

# Preface

*Geometry is nothing but an expression of a symmetry group.* Fortunately, geometry escaped this stifling straitjacket description, an urban legend formulation of Felix Klein's Erlangen program. Nonetheless, there is a valuable ge(r)m of truth in this vision of geometry. Arithmetic and geometry have been intertwined since Euclid's development of arithmetic from geometric constructions. A group, in the abstract, is a set of elements, devoid of concrete form, with just one operation satisfying a minimalist set of axioms. Representation theory is the study of how such an abstract group appears in different avatars as symmetries of geometries over number fields or more general fields of scalars. This book is an initiating journey into this subject.

A large part of the route we take passes through the representation theory of semisimple algebras. We will also make a day-tour from the realm of finite groups to look at the representation theory of unitary groups. These are infinite, continuous groups, but their representation theory is intricately interlinked with the representation theory of permutation groups, and hence this detour from the main route of the book seems worthwhile.

Our navigation system is set to avoid speedways as well as slick shortcuts. Efficiency and speed are not high priorities on this journey. For many of the ideas we view the same set of results from several vantage points. Sometimes we pause to look back at the territory covered or to peer into what lies ahead. We stop to examine glittering objects – specific examples – up close.

The role played by the characteristic of the field underlying a representation is described carefully in each result. We stay almost always within the semisimple territory, etched out by the requirement that the characteristic of the field does not divide the number of elements of the group. By not making any special choice for the field  $\mathbb{F}$ , we are able to see the role of semisimplicity at every stage and in every result.

Authors generally threaten readers with the admonishment that they *must* do the exercises to appreciate the text. This could give rise to insomnia if one wishes to peruse parts of this text at bedtime. However, for daytime readers, there are several exercises to engage in, some of which may call for breaking intellectual sweat, if the eyes glaze over from simply reading.

The style of presentation I have used is unconventional in some ways. Aside from the very informal tone, I have departed from rigid mathematical custom by repeating definitions instead of sending the reader scurrying back and forth to consult them. I have also included all hypotheses (such as those on the ground field  $\mathbb{F}$  of a representation) in the statement of every result, instead of stating them at the beginnings of sections or chapters. This should help the reader who wishes to take just a quick look at some result or sees the statement on a sample page online.

For whom is this book intended? For graduate and undergraduate students, for teachers, for researchers, and also for those who simply want to explore this beautiful subject for itself. This book is an introduction to the subject; at the end, or even part way through, the reader will have enough equipment and experience to read more specialized monographs to pursue roads not traveled here.

A disclaimer on originality needs to be made. To the best of my knowledge, there is no result in this book that is not already “known.” Mathematical results evolve in form, from original discovery through mutations and cultural forces, and I have added historical remarks or references only for some of the major results. The reader interested in a more thorough historical analysis should consult works by historians of the subject.

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