Observations	8

Variance analysis

Variance analysis						
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>	
Regression	1	0.001903884	0.001903884	1.51044566	0.265081063	
Residual	6	0.007562868	0.001260478			
Total	7	0.009466752				
			Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection			0.041023376	0.054879486	0.747517499	0.48300787
Changes of prices of consumer goods and services			0.680840916	0.553978736	1.229001895	0.26508106

This model shows that the yearly inflation rate in this macro-system can be explained by the dynamics of *consumer* prices by approximately 20%, although the high values of *p*-level do not allow us to judge reliably the specific values of the coefficients derived by the linear regression method.

2.5.2.6 Dependence of the Inflation Rate on Wear and Tear of Capital Assets

$$d \text{ Infl} = 1.7868 \times d \text{ Rd} - 0.1167$$

Regression statistics					
Multiple R					0.68692623
Squared R					0.471867645
Normalized squared R					0.383845586
Standard error					0.028866641
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	1	0.004467054	0.004467054	5.36078854	0.059829763
Residual	6	0.004999698	0.000833283		
Total	7	0.009466752			
		Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection		−0.116679487	0.097008911	−1.202770814	0.27436974
Wear and tear of capital assets		1.786895333	0.771764462	2.315337673	0.05982976

This model confirms the fact that the technological backwardness and high wear and tear of capital assets are significant inflation factors that can strongly contribute to the development of inflationary processes [28, 29].

2.5.2.7 Dependence of the Inflation Rate on Index of Physical Volume of Industrial Production

$$d\text{ Infl} = -3.0344 \times d\text{ Fv} + 0.4464$$

Regression statistics					
Multiple R					0.325197844
Squared R					0.105753638
Normalized squared R					−0.043287422
Standard error					0.037562411
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	1	0.001001143	0.001001143	0.70956042	0.431878752
Residual	6	0.008465608	0.001410935		
Total	7	0.009466752			
		Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection		0.446409333	0.403525118	1.106273969	0.31097982
Index of physical volume of industrial production of the Republic of Kazakhstan		−3.034474979	3.602374574	−0.842354096	0.43187875

2.5.2.8 Dependence of the Inflation Rate on the Volume of Investment in Capital Asset

$$d\text{ Infl} = 0.22 \times d\text{ Invst} + 0.0817$$

Regression statistics					
Multiple R					0.082469621
Squared R					0.006801238
Normalized squared R					−0.158731889
Standard error					0.039586121
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	1	6.43856E-05	6.43856E-05	0.04108687	0.846069149
Residual	6	0.009402366	0.001567061		
Total	7	0.009466752			
	Coefficients		Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.081785455		0.123622153	0.661576044	0.53281154
Investment in capital assets	0.220005931		1.085382584	0.202698969	0.84606915

This model expresses a fact that is encouraging for the economy of Kazakhstan. Investments in the main capital assets pose an inflationary danger of less than 1%. In other macro-systems, this may not be so, the investment in capital assets in any case stimulates aggregate demand, which unavoidably leads to growth of the common level of prices.

For comparison, let us recall that the net export, which is also a part of aggregate demand, explains the dynamics of the inflation rate in Kazakhstan by almost 50% (model 2.5.2.1 above).

Under these conditions, it is short-sighted for a number of reasons to state the problem as one of maximizing exports. The main reason is that this maximization can make the national economy less controllable and more subject to the effect of external shocks.

2.5.2.9 Dependence of the Inflation Rate on the Ratio of Withdrawal Coefficient to Renewal Coefficient

$$d\text{ Infl} = 0.0886 \times d\text{ Rlr} + 0.1055$$

Regression statistics	
Multiple R	0.544831904
Squared R	0.296841803
Normalized squared R	0.17964877
Standard error	0.033308217
Observations	8

(continued)

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	1	0.002810128	0.002810128	2.53293047	0.162598324
Residual	6	0.006656624	0.001109437		
Total	7	0.009466752			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.105558889	0.011797374	8.947659617	0.00010879
Ratio of withdrawal coefficient to renewal coefficient	0.088640382	0.055695484	1.59151829	0.16259832

2.5.2.10 Dependence of the Inflation Rate on the Currency Exchange Rate

$$d\text{Infl} = -0.0772 \times dCr + 0.1054$$

Regression statistics

Multiple R	0.200867874
Squared R	0.040347903
Normalized squared R	-0.119594113
Standard error	0.038911841
Observations	8

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	1	0.000381964	0.000381964	0.25226581	0.633380857
Residual	6	0.009084788	0.001514131		
Total	7	0.009466752			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.10543109	0.013981186	7.540926073	0.00028201
Currency exchange rate	-0.07722679	0.153758376	-0.502260702	0.63338086

This last model is not statistically significant. In addition to the low value of R^2 (a little more 4%) and high p -level, this model also does not satisfy the Fisher criterion, i.e., applying econometric methods gives grounds to conclude that the form of dependence of a variable on the parameter explaining its dynamics has been chosen incorrectly.

