Chapter 2
Environment and Equilibrium

Abstract This chapter models the connections between the redistributive tax rate and income inequality in a dynamic tax game embedded in an overlapping generation model with heterogeneous agents. In the first period, agents vote and work; in the second period, they consume. The source of heterogeneity within the same generation is endorsed due to differences in labor efficiency among the agents. The existence and stability of the political–economic equilibrium is shown to exist. In accordance to the wishes of the median voter, government redistributes the tax revenue completely to the young. This process is repeated in subsequent time periods. Each agent, based on his selection of policy, maximizes his lifetime indirect utility, subject to his personal constraints. Nash equilibria are sub optimal, under the assumption of sequential rationality. Hence, agents have an incentive to cooperate.

Keywords Cooperation • Coordination • Dynamic general equilibrium • Median voter • Nash equilibrium • Policy commitment • Ramsey equilibrium • Two-period overlapping generations model

In a two–period–lived overlapping generations model, we consider heterogeneous agents who vote and work when young and consume when they are old. Assuming no population growth, agents born in period t (superscript) maximize a time-separable utility function:

$$U(l_t^t) + \beta U(c_{t+1}^t), \tag{2.1}$$

where subscripts denote calendar time. Specifically, l_t^t and c_{t+1}^t denote, respectively, an individual agent's leisure at period t and his consumption at period $t+1$.

Agents have identical preferences. It is further assumed that the period utility function $U(\cdot)$ is continuous, twice differentiable, and strictly concave. Agents discount future utility by a subjective discount rate $\beta \in (0,1)$. Both consumption and leisure are normal goods. Moreover, we assume that the marginal utility of each argument tends to infinity as the limit of the argument tends to zero.

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The only source of heterogeneity within the same generation is the index of agents' ability endowment of labor efficiency units, e. It is distributed in the population according to a known and time–invariant distribution, $\Gamma(e)$, which satisfies the assumptions noted next.

Assumption 2.1: Heterogeneity.

$$\int_{\underline{e}}^{\bar{e}} \mathrm{d}\Gamma(e) = 1,$$

$$\int_{\underline{e}}^{\bar{e}} e\mathrm{d}\Gamma(e) = 1,$$

$$\int_{\underline{e}}^{e^m} \mathrm{d}\Gamma(e) = 1/2,$$

where $e^m < 1$.

The first equation of Assumption 2.1 indicates that $\Gamma(e)$ is well–defined on the interval $[\underline{e}, \bar{e}]$, where $\underline{e} > 0$.[1] The second equation normalizes the mean of the distribution to one. The third equation restricts our attention to the class of distributions where the median is less than the mean.

Each agent is endowed with a normalized unit of time to be allocated between leisure, l_t^t, and work, n_t^t, in his first period of life. Depending on his ability endowment, e, each unit of n_t^t earns an effective wage rate, eW_t. The only store of value in this economy is capital investment in a competitive capital market. Each unit of consumption good invested yields a (pre-tax) factor rate, R_{t+1}. We note that there are no gains from trade between agents.[2] An agent's asset holding at date t is denoted by a_t^t. We write his time constraint, first–, and second–period budget constraints, respectively, as

$$\begin{aligned} l_t^t + n_t^t &\leq 1, \\ a_t^t &\leq eW_t n_t^t (1 - \tau_t) + T_t, \\ c_{t+1}^t &\leq a_t^t R_{t+1} (1 - \tau_{t+1}), \end{aligned} \tag{2.2}$$

where W_t, R_{t+1}, τ_t, and T_t are , respectively, the wage rate, the interest factor, income tax rate, and a lump–sum transfer—all suitably time–subscripted.

[1] The support of the distribution need not be finite or bounded for our results to hold.

[2] Since young agents are identical up to endowment, it is clear that in an environment without uncertainty there are no gains from trade among agents of the same generation. We further assume that this is a classical economy, so there is no role for paper money.

There is a continuum of individuals in this economy. Factor markets are competitive. Capital, labor markets, and the political process determine interest, wage rates, and fiscal policies, respectively. Agents take prices and policies parametrically.

Firms possess a constant–returns–to–scale (CRS) production technology, specified as $F(K_t, N_t) = Y_t$. Without loss of generality, we assume that there is a continuum of firms, normalized to size one. These firms produce consumption goods by employing efficiency labor and capital. The two–factor aggregates are denoted as:

$$N_t = \int_{\underline{e}}^{\bar{e}} n_t^t e \mathrm{d}\Gamma(e),$$

and

$$K_t = \int_{\underline{e}}^{\bar{e}} k_t \mathrm{d}\Gamma(e).$$

We further assume that both the wage rate, $W_t = \frac{\partial F(\cdot)}{\partial N_t}$, and the interest factor for all t is given as $R_t = \frac{\partial F(\cdot)}{\partial K_t}$.

