Preface

Algebraic geometry is the geometric study of sets of solutions to polynomial equations over a field (or ring). These objects, called algebraic varieties (or schemes or ...), can be studied using tools from commutative and homological algebra. When the field is the field of complex numbers, these methods can be supplemented with transcendental ones, that is, by methods from complex analysis, differential geometry, and topology. Much of the beauty of the subject stems from the rich interplay of these various techniques and viewpoints. Unfortunately, this also makes it a hard subject to learn. This book evolved from various courses in algebraic geometry that I taught at Purdue. In these courses, I felt my job was to act as a guide to the vast terrain. I did not feel obligated to cover everything or to prove everything, because the standard accounts of the algebraic and transcendental sides of the subject by Hartshorne [60] and Griffiths and Harris [49] are remarkably complete, and perhaps a little daunting as a consequence. In this book I have tried to maintain a reasonable balance between rigor, intuition, and completeness. As for prerequisites, I have tried not to assume too much more than a mastery of standard graduate courses in algebra, analysis, and topology. Consequently, I have included discussions of a number of topics that are technically not part of algebraic geometry. On the other hand, since the basics are covered quickly, some prior exposure to elementary algebraic geometry (at the level of say Fulton [40], Harris [58, Chapters 1–5] or Reid [97]) and calculus with manifolds (as in Guillemin and Pollack [56, Chapters 1 & 4] or Spivak [109]) would certainly be desirable, although not absolutely essential.

This book is divided into a number of somewhat independent parts with slightly different goals. The starred sections can be skipped without losing much continuity. The first part, consisting of a single chapter, is an extended informal introduction illustrated with concrete examples. It is really meant to build intuition without a lot of technical baggage. Things really get going only in the second part. This is where sheaves are introduced and used to define manifolds and algebraic varieties in a unified way. A watered-down notion of scheme—sufficient for our needs—is also presented shortly thereafter. Sheaf cohomology is developed quickly from scratch in Chapter 4, and applied to de Rham theory and Riemann surfaces in the next few chapters. By Part III, we move into Hodge theory, which is really the heart of transcendental algebraic geometry. This is where algebraic geometry meets differential geometry on the one hand, and some serious homological algebra on the other. Although I have skirted around some of the analysis, I did not want to treat this entirely as a black box. I have included a sketch of the heat equation proof of the Hodge theorem, which I think is reasonably accessible and quite pretty. This theorem along with the weak and hard Lefschetz theorems have some remarkable consequences for the geometry and topology of algebraic varietes. I discuss some of these applications in the remaining chapters. From Hodge theory, one extracts a set of useful invariants called Hodge numbers, which refine the Betti numbers. In the fourth part, I consider some methods for actually computing these numbers for various examples, such as hypersurfaces. The task of computing Hodge numbers can be converted to an essentially algebraic problem, thanks to the GAGA theorem, which is explained here as well. This theorem gives an equivalence between certain algebraic and analytic objects called coherent sheaves. In the fifth part, I end the book by touching on some of the deeper mysteries of the subject, for example, that the seemingly separate worlds of complex geometry and characteristic p geometry are related. I will also explain some of the conjectures of Grothendieck, Hodge, and others along with a context to put them in.

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