

Chapter 2

Characteristics of Whole Evolutions

After introducing the brief history of the concept of systems and the systemic yoyo model in the previous chapter, we now in this chapter look at the theoretical foundation on why such an intuitive model of general systems holds for each and every system that is either tangible or imaginable.

2.1 Blown-Ups: Moments of Transition in Evolutions

When we study nature and treat everything we see as a system (Klir 2001), then one fact we can easily see is that many systems in nature evolve in concert. When one thing changes, many other seemingly unrelated things alter their states of existence accordingly. If we attempt to make predictions regarding an evolving event, such as the price of a stock in the financial markets, where the market behavior is dependent on the behaviors of the market participants, or what is forthcoming in the weather system, where nature goes through its normal course of evolution without being affected by how we think it would come out to be, we may very well find ourselves in a situation that is somewhere in between whereby we do not have enough information or we have too much information. It has been shown (Soros 2003; Liu and Lin 2006; Lorenz 1993) that no matter which situation we are in, too much information or too little, we face with uncertainties. That is why we propose (OuYang et al. 2009) to look at the evolution of the system or event whose future outlook we need to predict as a whole. That is, when developments and changes naturally existing in the natural environment are seen as a whole, we have the concept of whole evolutions. And, in whole evolutions, other than continuities, also studied in modern mathematics and science, what seems to be more important and more common is discontinuity, with which transitional changes (or blown-ups) occur. These blown-ups reflect not only the singular transitional characteristics of the whole evolutions of nonlinear equations, but also the changes in old structures being replaced by new structures.

Thousands of case studies have shown the fact (Lin 2008b) that reversal and transitional changes are the central issue and an extremely difficult open problem of prediction science, since the well-developed method of linearity, which tries to extend the past rise-and-fall into the future, does not have the ability to predict forthcoming transitional changes and what will occur after the changes. In terms of nonlinear evolution models, blown-ups reflect destructions of old structures and establishments of new ones. Although studies of these nonlinear models reveal the limits and weaknesses of calculus-based theories, we can still make use of their analytic forms to describe to a certain degree the realisticity of discontinuous transitional changes of objective events and materials. So, the concept of blown-ups is not purely mathematical; it is also realistic. By borrowing the form of calculus, we can write the concept of blown-ups as follows: For a given (mathematical or symbolic) model that truthfully describes the physical situation of our concern, if its solution $u = u(t; t_0, u_0)$, where t stands for time and u_0 the initial state of the system, satisfies

$$\lim_{t \rightarrow t_0} |u| = +\infty, \quad (2.1)$$

and at the same time moment when $t \rightarrow t_0$, the underlying physical system also goes through a transitional change, then the solution $u = u(t; t_0, u_0)$ is called a blown-up solution and the relevant physical movement expresses a blown-up. Our analysis of thousands of real-life cases of various evolutionary systems (Lin 2008b) indicate that disastrous events appear at the moments of blown-ups in the whole evolutions of systems.

For nonlinear models in independent variables of time (t) and space (x, x , and y , or x, y , and z), the concept of blown-ups is defined similarly, where blow-ups in the model and the underlying physical system can appear in time or in space or in both. For instance, if the time-space evolution equation is written as follows:

$$\partial_t u = g(t, u, \partial_x u), \quad (2.2)$$

where u is an $n \times 1$ matrix of the state variables, $g(t, u, \partial_x u)$ an $n \times 1$ matrix of functions in t, u , and $\partial_x u$, ∂ and ∂_t stand for the differential operations $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t}$, respectively. Assume that the initial (or boundary) condition is

$$u(t_0, x) = u_0(x). \quad (2.3)$$

Then, when the solution $u = u(t, x; t_0, u_0)$ or $u_x = u_x(t, x; t_0, u_0)$ changes, for $t \in [t_0, t_b)$, continuously, and when $t \rightarrow t_b$, the following holds true