Government exists to implement the wishes of the pivotal voters. It only redistributes income, i.e., it issues neither debt nor provides any public goods. Government levies a proportional tax, τ_t, against output, Y_t. It balances its budget by providing a lump–sum transfer, T_t, to the existing young. We express the government's budget constraint as:

$$T_t = \tau_t Y_t. \tag{2.3}$$

Policies are said to be feasible if equation (2.3) holds.

2.1 The Political–Economic Equilibrium

A political–economic equilibrium satisfies three conditions:

1. *Competitive economic equilibrium:* Given any feasible policies and factor prices, economic decisions are optimal for agents; given feasible policies, prices are set to clear markets;
2. *Political equilibrium:* The policy implemented in each period is (weakly) preferred to any other feasible policy by a majority of the voters; and
3. *Rationality of expectations:* The expectations of individuals in their roles as economic agents and voters are fulfilled.

2.2 Competitive Economic Equilibrium

Since capital investment is the only asset in this economy, a perfect foresight competitive equilibrium is defined as follows:

Definition 2.1: A Competitive Equilibrium. Given a vector of policy, $\{\tau_t, T_t\}_{t=1}^{\infty}$, a competitive economic equilibrium consists of a vector of prices, $\{W_t, R_t\}_{t=1}^{\infty}$, and a vector of allocations, $\{l_{i,t}^t, n_{i,t}^t, c_{i,t+1}^t\}_{t=1}^{\infty}$, $i \in [\underline{e}, \bar{e}]$, such that (a) given the policy and price vectors, the vector of allocations maximizes both agents' utilities for all i and firms' profits, (b) the vector of prices is consistent with cleared goods and factor markets, and (c) the government budget is balanced in each period.

Assuming non-satiation, equation (2.2) hold as strict equalities in equilibrium. Taking parametrically the after–tax wage rate, $\tilde{W}_t \equiv W_t(1-\tau_t)$, the after–tax interest factor, $\tilde{R}_{t+1} \equiv R_{t+1}(1-\tau_{t+1})$, and transfer, T_t, the optimal labor supply is the solution to the first order condition,

$$U'(1-n_t^t) = \beta U'\left(\left(n_t^t e\tilde{W}_t + T_t\right)\tilde{R}_{t+1}\right) e\tilde{W}_t\tilde{R}_{t+1}.$$

The optimal labor–leisure and saving decisions are respectively described by two functions:

$$n_t^{*t} = \mathbf{n}\left(\tilde{R}_{t+1}, \tilde{W}_t, T_t\right),$$

and

$$a_t^{*t} = \mathbf{z}\left(\tilde{R}_{t+1}, \tilde{W}_t, T_t\right).$$

Because fiscal decisions distort labor supply decisions, there is a threshold ability, e_t° (below), which allows agents to subsist on welfare alone and provide zero hours of work, i.e.,

$$e_t^{\circ} = \frac{1}{\beta\tilde{W}_t\tilde{R}_{t+1}}\frac{U'(1)}{U'\left(T_t\tilde{R}_{t+1}\right)}.$$

We express $e_t^{\circ} = \mathbf{e}(\tilde{R}_{t+1}, \tilde{W}_t, T_t)$. The aggregate labor supply comes from the population that remains in the work force, given factor prices and polices, i.e.,

$$N_t = \int_{e^o}^{\bar{e}} n_t e \mathrm{d}\Gamma(e).$$

One additional equation, the capital market clearing condition, is necessary to complete the description of an economic equilibrium is the capital market clearing condition. Recall that a_t^t is the asset holding for an agent at time t. Thus, the aggregate saving at time t is denoted by:

$$A_t \equiv \int_{\underline{e}}^{\bar{e}} a_t^t \mathrm{d}\Gamma(e).$$

Capital market clearing implies that today's aggregate saving equals tomorrow's aggregate capital stock, K_{t+1}—i.e.,

$$K_{t+1} = A_t. \tag{2.4}$$

2.3 A Perfect Foresight Political Equilibrium

This section considers any voting rule that allows a pivotal individual to choose a tax rate τ_t for $t > 0$. Before the definition of an equilibrium policy, we describe the policy choice problem confronting young voters. In an environment where future decisions and allocations cannot be committed, these policy choices and allocations must be individually optimal at each period. The sequentially optimal nature of the equilibrium makes the timing of the actions of agents and the government crucial to this analysis. Events unfold thus: Once all young agents in period t have made their allocation decisions, the government, according to the wishes of the pivotal voter in the same period, will impose tax τ_t upon the current output, Y_t. The tax revenue, T_t, is instantaneously redistributed wholly to the young, while the existing old consume. In period $t+1$ the game repeats.[3] The policy voted by each agent is individually optimal, in the sense that such a policy maximizes his lifetime (indirect) utility subject to his personal constraints and the feasibility of policy.