Model 2.5.2.1 is of the highest quality among the models constructed here; model 2.5.2.6 is the next in quality. This reflects both the explanatory ability of these models expressed by the determination coefficient and the quality (degree of reliability) of the coefficients derived as a result of computations.

Table 2.22 Results of constructing one-factor linear regression models

Factor	<i>p</i> -Level	Squared <i>R</i>
2.1. Net export	0.052	0.494
2.6. Wear and tear of capital assets	0.060	0.472
2.9. Ratio of withdrawal coefficient to renewal coefficient	0.163	0.297
2.5. Change in prices of consumer goods and services	0.265	0.201
2.3. Costs of R&D and innovation	0.335	0.155
2.4. Income of households used for consumption, yearly mean value per capita	0.376	0.132
2.7. Index of physical volume of industrial production	0.432	0.106
2.10. Currency exchange rate	0.633	0.040
2.2. Monetary aggregate volume	0.831	0.008
2.8. Investment in capital assets	0.846	0.007

The next step consists in organizing the derived models in order of decreasing R^2 . This gradation miraculously (but not randomly) coincides with increasing p -level of the coefficient of the explanatory parameter of the regression (Table 2.22).

Combining the factors that best explain the dynamics of the inflation rate and avoiding multicollinearity, let us construct the multifactor regression inflation models.

2.5.3 Construction of Multifactor Regression Models of Inflation

Combining three of the most significant factors, we derive model 2.5.3.1.

2.5.3.1 Dependence of the Inflation Rate on Net Export Volume, Wear and Tear of Capital Assets, and Ratio of Withdrawal Coefficient to Renewal Coefficient

$$d\,Infl = 0.00038 \times d\,Nx + 0.9021 \times d\,Rd + 0.0192 \times d\,Rlr$$

Regression statistics	
Multiple R	0.983824162
Squared R	0.967909982
Normalized squared R	0.755073974
Standard error	0.025399041
Observations	8

(continued)

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	3	0.097290321	0.032430107	50.2705633	0.001240905
Residual	5	0.003225556	0.000645111		
Total	8	0.100515878			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Net export	0.000380604	0.000239745	1.587538531	0.173254
Wear and tear of capital assets	0.902141768	0.078494697	11.49302828	8.7405E-05
Ratio of withdrawal coefficient to renewal coefficient	0.019216022	0.050395611	0.381303485	0.71864334

With a very good determination coefficient (almost 97%), the *p*-level of the latter regression parameter evidently exceeds its admissible value. Obviously, the reason for this is because the ratio of the withdrawal coefficient to the renewal coefficient is closely connected with the wear and tear of capital assets, so one of these parameters should be considered unnecessary. Thus, the regression parameter with the coefficient having the highest *p*-level in model 2.5.3.1 should be removed from this model.

2.5.3.2 Dependence of the Inflation Rate on Net Export Volume and Wear and Tear of Capital Assets

$$d \text{ Infl} = 0.00042 \times d N_x + 0.9111 \times d R_d$$

Regression statistics

Multiple R	0.983349812
Squared R	0.966976853
Normalized squared R	0.794806328
Standard error	0.023520738
Observations	8

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	2	0.097196527	0.048598263	87.8453696	0.000127375
Residual	6	0.003319351	0.000553225		
Total	8	0.100515878			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Net export volume	0.000429264	0.000187949	2.283935649	0.06246151
Wear and tear of capital assets	0.911137115	0.069329295	13.14216614	1.1977E-05

The correctness of the undertaken step is confirmed by the fact that the coefficient R^2 of the model does not change significantly (the difference is one-tenth of a percent). Nevertheless, the rest of the model's statistical parameters demonstrate the complete adequacy of the model.

Attempts to enter some additional factors into this model do not produce good results. Indeed, at each attempt the p -level of the entered factor exceeds 0.5, which of course is too high assuming normally distributed regression coefficients.

Construction of the models using another explaining factor leads to worse results in comparison to model 2.5.3.2. Usually such models have a lower determination coefficient and higher p -level of the coefficients of the explanatory parameters.

Let us consider model 2.5.3.3 as one of the typical (and quite successful) attempts of this kind.