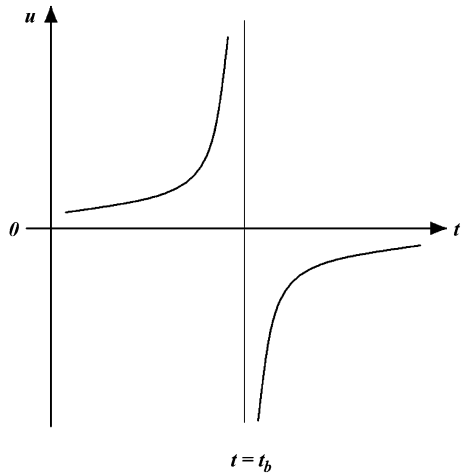
$$\lim_{t \rightarrow t_0} |u| = +\infty, \quad (2.4)$$

or

$$\lim_{t \rightarrow t_0} |u_x| = +\infty, \quad (2.5)$$

then either u or u_x is called a blown-up solution, if the boundary value problem, consisting of Eqs. 2.2 and 2.3, truly describes a physical system and at the time

Fig. 2.1 A transitional blown-up



moment $t = t_b$, this physical system goes through a transitional change. In the rest of this chapter, we assume that this assumption holds true.

Based on the evolutionary behaviors of the physical system before and after the transition, we can classify blown-ups into two categories: transitional and non-transitional blown-ups. A blown-up is transitional, if the development of the physical system after the special blow-up moment in time or space is completely opposite to that before the blow-up. For example, if the evolution of the system grows drastically before the blow-up and the development of the system after the blow-up starts from nearly ground “zero,” then such blown-up is transitional (Fig. 2.1). Otherwise, it is called non-transitional (Fig. 2.2).

2.2 Mathematical Properties of Blown-Ups

To help us understand the mathematical characteristics of blown-ups, let us look at the following constant-coefficient equation:

$$\dot{u} = a_0 + a_1 u + \dots + a_{n-1} u^{n-1} + u^n = F, \quad (2.6)$$

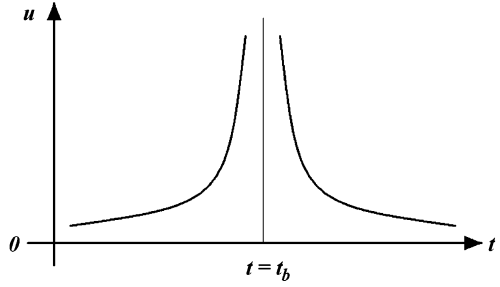
where u is the state variable, and a_0, a_1, \dots, a_{n-1} are constants.

2.2.1 Blown-Up Properties of Quadratic Nonlinear Models

When $n = 2$, Eq. 2.6 becomes

$$\dot{u} = a_0 + a_1 u + u^2, \quad (2.7)$$

Fig. 2.2 A non-transitional blown-up



which is one of the simplest general nonlinear evolution models and is also a special Riccati equation [for all omitted technical details in this chapter, please consult with (Wu and Lin 2002)]. This equation has found a wide range of applications in the prediction science. Let us now first discuss this equation's properties of blown-ups, which are important in terms of both theoretical and practical implications. Here, we will not apply Riccati variable transformations to solve Eq. 2.7. Instead, we will present our analysis with different values of a_0 and a_1 .

1. When $\Delta = a_1^2 - 4a_0 = 0$, Eq. 2.7 becomes

$$\dot{u} = \left(u + \frac{1}{2}a_1\right)^2. \quad (2.8)$$

Now, integrating by separating variables produces

$$u = -\frac{1}{2}a_1 - (A_0 + t)^{-1} \quad (2.9)$$

where A_0 is the integrating constant, determined by the given initial value. Evidently, the solution Eq. 2.9 contains a blown-up. To this end, let us analyze the details as follows.