We first examine an open-loop rational expectations policy equilibrium. In this equilibrium, voters take the policy decisions of their predecessors as given. In addition, voters at period t will limit their voting decisions to those they *perceive* to be consistent with their expectations of future policies.[4]

Definition 2.2: An Open-Loop Rational Expectations Equilibrium. Given a record of all the previous policies, and the rational expectations of future policies, an agent chooses $\tau_{i,t}$, $\forall i \in [\underline{e}, \bar{e}]$, $t = 1, 2, \ldots$ such that his lifetime utility is maximized and $\tau_{i,t}$ is feasible.

We introduce several key accounting notations. Recall that at each date, t, there exists a continuum of voters, defined on a closed interval $[\underline{e}, \bar{e}]$. Let i be an individual index, i.e., $i \in [\underline{e}, \bar{e}]$. Each young agent i at date t chooses a tax rate between zero and one. Let us denote an individual voter's strategy set as the closed interval $I_{i,t} \equiv [0, 1]$. Let $I_t \equiv \prod_{i \in [\underline{e}, \bar{e}]} I_{i,t}$ be the joint strategy set of all voters at time t, and likewise let $I \equiv \prod_{t=1,2,\ldots} I_t$ be the product of these joint strategy sets across the infinite horizon. The dynamic tax game is defined by four elements.

[3]The games played at period t and $t+1$ are not identical, in that the payoffs are different. That is, this super-game is dynamic, not just repeating.

[4]To derive the policy decisions in this equilibrium, we make use of the language of dynamic game theory. In particular, we employ the notation devised by Friedman (1990).

Definition 2.3: A Dynamic Tax Game.

$$G = ([\underline{e}, \bar{e}], I, P, \beta).$$

Denote an agent's subjective discount factor as β, and the lifetime payoff function as P. Before the definition of P, recall that in the two–period, overlapping generations model, agents maximize a utility function of the following form:

$$U(l_t^t) + \beta U(c_{t+1}^t),$$

subject to equation (2.2). In the most general version of the game, an agent's single-period payoff functions, defined over strategies, take all previous actions as arguments. Denote a vector of all previous actions at date t as a history of the game, $h_t = \{\tau_0, \tau_1, \ldots, \tau_{t-1}\}$, where $\tau_s \in I_s$, and $s = 0, \ldots, t-1$. Correspondingly, we define the single–period payoff function of voter i at date t as:

$$\pi_{i,t}(\tau_t, h_t),\ \forall i \in [\underline{e}, \bar{e}],\ t = 1, 2, \ldots, \tag{2.5}$$

where $\pi_{i,t}(\tau_t, h_t)$ is assumed to be increasing and concave in $\tau_{i,t}$ for all i and t. Equation (2.5) signifies that an agent's period payoff is a function of his cohort's actions and *all* the actions taken in the past.

2.4 Capital Stock as a State Variable

Next, we provide a sufficient condition under which the single-period payoff function for voters can be specialized to one which takes the level of current capital stock, K_t, as a summary statistic for the history of the game. This restriction substantially reduces the dimensionality of the single-period payoff function.

Consider a version of the payoff functions that take as arguments today's and yesterday's policy decisions. Replacing h_t by its last element, the payoff functions in equation (2.5) can be expressed as:

$$\pi_{i,t}(\tau_t, \tau_{t-1}),\ \forall i \in [\underline{e}, \bar{e}],\ t = 1, 2, \ldots. \tag{2.6}$$

We define the super–game payoff, P, as a discounted stream of these restricted single–period payoffs.

Definition 2.4: Game Payoff Function. Let $P_{i,t}(\tau_{t-1}, \tau_t, \tau_{t+1}) = \pi_{i,t}(\tau_\mathbf{t}, \tau_{t-1}) + \beta \pi_{i,t+1}(\tau_{t+1}, \tau_t)$ be the dynamic game payoff function, $\forall i \in [\underline{e}, \bar{e}]$, $\forall t = 1, 2, \ldots$.

The following stationarity assumption further simplifies the super–game payoff:

Assumption 2.2: Stationarity. Individual preferences, $U(\cdot)$, and the distribution of abilities, $\Gamma(e)$, are stationary.