2.5.3.3 Dependence of the Inflation Rate on Costs of R&D and Innovation, Index of Physical Volume of Industrial Production, and Mean Yearly Income of Households per Capita Used for Consumption

$$d\text{ Infl} = -0.0269 \times d\text{ Innov} + 0.8193 \times d\text{ Fv} + 0.0861 \times d\text{ Mh}$$

Regression statistics					
Multiple R					0.959674566
Squared R					0.920975272
Normalized squared R					0.689365381
Standard error					0.039857847
Observations					8
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	3	0.092572638	0.030857546	19.4237782	0.007568807
Residual	5	0.00794324	0.001588648		
Total	8	0.100515878			
	Coefficients		Standard error	<i>t</i> -statistics	<i>P</i> -Level
Costs of R&D and innovations	-0.026926846		0.049077561	-0.548659005	0.60683873
Index of physical volume of industrial production	0.819367611		0.242821599	3.374360495	0.0197954
Mean yearly income of households per capita used for consumption	0.086165843		0.108645937	0.793088499	0.46368084

So, model 2.5.3.2 is most adequate among the constructed models. It allows us to explain almost 97% of the dynamics of the inflation rate in the economy of the Republic of Kazakhstan during the last ten years by means of two explaining factors.

2.5.4 Construction of Autoregression Models of Inflation Rate

2.5.4.1 Dependence of the Inflation Rate on Two Preceding Years

$$Infl(t) = 0.875 \times Infl(t-1) - 0.701 \times Infl(t-2) + 0.087$$

Regression statistics					
Multiple R					0.810422603
Squared R					0.656784795
Normalized squared R					0.427974658
Standard error					0.030184345
Observations					6
Variance analysis					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of F
Regression	2	0.005230478	0.002615239	2.870435745	0.201071158
Residual	3	0.002733284	0.000911095		
Total	5	0.007963762			
	Coefficients		Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y-intersection	0.087182255		0.042416364	2.055392011	0.132078479
Inflation (t-1)	0.875585255		0.372107188	2.353045795	0.100028831
Inflation (t-2)	0.701332319		0.444901675	-1.576375992	0.213029975

The derived model shows that 66% of variations of the yearly inflation rate in Kazakhstan can be explained by the dynamics of the same rate for two preceding years.

2.5.4.2 Dependence of Inflation Rate on Three Preceding Years

$$Infl(t) = 0.078 \times Infl(t-1) + 0.349 \times Infl(t-2) - 0.524 \times Infl(t-3) + 0.128$$

Regression statistics	
Multiple R	0.812898732
Squared R	0.660804348
Normalized squared R	-0.356782608
Standard error	0.030588451
Observations	5

(continued)

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	3	0.001822794	0.000607598	0.649383666	0.697175376
Residual	1	0.000935653	0.000935653		
Total	4	0.002758447			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y- intersection	0.128852949	0.066712285	1.931472569	0.304137731
Inflation (t-1)	0.078035028	0.707204219	0.11034299	0.930036504
Inflation (t-2)	0.349251506	0.907146362	0.385000173	0.766036509
Inflation (t-3)	−0.524946165	0.628028501	−0.835863602	0.556766413

The constructed model is insignificant by all criteria. Such would be expected, because it has four parameters, and no correct model can be constructed using five observations.

2.5.4.3 Dependence of the Inflation Rate on the Values of Two and Three Preceding Years

$$Infl(t) = 0.436 \times Infl(t-2) - 0.571 \times Infl(t-3) + 0.133$$

Regression statistics

Multiple R	0.810354519
Squared R	0.656674446
Normalized squared R	0.313348891
Standard error	0.021760577
Observations	5

Variance analysis

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance of <i>F</i>
Regression	2	0.001811402	0.000905701	1.912687353	0.343325554
Residual	2	0.000947045	0.000473523		
Total	4	0.002758447			

	Coefficients	Standard error	<i>t</i> -statistics	<i>P</i> -Level
Y- intersection	0.133571731	0.036425493	3.666984809	0.066981695
Inflation (t-2)	0.436105273	0.320799038	1.359434479	0.306993601
Inflation (t-3)	−0.571758565	0.329429306	−1.735603225	0.224769026

We derive a model of approximately the same quality as model 2.5.4.1. This means that this model can be used for purposes of analysis and (in many cases) prediction of the inflationary dynamics of this macro-system almost equally well.

Nevertheless, let us point out a difference between these two models. In the latter (model 2.5.4.3), the free term characterizing the “autonomous” inflation rate independent of the preceding dynamics of that parameter is the most reliable. In model (2.5.4.1), the values of all three coefficients are reliable almost equally.

2.6 Conclusion

Finally, let us recall that the methodology of the construction of the factor regression models derives its strength and foundation from the concept of rational expectations. In other words, to estimate approximately the inflation rate in Kazakhstan in the future (for instance, the next year) by means of models of this kind, one should ask the following question: What are the values of the parameters on which the inflation rate function strongly depends? In this case, these are the net export volume and wear and tear of capital assets (if one considers model 2.5.3.2 as the basis, which is the best model in view of the econometric indexes).

The methodology of the construction of linear autoregression models originates from the correctness of the concept of adaptive estimation. The inflation rate of the next period strongly depends on the past history, and its subsequent dynamics can be predicted on exactly this basis.

We have proved that methods based on both the concept of rational expectation and the concept of adaptive expectation are suitable for estimation and analysis of inflationary processes in the Republic of Kazakhstan.

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