- (a) If $A_0 > 0$, then u gradually stabilizes at the equilibrium state $-\frac{1}{2}a_1$ as time t approaches $+\infty$. So, in the interval $t \in [0, +\infty)$, the evolution of u is continuous.
 - (b) If $A_0 < 0$, then when $t \rightarrow t_0 = -A_0$, the state $u \rightarrow +\infty$. At this moment, u experiences a discontinuous singularity and a transitional change after the singularity. This kind of discontinuity has nothing to do with the smoothness of the given initial field, showing the essential characteristics of nonlinear evolutions. See Fig. 2.3.
2. When $\Delta = a_1^2 - 4a_0 > 0$, Eq. 2.7 becomes

$$\dot{u} = \left(u + \frac{1}{2}a_1\right)^2 - \frac{1}{4}(a_1^2 - 4a_0). \quad (2.10)$$

By separating the variables and integrating the previous equation, we obtain

Fig. 2.3 Curve #2 stands for the integral curve when $A_0 > 0$, while curves #1 and #3 for $A_0 < 0$, where the u -axis is located at $t = -A_0$

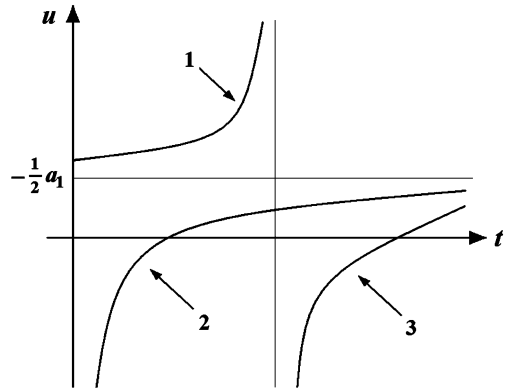
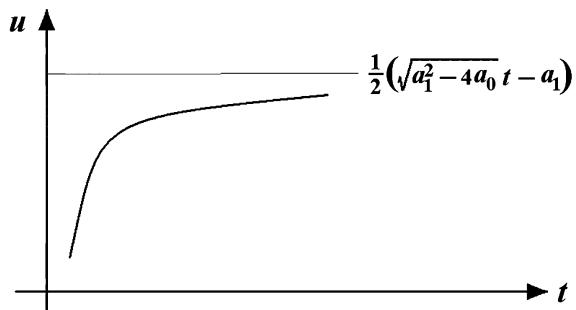


Fig. 2.4 Continuous evolution without experiencing any blown-up for the case $a_1 < 0$



$$\ln \left| \frac{u + \frac{1}{2}a_1 - \frac{1}{2}\sqrt{a_1^2 - 4a_0}}{u + \frac{1}{2}a_1 + \frac{1}{2}\sqrt{a_1^2 - 4a_0}} \right| = \sqrt{a_1^2 - 4a_0}t + A_0, \quad (2.11)$$

where A_0 is the integration constant. For this scenario, we can analyze the blown-up situation as follows.

- (a) If the movement of our interest is limited to $|u + \frac{1}{2}a_1| < \frac{1}{2}\sqrt{a_1^2 - 4a_0}$, which is a bounded movement, then the solution of Eq. 2.10 is given as follows:

$$u = \frac{1}{2}\sqrt{a_1^2 - 4a_0} \tanh\left(\frac{1}{2}\sqrt{a_1^2 - 4a_0}t + \frac{1}{2}A_0\right) - \frac{1}{2}a_1. \quad (2.12)$$

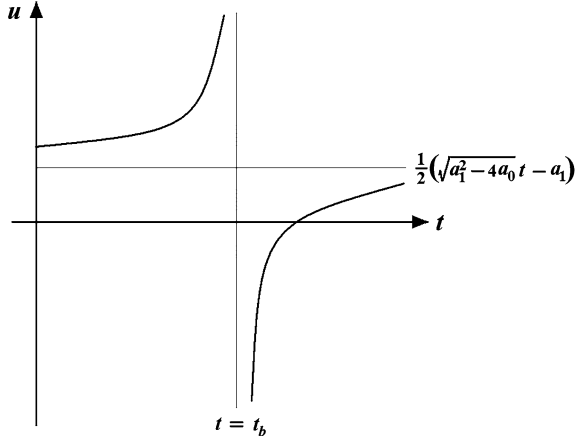
In this case, the development of the evolution is continuous without experiencing any blown-up, Fig. 2.4.