These stationarity assumptions and the hypothesis of the uniqueness of the pivotal individual enable us to drop both the individual index and the time subscripts, which are associated with the super–game payoff, i.e.,

$$P(\tau_{t+1}, \tau_t, \tau_{t-1}),$$

where $\tau_t \in [0,1]$.[5]

The following statement characterizes the nature of an open–loop, non–cooperative equilibrium for this dynamic tax game:[6]

Proposition 2.1: A Characterization Theorem. $\{\tau_0^*, \tau_1^*, \tau_2^*, \ldots\} \in \tau_0^* \times \prod_{t=1,2,\ldots} I_t$ *is an open-loop non-cooperative equilibrium for a game satisfying Assumption 2.2 if and only if*

$$P(\tau_{t+1}^*, \tau_t^*, \tau_{t-1}^*) = \mathrm{Max}_{\tau_t \in [0,1]} P(\tau_{t+1}^*, \tau_t, \tau_{t-1}^*),\ \forall t = 1, 2, \ldots. \tag{2.7}$$

In addition to providing an algorithm to compute the optimal tax rate for the pivotal voter, Proposition 2.1 asserts that his individually optimal tax rate is a function of two arguments: the tax rate decided in the last period, τ_{t-1}, and the equilibrium tax rate to be selected in the succeeding period, τ_{t+1}. Therefore, the subset of open–loop, non–cooperative solutions of the game G with payoff function P has the form:

$$\Psi(\tau_{t+1}, \tau_{t-1}) \subset [0,1].$$

In particular, a solution to a sequence of tax rates is an element of the above correspondence,

$$\tau_t^* \in \Psi(\tau_{t+1}^*, \tau_{t-1}^*),\ \forall t = 1, 2, \ldots. \tag{2.8}$$

Since we are ultimately concerned with such a political economy in the steady state, the dynamic properties of the open–loop, non–cooperative equilibrium sequence are important for our analysis. We adopt the following technical assumption, which ensures the existence of a uniquely stable steady state for equation (2.8).

Assumption 2.3: The Lipschitz Condition. $\Psi(\tau', \tau'')$ is a single–value function that obeys the following:

$$\|\Psi(\tau_{t+1}, \tau_{t-1}) - \Psi(\tau'_{t+1}, \tau''_{t-1})\| \leq \lambda_1 \|\tau_{t-1} - \tau''_{t-1}\| + \lambda_2 \|\tau_{t+1} - \tau'_{t+1}\|,$$

where $\lambda_1 + \lambda_2 < 1$, $\forall \tau_{t-1}, \tau''_{t-1}, \tau_{t+1}, \tau'_{t+1} \in [0,1]$.

[5]The arguments of P are now scalars, which are policy decisions of the next period, the current period, and the last period, respectively. Since it is understood that only the pivotal individual's decision matters to the policy choice, we suppress his individual subscripts. Also, the pivotal voter's characteristics are stationary, as the distribution of abilities is stationary. Hence, the time–subscripts are irrelevant.

[6]The proofs of the following Proposition 2.1 and other propositions are in Appendix A.

Heuristically, Assumption 2.3 requires that the changes in the equilibrium policy cannot be too rapid.

The following proposition guarantees the existence of an open–loop, non–cooperative equilibrium.

Proposition 2.2: An Existence Theorem.

$$\Psi(\tau_{t+1},\tau_{t-1}) \neq \emptyset,\ \tau_{t+1},\tau_{t-1} \in [0,1],\ \forall t = 1,2,\ldots.$$

The next statement, which characterizes the dynamic properties of the open–loop equilibrium, is due to Friedman (1990).[7]

Proposition 2.3: The Existence of a Uniquely Stable Steady State. *If the equilibrium correspondence of a dynamic game G satisfies the Assumption 2.3, then there exists a unique, steady–state equilibrium. In particular, there exists a unique* $\tau' \in [0,1]$ *such that if* $\tau_0 = \tau'$*, then* $\{\tau', \tau', \ldots\}$ *is an open–loop, non–cooperative equilibrium. Moreover, given any initial tax rate,* $\tau_0 \in [0,1]$*,*

$$\lim_{t\to\infty} \tau_t^* = \tau'.$$

The steady–state tax rate is the solution to the following fixed–point problem:

$$\tau' = \Psi(\tau'). \tag{2.9}$$

If equation (2.9) satisfies the Lipschitz condition, then the equilibrium correspondence under consideration constitutes a contraction mapping, which maps from the closed interval [0,1] to itself. It is well known that a uniquely stable fixed point exists for this type of contraction mapping.[8,9]

Once the existence and the dynamic properties of an open–loop, non–cooperative tax sequence of this simpler version of the tax game are ascertained, we turn to the issue of the decision aggregating mechanism at each stage of game t. In particular, we identify the characteristics of the pivotal voter.

