Preface

This book aims to be useful. This might appear to be a trivial statement (after all, what would the alternative be?), but let us explain how this simple motivation provides us with a rather ambitious goal. The central theme of the book revolves around Stokes's theorem, and it deals with the following associated paradox. There are clear intuitive notions coming from the physical world and our own visual geometric insight that tell us what a closed surface is, what the interior and exterior of that surface are, what is meant by a flux across it, what a normal vector to it is, and whether it points in or out—in other words, how to orient that surface. The student of vector calculus is usually provided with a clear and useful set of rules as to how to orient a surface in applying the divergence theorem, and how to orient the boundary of a surface in the classical Stokes's theorem. However, when this student undertakes a formal study of orientation through mathematical analysis and/or differential geometry, she or he then realizes that orientation is defined in terms of the tangent space at each point of the surface, and the connection with the practical rules of vector calculus is far from clear. To make things worse, the usual closed surfaces used in $\mathbb{R}^3$, that is, those that are required for practical purposes, have vertices and edges, the most natural example being the cube, and they are not regular surfaces. Hence, a student of mathematics, formally at least, cannot apply Stokes's theorem to most natural situations in which it is required.

There is another element that deeply concerned the authors when they were introduced to the subject, and that is the various notational conventions. Actually, the problem is not just with notation. This is a subject that can be approached in many different ways, all of which are equally valid, and each of which has its own particular merits. There is undoubtedly an advantage in seeing a topic treated in different ways, but unfortunately in this instance, it is all too common for a student to become trapped with the particular notation and/or point of view used by one author on the subject. For example, a recommended textbook may choose to use a vectorial point of view, and employ integrals of differential forms, while another may opt for scalar integrals and the use, or not, of tensors. There are several possibilities for the definition of regular surface, from the very abstract notion of differential manifold to the more familiar concept of differential submanifold of $\mathbb{R}^n$, and so on. A student will follow one approach, which uses one of the more or less equivalent definitions available, but when the student tries to clarify an obscure point by studying another good exposition, very frequently the notation is alien, or worse, inconsistent with what the student already knows, so that the only option is to begin from scratch with the alternative approach. Most of the time, the student becomes frustrated and simply gives up.

This book is intended as a text for undergraduate students who have completed a standard introduction to differential and integral calculus of functions of several variables. We have written the book principally having in mind students of mathematics who need a precise and rigorous exposition of Stokes's theorem. This has led us to choose a differential-geometric point of view. However, we have taken great care to bridge the gap between a formal rigorous approach and a concrete presentation of applications in two and three variables. We show how to use the tools from vector calculus and modern methods that help to check, for example, whether a particular set in $\mathbb{R}^3$ is an orientable surface with boundary. In a less formal way, we show how to apply the obtained results on integration over regular surfaces to less amenable (but more practical) situations like the cube. We have looked at most of the definitions of regular surface and shown the equivalence of them. We discuss how one definition may be more convenient for solving exercises while another, equivalent, definition may be more suitable in proving a theorem. We have tried to include in each chapter as many examples and solved exercises as possible.

We have chosen the point of view of k-forms, but in each possible instance we switch to employing vector fields and the classical notation coming from physics. In general, we have made an effort to explain the connection between the usual practical rules from vector calculus and the rigorous theory that is at the core of vector analysis. This means that the book is also addressed to engineering and physics students, who know quite well how to handle the familiar theorems of Green, Stokes's, and Gauss, but who would like to know why they are true and how to recover these familiar useful tools in $\mathbb{R}^2$ or $\mathbb{R}^3$ from the mighty formal Stokes's theorem in $\mathbb{R}^n$. In summary, we have tried hard to show that vector analysis and vector calculus are not always at odds with one another. Perhaps we should have appended a question mark to the title.

The book contains some appendices that are not necessary for the rest of the book, but will offer the student the opportunity to get more deeply involved in the subject at hand.

While we are in great debt to many authors, including Do Carmo, Edwards, Fleming, Rudin, and Spivak, we do believe that our approach is quite original. However, we do not pretend that originality is our principal motivation. Only to be useful.

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