- (b) If the range of movement satisfies $|u + \frac{1}{2}a_1| > \frac{1}{2}\sqrt{a_1^2 - 4a_0}$, which stands for a bounded motion, then we have

$$u = \frac{1}{2}\sqrt{a_1^2 - 4a_0} \coth\left(-\frac{1}{2}\sqrt{a_1^2 - 4a_0}t - \frac{1}{2}A_0\right) - \frac{1}{2}a_1. \quad (2.13)$$

If $A_0 < 0$ and when

Fig. 2.5 A blown-up solution for the case of $a_1 < 0$



$$t \rightarrow t_b = -\frac{A_0}{\sqrt{a_1^2 - 4a_0}}, \quad (2.14)$$

the solution in Eq. 2.13 experiences a blown-up, Fig. 2.5. What is analyzed indicates that under a fixed set of conditions, whether or not a model experiences discontinuities is dependent on the given initial field. So, it is difficult for numerical solutions of nonlinear evolution equations to be bounded in each iteration step, no matter which smoothing scheme is employed. In this sense, it can be said that various integration schemes, designed to solve nonlinear evolution equations, cannot really avoid experiencing “explosive” growth or being trapped in “error-value spiral” computations due to the evolutionary singularities. This is where many of the paradoxes and “fascinating” conclusions in the Lorenz’s chaos theory are originated.

3. When $\Delta = a_1^2 - 4a_0 < 0$, Eq. 2.7 becomes

$$\dot{u} = \left(u + \frac{1}{2}a_1\right)^2 + \frac{1}{4}(4a_0 - a_1^2). \quad (2.15)$$

Integrating this equation provides the following:

$$u = \frac{1}{2}\sqrt{4a_0 - a_1^2} \tan\left(\frac{1}{2}\sqrt{4a_0 - a_1^2}t + A_0\right) - \frac{1}{2}a_1. \quad (2.16)$$

When

$$t \rightarrow t_b = \frac{2}{\sqrt{4a_0 - a_1^2}}\left(\frac{\pi}{2} + n\pi - A_0\right),$$

Eq. 2.16 contains periodic transitional blown-ups.

From this detailed analysis it can be seen that under different conditions, the solution of the same nonlinear model can either be continuous and smooth or experience either blown-ups or periodic transitional blown-ups. So, even for the simplest nonlinear evolution equations, the well-posedness of their evolutions in terms of differential mathematics is conditional. Here, the requirements for well-posedness are: the solution exists; it is unique, and stable (or continuous and differentiable).

A side note: please be aware that the condition of well-posedness is very important for the majority of the applied mathematics to work.

2.2.2 Blown-Up Properties of the Cubic and n th Degree Polynomial Models

When $n = 3$, other than the situation that $F = 0$ has two real solutions, one of which is of multiplicity 2, Eq. 2.6 experiences blown-ups. For details, please consult with (Wu and Lin 2002). For the general case n , even though the analytical solution of Eq. 2.6 cannot be found exactly, the blown-up properties of the solution can still be studied through qualitative means. To this end, based on the fundamental theorem of algebra, let us assume that Eq. 2.6 can be written as

$$\dot{u} = F = (u - u_1)^{p_1} \dots (u - u_r)^{p_r} (u^2 + b_1 u + c_1)^{q_1} \dots (u^2 + b_m u + c_m)^{q_m}, \quad (2.17)$$

where p_i and q_j , $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, m$, are positive whole numbers, and $n = \sum_{i=1}^r p_i + 2 \sum_{j=1}^m q_j$, $\Delta = b_j^2 - 4c_j < 0$, $j = 1, 2, \dots, m$. Without loss of generality, assume that $u_1 \geq u_2 \geq \dots \geq u_r$, then the blown-up properties of the solution of Eq. 2.17 are given in the following theorem.