Given the knowledge of τ_{t-1}^*, and a perfect foresight of τ_{t+1}^*, an agent i at date t will vote for a tax rate to maximize his lifetime payoffs—i.e.,

$$\tau_{i,t}^* = \text{ArgMax}_{\tau_t \in [0,1]} P_{i,t}.$$

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[7] See Friedman (1990), page 165, Theorem 5.1.

[8] See, for example, Stokey and Lucas (2005). The existence and stability of a long–run equilibrium tax rate confine our focus to the consideration of the steady–state tax rate.

[9] The existence, uniqueness, and stability of the steady state equilibrium enable us to calibrate a parameterized version of this model to match long–run features of actual economies.

By a modified version of the Median Voter Theorem, we infer that the pivotal voter is at the median of the distribution of abilities.[10] This result is stated formally as follows:

Proposition 2.4: A Median Voter Theorem. *At each period t, given τ^*_{t-1}, a perfect foresight value for τ^*_{t+1}, and a distribution of productivity, $\Gamma(e)$, satisfying Assumptions 2.1 and 2.2,*

$$\tau^*_t = \tau^*_{m,t} > 0, \; \forall t = 1,2,\ldots,$$

where $\tau^*_{m,t}$ denotes the individually optimal choice of the voter endowed with the median ability.

As in Meltzer and Richard (1981), we have established that in the open–loop equilibrium the relevant measure of inequality is the mean–median ratio. Therefore, an economy endowed with a positively skewed distribution of abilities suffers from a higher equilibrium tax rate than a symmetric economy.

2.5 Markovian Payoff Functions

This section shows that the restricted model is identical to single–period payoff functions that take K_t as a state variable. Before the reintroduction of the unrestricted, single–period payoff function for agent i at date t, recall that h_t is the history of the game at date t, h_t. In this environment, an agent's payoff is a function of the current decision, as well as all of the past pivotal, which are summarized in h_t. Consider the pivotal voters' payoff functions,

$$P = \pi_t(\tau_t, h_t) + \beta \pi_{t+1}(\tau_{t+1}, h_{t+1}), \; \forall t = 1,2,\ldots.$$

Recall that at each date, t, capital stock, K_t, is a state variable with an associated law of motion, $\Phi(\cdot)$, where

$$\begin{aligned} \Phi:\ & \kappa \times [0,1] \mapsto \kappa, \\ & \Phi(K_t, \tau_t) \mapsto K_{t+1}, \end{aligned}$$

with $\kappa \subset \Re_+$. Therefore, it is assumed to be compact and convex.

With the inelastic saving assumption, in conjunction with a linear CRS production function, the law of motion can be specialized as:

Proposition 2.5: The Law of Motion of Capital.

$$\Phi(K_t, \tau_t) \equiv K_{t+1} = WN_t(K_t, \tau_t) + \tau_t K_t R. \tag{2.10}$$

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[10] See Meltzer and Richard (1981) and Roberts (1975).

Equation (2.10) describes the nature of capital accumulation in this economy. Since agents save their entire share of after–tax income plus the lump sum welfare transfers, and because these transfers are financed by a redistributive tax rate levied against the total output, the capital stock next period is the sum of the aggregate wage payment and the fraction of welfare transfers that is financed by the old–to–young redistribution. Therefore, this form of redistributive taxation behaves like a reverse social security program that subsidizes the saving of the young.

Assumption 2.4: Invertibility. Φ is continuously invertible with respect to τ_t. The inverse is written as:

$$\tau_t = \Phi^{-1}(K_t, K_{t+1}).$$

Given equation (2.10), a sequence of tax rates, $\{\tau_1, \tau_2, \ldots, \tau_T\}$, and the initial conditions $\{K_0, \tau_0\}$, by iterating the law of motion forward, we obtain K_1 and, recursively, $\{K_2, \ldots, K_{T+1}\}$.

Since the inverse of the law of motion is assumed to hold, given the same pair of initial conditions and a sequence of capital stocks, $\{K_1, \ldots, K_{T+1}\}$, we obtain the sequence of tax rates, $\{\tau_0, \tau_1, \ldots, \tau_T\}$, by iterating the inverse mapping. Therefore, given these initial conditions—the law of motion and its inverse mapping—the two sequences contain the same information. In particular, K_t and τ_{t-1} are informationally equivalent for all $t = 1, 2, \ldots$.

