Preface

Things in life break, and as my son used to say after being asked why he broke one of his toys, *"It happens."* This monograph is mainly aimed at providing mathematical insight into why "it happens," especially when brittle materials are of interest. We are interested also in investigating whether "nature is acquainted with the mathematical solution," i.e., does the experimental evidence correspond to the mathematical predictions?

We are motivated by the theory of fracture mechanics, which has matured over the past half century and is able nowadays to predict failure incidents in mechanical components due to an existing *crack*. The classical approach to fracture mechanics is based on a simplified postulate, namely the correlation of a parameter characterizing the linear elastic solution in a neighborhood of the crack tip to experimental observations. It is well known that the linear elastic solution is singular at the crack tip, i.e., its gradient (associated with the stress field) tends to infinity. Thus, from an engineering viewpoint, the linear elastic solution is meaningless in the close vicinity of the crack tip, because of evident nonlinear effects such as large strains and plastic deformations.

Nevertheless, when the nonlinear behavior is confined entirely to some small region inside an elastic solution, then it can be determined through the solution of the linear elastic problem. Consequently, experimental observations on failure initiation and propagation in the neighborhood of a crack tip have been shown to correlate well with the linear elastic solution in many engineering applications.

Although attracting much attention, a crack tip is only a special, and rather simple case of singular points. In a solid body, singular solutions occur at reentrant corners, where material properties abruptly change along a free surface; at interior points where three or more zones of different materials intersect; or where an abrupt change in boundary conditions occurs. In the introduction we show some examples of the aforementioned singularities in "two-dimensional" domains.

From the mathematical viewpoint, the linear elastic solution in the vicinity of any of the above cases has the same characteristics as the solution in the neighborhood of a crack tip. Thus, an unavoidable question comes to mind: *Can one predict failure initiation at the singular points based on parameters of the elastic solution?*

The answer to this question is of major engineering importance due to its broad applicability to failures in electronic devices, composite materials and metallic structures. As in linear elasticity, the solution to heat-conduction problems has similar behavior near singularities, and the coupled thermo elastic response is crucial in understanding failure-initiation events in electronic components.

The first step toward a satisfactory answer is the capability to reliably compute the singular solution and/or functionals associated with it in the neighborhood of any singularity. This is one of the main motivations in writing this monograph. We also wanted to gather as many *explicit* mathematical results as possible on the linear elastic and heat-conduction solutions in the neighborhood of singular points, and present these in engineering terminology for practical usage. This means that we will rigorously treat the mathematical formulations from an engineering viewpoint. We present numerical algorithms for the computation of singular solutions in anisotropic materials and multi material interfaces, and advocate for the proper interpretation of the results in engineering practice, so that these can be correlated to experimental observations.

In the third part of the book, three-dimensional domains and singularities associated with edges and vertices are addressed. These have been mostly neglected in the mathematical analysis due to the tedious required treatment. In the past ten years, major achievements have been realized in the mathematical description of the singular solution in the vicinity of 3-D edges, with new insights into these realistic 3-D solutions. These are summarized herein together with new numerical methods for the extraction of so-called edge stress intensity functions and their relevance to fracture initiation. We also derive exact solutions in the vicinity of vertex singularities and extend the numerical methods for the computation of these solutions when analytical methods become too complex to be applied.

I have tried to make this book introductory in nature and as much as possible self-contained, and much effort has been invested to make the text uniform in its form and notation. Nevertheless, some preliminary knowledge of the finite element method is advised (see, e.g., [178]) but not mandatory, because we use the method for the solution of example problems (a short chapter is devoted to finite element fundamentals). It is aimed at the postgraduate level and to practitioners (engineers and applied mathematicians) who are working in the field of failure initiation and propagation. *Many examples of engineering relevance are provided and solved in detail.* We apologize to authors of relevant works that have not been cited; this is the result of my ignorance rather than my judgment.

The book is divided into fourteen chapters, each containing several sections. Most of it (the first nine chapters) addresses two-dimensional domains, where only singular *points* exist. The thermo elastic system and the feasibility of using the eigen pairs and GSIFs for predicting failure initiation in brittle material in engineering practice are addressed. Several failure laws for two-dimensional domains with V-notches and multi material interfaces are presented, and their validity is examined by comparison to experimental observation. A sufficient simple and reliable condition for predicting failure initiation (crack formation) in micron-level electronic devices, involving singular points, is still a topic of active research and interest, and we address it herein. Three-dimensional problems are addressed in the next five chapters, discussing the singular solution decomposition into edge, vertex, and edge-vertex singular solutions. I conclude with circular edges in 3-D domains and some remarks on open questions.

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