Theorem 2.1 The condition under which the solution of an initial value problem of Eq. 2.3 contains blown-ups is given by

1. When u_i , $i = 1, 2, \dots, r$, does not exist, that is, $F = 0$ does not have any real solution; and
2. If $F = 0$ does have real solutions u_i , $i = 1, 2, \dots, r$, satisfying $u_1 \geq u_2 \geq \dots \geq u_r$,
 - (a) When n is an even number, if $u > u_1$, then u contains blow-up(s);
 - (b) When n is an odd number, no matter whether $u > u_1$ or $u < u_r$, there always exist blown-ups.

A detailed proof of this theorem can be found in (Wu and Lin 2002, pp. 65–66) and is omitted here.

For higher order nonlinear evolution systems, in general, each of them can be simplified into systems of nonlinear evolution equations in a form similar to that of Eq. 2.6. Since the general case is difficult to analyze, let us look at the following second order nonlinear evolution equation:

$$\ddot{u} = a_0 + a_1 u + a_2 u^2 + u^3, \quad (2.18)$$

where $\ddot{u} = \frac{d^2 u}{dt^2}$ and a_i ($i = 0, 1, 2$) are constants. Now, this equation can be reduced to the following system of first order equations in two variables:

$$\begin{cases} \dot{u} = v \\ \dot{v} = a_0 + a_1 u + a_2 u^2 + u^3 \end{cases} \quad (2.19)$$

Multiplying the left-hand side of the second equation by v and the right-hand side by u and then integrating the resultant equation provides the following:

$$v^2 = E = 2 \left(\frac{1}{4} u^4 + \frac{1}{3} a_2 u^3 + \frac{1}{2} a_1 u^2 + a_0 u + h_0 \right), \quad (2.20)$$

where h_0 is the integration constant. Now, the system in Eq. 2.20 can be reduced into

$$\dot{u} = \pm \sqrt{E}. \quad (2.21)$$

Now, we study the blown-up properties of Eq. 2.21 as follows.

1. If $E = 0$ has a real solution of multiplicity four, then based on Eq. 2.8, it can be shown that u contains blown-ups.
2. If $E = 0$ contains two distinct real solutions of multiplicity two, then $E = \frac{1}{4}(u^2 + b_1 u + c_1)^2$ and $b_1^2 - 4c_1 \neq 0$. Now, the analysis is done in two cases:
 - (a) When $b_1^2 - 4c_1 > 0$, based on Eq. 2.10 it follows that u contains a continuous bounded solution and a blown-up solution.
 - (b) When $b_1^2 - 4c_1 < 0$, based on Eq. 2.14 it follows that u contains both a periodic blown-up solution.
3. If $E = 0$ contains only one real solution, which has multiplicity two, and two conjugate complex solutions, then $E = u^2(u^2 + b_2 u + c_2)$, where we assumed that the real solution of multiplicity two is zero, and $b_2^2 - 4c_2 < 0$. In this case, Eq. 2.21 becomes

$$\dot{u} = \pm u \sqrt{u^2 + b_2 u + c_2}. \quad (2.22)$$

Let $u = \frac{1}{v}$. Then the previous equation can be reduced into

$$\frac{dv}{\sqrt{C_2 \left(v + \frac{b_2}{2c_2} \right)^2 + \frac{4c_2 - b_2^2}{4c_2}}} = \pm dt. \quad (2.23)$$

Now, let us analyze the scenario in two cases.