Assumption 2.4 and the law of motion of capital establishes a correspondence between the history of the game and the history of capital stocks. Under the restriction that replaces h_t with τ_{t-1} in the single–period payoff functions, it is equivalent to consider single-period payoff functions that use the capital stock as a state variable. We summarize the preceding discussion in Proposition 2.6:

Proposition 2.6: The Equivalence of Payoff Functions.

$$\pi_{i,t}(\tau_{t-1}, \tau_{i,t}) = \pi_{i,t}(K_t, \tau_{i,t}),\ \forall i \in [\underline{e}, \bar{e}], t = 1, 2, \ldots$$

Since the restriction sets are equivalences, it follows:

Corollary 2.1: Single Period Payoff Function. *The single–period payoff functions, $\pi_{i,t}(K_t, \tau_{i,t})$, are continuous, increasing, and concave in $(K_t, \tau_{i,t})$, $\forall i \in [\underline{e}, \bar{e}]$, $\forall t = 1, 2, \ldots$.*

Since the restrictions on the sets and the functional forms are identical in the two cases, equilibrium and its dynamic properties are preserved, and the two formulations of the payoff functions are identical. Given such a restriction, it therefore is legitimate to express the single–period (Markovian) payoff at date t as $\pi(K_t, \tau_t)$.

2.6 Static Versus Dynamic

The dynamic tax game, which we have considered thus far, is analogous to a dynamic infinite Prisoners' Dilemma problem. Here we recall that the equilibrium correspondence is a function of the tax rates of the prior and the next periods. In an environment where there is no policy commitment, this equilibrium correspondence essentially requires the period decisions to be subgame perfect. In other words, the equilibrium tax rate chosen at period t must satisfy rationality restrictions imposed by *all* subsequent periods. Typically, an infinite Nash equilibrium delivers the lowest possible utility.[11]

This last point highlights the importance of policy commitment in this type of tax game. Persson and Tabellini (2000, 2003, 2004) study endogenous taxation is an overlapping generations model with parametric factor prices and policy commitment. These two assumptions jointly eliminate a subset of equilibrium policies that can be supported by trigger–type strategies. In particular, because of these assumptions the individually optimal decisions by the median voters are repeating (up to the level of existing capital stock) but not dynamic, in that subgame perfection of policy choices is irrelevant. That is, for Persson and Tabellini (2000, 2003, 2004) future decisions are of no concern to the formulation of today's policy.

2.7 A Ramsey Solution

Because of sequential rationality, infinite Nash equilibria are typically sub optimal. This gives agents incentives to cooperate. To illustrate, let us consider an analogous economy where policy commitments are possible. Following Chari et al. (1989), we model policy commitments by allowing the government to set the tax rates once and for all, after which private agents make their economic decisions accordingly. We adopt the following definition of Ramsey equilibrium for our economy:

Definition 2.5: A Ramsey Equilibrium. A Ramsey Equilibrium is a vector of policies, $\{\tau_t, T_t\}_{t=0}^{\infty}$, and a vector of allocations, $\{l_{i,t}^t, n_{i,t}^t, c_{i,t+1}^t\}_{t=0}^{\infty}$, $i \in [\underline{e}, \bar{e}]$, such that:

1. Given a vector of policies, individual allocations maximize each agent's utility subject to his budget constraints;
2. The vector of policies maximize $\sum_{t=0}^{\infty} \beta^t V(\tau_t, K_t)$, where $V(\tau_t, K_t)$ is the indirect utility function of the median voter at period t; and
3. The vector of policies is feasible at each period t.

[11] See Abreu (1988).

In particular, government solves the following policy programming problem for the steady state policy,

$$\mathrm{MAX}_{\tau\in[0,1]} \sum_{t=0}^{\infty} \beta^t V(\tau, K_t),$$

subject to

$$N_t = \mathbf{N}(\tau, K_t), \tag{2.11}$$

$$T_t = \mathbf{T}(\tau, K_t), \tag{2.12}$$

$$K_{t+1} = \boldsymbol{\Phi}(\tau, K_t). \tag{2.13}$$

$V(\tau, K_t)$ is as defined above.[12] Boldface capital letters denote the aggregate reduced forms that determine, in the order of appearance, the aggregate labor participation, the lump sum transfers (or alternatively, the government budget constraint at date t), and the law of motion for the capital stock.

This policy programming with a commitment technology essentially requires median voters of different generations to both cooperate *and* coordinate to achieve the best stationary policy.[13] Let the solution of the preceding program be denoted by τ^R. By construction, we note the following:

Lemma 2.1: The Sustainability of the Open–Loop Equilibrium. *The open-loop non-cooperative equilibrium, τ^*, is sustainable.*

This non–cooperation equilibrium is analogous to an infinite Nash equilibrium, which offers the lowest utility for median voters.

Recall that $V(\cdot)$ denotes the indirect utility of median voters.