- (a) When $c_2 < 0$, we have $b_2^2 - 4c_2 < 0$ and an integration of Eq. 2.23 can produce

$$-\arcsin \frac{v + \frac{b_2}{2c_2}}{\sqrt{\frac{b_2^2 - 4c_2}{4c_2}}} = \sqrt{-c_2}(\pm t + A_0), \quad (2.24)$$

where A_0 is a constant. When t takes on a certain special value, $v = 0$ and $u = \frac{1}{v} \rightarrow \infty$. So, the solution of Eq. 2.22 experiences blown-ups.

- (b) When $c_2 > 0$ and either $b_2^2 - 4c_2 > 0$ or $b_2^2 - 4c_2 < 0$, integrating Eq. 2.23 provides

$$-\ln \left[\sqrt{c_2} \left(v + \frac{b_2}{2c_2} \right) + \sqrt{c_2 \left(v + \frac{b_2}{2c_2} \right)^2 + \frac{4c_2 - b_2^2}{4c_2}} \right] = \sqrt{c_2}(\pm t + A_0). \quad (2.25)$$

which evidently contains blown-ups.

4. If $E = 0$ has four distinct real solutions, let us assume $u_1 > u_2 > u_3 > u_4$, then Eq. 2.21 becomes

$$\dot{u} = \pm \sqrt{(u - u_1)(u - u_2)(u - u_3)(u - u_4)}. \quad (2.26)$$

So, the continuous bounded movements of Eq. 2.26 can be written by using an elliptic function. As for the unbounded movement, we will have to apply the method of estimation to prove that it contains blown-ups. As a matter of fact, when $u > u_1$ and take the “+” on the right hand side of Eq. 2.26, we can obtain the following estimate:

$$\dot{u} \geq (u - u_1)^2. \quad (2.27)$$

It is not hard to prove that in this case, u contains blown-ups. Similarly, it can be shown that when $u < u_4$, u also contains blown-ups.

Example 2.2.1 To see how the previous procedure actually works out, let us look at the following example of the nonlinear elasticity model

$$\ddot{X} = -fX + X^3, \quad (2.28)$$

where X stands for displacement of position, f the linear elasticity coefficient. As what was done to Eq. 2.18, we have

$$E^2 = \dot{X}^2 = \frac{1}{4}X^4 - \frac{1}{2}fX^2 + h_0. \quad (2.29)$$

Now, Eq. 2.28 can be reduced to the following first order nonlinear evolution equation

$$\dot{X} = \pm\sqrt{E}. \quad (2.30)$$

Our earlier discussion indicates that under certain conditions, the evolution of X contains blown-ups. Therefore, the nonlinear elasticity change is fundamentally different from that of linear elasticity change. That is, within the range of elasticity, linear elasticity models evolve continuously and smoothly. And, the evolution of nonlinear elasticity models contains discontinuous singular blown-ups.

2.2.3 Blown-Up Properties of Nonlinear Time–Space Evolution Equations

As for evolution equations involving changes in space, they can directly and intuitively reflect the physical meanings of blown-ups. The one-dimensional advection equation is one of the simplest nonlinear evolution equations involving changes in space. Its Cauchy problem can be written as follows:

$$\begin{cases} u_t + uu_x = 0 \\ u|_{t=0} = u_0 \end{cases}, \quad (2.31)$$

where u stands for the speed of a flow, $u_t = \frac{\partial u}{\partial t}$ and $u_x = \frac{\partial u}{\partial x}$ with t representing time and x the one-dimensional spatial location. By using the method of separating variables without expansion, we let

$$u = A(t)v(x) \text{ and } u_0(x) = A(0)v(x). \quad (2.32)$$

So, Eq. 2.31 can be reduced to the following:

$$\frac{\dot{A}}{A^2} = -v_x = -\lambda, \quad (2.33)$$

where λ is a constant. Then we have

$$\dot{A} + \lambda A^2 = 0 \text{ and } v_x = \lambda. \quad (2.34)$$

Integrating Eq. 2.34 by separating the variables and taking $A(0) = A_0$ provides