Proposition 2.7: An Inequality. *For a given level of capital stock, $K_t > 0$, any sustainable steady–state equilibrium τ must have a utility level $V(\tau, K_t)$ greater than or equal to the utility level $V(\tau^*, K_t)$ of the non–cooperation equilibrium, i.e.,*

$$V(\tau, K_t) \geq V(\tau^*, K_t).$$

Corollary 2.2 addresses the essential difference between the Ramsey solution and the open-loop equilibrium. In an environment where a commitment technology is unavailable, the Ramsey solution is not credible. In another words, there exists at least one binding, sequentially rational restriction that rules the Ramsey solution inadmissible for the set of subgame perfect solutions. Consider Propositions 2.7 and 2.4 jointly. Immediately apparent is the following corollary:

Corollary 2.2: Inequality Condition. $\tau^* > \tau^R$, *which induces an inequality in utility $V(\tau^*, K_t) < V(\tau^R, K_t)$ for any $K_t > 0$.*

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[12] Since we assume stationarity of $\Gamma(e)$, the indirect utility functions of each generation's median voter are identical. Therefore, the associated reduced–form function is time–subscript free.

[13] We could have considered tax sequences that are not stationary. However, in economies with constant growth rates, it has been shown that optimal policies are stationary. See Krusell et al. (1997).

Recall that h_t is a record of tax rates selected from date 0 up to date t, i.e., $h_t = \{\tau_s\}_{s=0}^{t-1}$. Following Chari et al. (1989), we define a set of revert–to–non–cooperation plans, τ^r. These plans specify the continuation of an existing policy if it has been consistently chosen in the past. Otherwise the plans specify reversion to the non–cooperation equilibrium, τ^*. Exploiting this folk–like strategy, the inequality discussed in the following section characterizes the entire set of time–consistent equilibria.

Proposition 2.8: A Folk Theorem. *As long as agents at each date t behave competitively given h_t, an arbitrary tax rate τ is a time–consistent equilibrium if and only if their lifetime utility under the arbitrary τ is higher than their lifetime utility under the non–cooperation equilibrium for any level of capital stock, i.e., $V(\tau, K_t) > V(\tau^*, K_t)$.*

Following from Proposition 2.8 is the observation that if agents care enough about their second period consumption, the Ramsey solution can be supported as a sustainable equilibrium. Formally, we state:

Corollary 2.3: Time Consistent Equilibrium. *There exists a $\bar{\beta} \in (0,1)$ such that for all $\beta \geq \bar{\beta}$ the Ramsey solution can be supported as a time–consistent equilibrium.*

Assuming the discount factor is large enough to support Ramsey equilibrium, a more substantive issue remains in the implementation of a trigger strategy. On the one hand, it has been shown that there is a great potential for fiscal institutions to foster inter–generational cooperation and thereby improve welfare. On the other hand, the theory is completely silent on the issue of coordination. Proposition 2.8 describes a large subset of possible equilibrium policies. Along with its corollaries, Proposition 2.8 specifies only the lower and upper bounds for supportable equilibrium policies, i.e., $\tau^r \in [\tau^*, \tau^R]$. These trigger–type strategies provide little practical guidance as to which policy to select.

Figure 2.1 summarizes the theoretical results by giving a graphical example of two hypothetical economies, A and B. In this depiction, economy A is endowed with a more skewed distribution of abilities. As a consequence, the Nash threat (A) renders the lifetime utility level of median voters closer to the origin. The upper right hand area bounded between the threat point (A) and the Ramsey frontier denotes all other sustainable equilibria, should the appropriate trigger strategies be applied. Let $\tau(a)$ denote the level of lifetime utility corresponding to the implemented tax policy, which is observed from data. The improvement from the threat point (A) to the actual policy, $\tau(a)$, is measured by the effectiveness of intergenerational cooperation, C(a). Similar notations describe economy B, which is endowed with a more symmetric distribution of abilities. Consequently, its threat point (B) is closer to the Ramsey frontier.

Generally, C$(\cdot)$ is not fixed. Given any threat point, the equilibrium, $\tau(.)$, is indeterminate. Thus, without knowing the extent of intergenerational cooperation, the connection between inequality and equilibrium tax rates is undefined in the absence of a commitment technology.

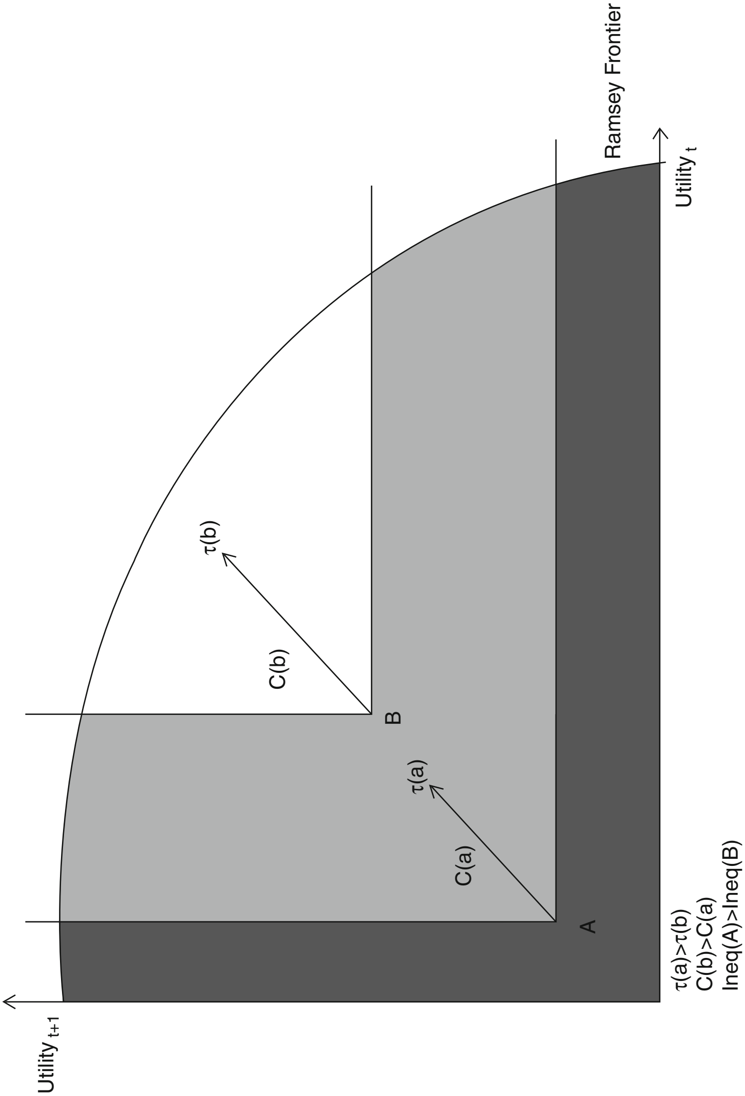

Fig. 2.1 Graphical description of two hypothetical economies

2.8 Inter–Versus Intra–Generation Interactions

There is an interesting correspondence between our model and the model in Alesina (1988). Similar to our framework, Alesina examines equilibrium policies in a dynamic environment in which commitment is unavailable. In a one–shot electoral game, rational voters expect and vote for a political party that will implement its ideological "bliss" policy without comprise. Because different political parties have different bliss points, equilibrium policies oscillate, depending on who holds power. In other words, the lack of commitment eliminates policy convergence—complete or partial—across periods in one–shot elections. However, in a repeated game, policy convergence is subgame perfect, depending on parties' discount rates and the distance between their respective bliss points. Alesina outlines conditions under which policy convergence can occur.

Similar to our model, Alesina also shows that in repeated games, efficient, first–best, and stable policy can be a time-consistent equilibrium in a bargaining game using one–shot Nash equilibria as threats. Like ours, the enforcement mechanism is folk–like: Any observed deviation from agreed–upon policies triggers permanent non–cooperation, whereby parties revert back to the one–shot Nash equilibria forever. Therefore, this mechanism produces credible policy if the discount factor is sufficiently close to one. Moreover, given that electoral victory is probabilistic, Alesina shows that the more equal the victory probabilities (i.e., 50% in a two–party system) the more likely cooperation and policy convergence are *ceteris paribus*. In fact, so long as each party's winning probability is positive, Pareto–improved policies (relative to one–shot Nash) are possible.

Despite these similarities, there are some obvious differences between our model and that of Alesina, particularly in the format and possible outcomes of elections. For example, the voting assumption of our model makes the young median voters pivotal in each period. It therefore could appear that these certain winners have no incentive to cooperate with the certain losers. In other words, it would seem unnecessary for the young pivotal voters to compromise with the contemporaneous old. Furthermore, in our model cooperation and coordination occur among different sets of winners *across* time periods.

These differences are stylistic and do not fundamentally alter the nature of the solution. This is because within a given generation the fraction of time a pivotal voter wins with certainty is 50%. These victories occur only when the pivotal voters are young. However, they lose with certainty the other 50% of their lives, when they are old. Probabilistically, this election framework is equivalent to Alesina's, since political parties within the same period have equal probability of electoral victory. In other words, the effects of inter–generational cooperation can be approximated by intra–generational bargaining, and vice versa.

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