$$A = \frac{A_0}{1 + A_0 v_x t}. \quad (2.35)$$

Multiplying both sides of this equation by u_x and taking $u_{0x} = A_0 v_x$ produces

$$u_x = \frac{u_{0x}}{1 + u_{0x}t}. \quad (2.36)$$

Based on the definition of the degree of divergence, u_x is the first-dimensional degree of divergence. When $u_{0x} > 0$, that is when the initial field is divergent, u_x declines continuously with time t until the diverging motion disappears. If $u_{0x} < 0$, that is when the initial field is convergent, then the solution of $u_x \rightarrow \infty$ and experiences a discontinuous singularity with time $t \rightarrow t_b = -\frac{1}{u_{0x}}$. Evidently, when $t < t_b$, $u_x < 0$ evolves continuously.

When $t > t_b$, $u_x > 0$. So, the convergent movement of the initial field ($u_{0x} < 0$) can be transformed into a divergent movement ($u_{0x} > 0$) through a blown-up. The characteristics of this kind of movement cannot be truly and faithfully described by linear analysis or statistical analysis. They reflect the fundamental characteristics of nonlinear evolutions.

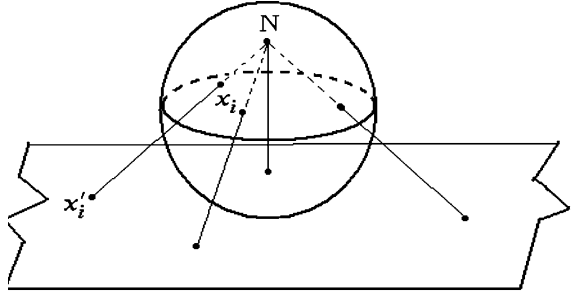
2.3 The Problem of Quantitative Infinity

One of the features of blown-ups is the quantitative infinity ∞ , which stands for indeterminacy mathematically. So, a natural question is how to comprehend this mathematical symbol ∞ , which in applications causes instabilities and calculation spills that have stopped each and every working computer.

2.3.1 *An Implicit Transformation Between Euclidean Spaces and Curvature Spaces*

To address the previous question, let us look at the mapping relation of the Riemann ball, which is well studied in complex functions (Fig. 2.6). This so-called Riemann ball, a curved or curvature space, illustrates the relationship between the infinity on the plane and the North Pole N of the ball. Such a mapping relation connects $-\infty$ and $+\infty$ through a blown-up. Or in other words, when a dynamic point x_i travels through the North Pole N on the sphere, the corresponding image x'_i on the plane of the point x_i shows up as a reversal change from $-\infty$ to $+\infty$ through a blown-up. So, treating the planar points $\pm\infty$ as indeterminacy can only be a product of the thinking logic of a narrow or low dimensional observ-control, since, speaking generally, these points stand implicitly for direction changes of one dynamic point on the sphere at the polar point N. Or speaking differently, the phenomenon of directionless, as shown by blown-ups of a lower dimensional space, represents exactly a direction change of movement in a higher dimensional curvature space.

Fig. 2.6 The Riemann ball—relationship between planar infinity and three-dimensional north pole



Therefore, the concept of blown-ups can specifically represent implicit transformations of spatial dynamics. That is, through blown-ups, problems of indeterminacy of a narrow observ-control in a distorted space are transformed into determinant situations of a more general observ-control system in a curvature space. This discussion shows that the traditional view of singularities as meaningless indeterminacies has not only revealed the obstacles of the thinking logic of the narrow observ-control (in this case, the Euclidean space), but also the careless omissions of spatial properties of dynamic implicit transformations (bridging the Euclidean space to a general curvature space).

Corresponding to the previous implicit transformation between the imaginary plane and the Riemann ball, let us now look at how we can relate quantitative $\pm\infty$ (symbols of indeterminacy in one-dimensional space) to a dynamic movement on a circle (a curved space of a higher dimension) through the modeling of differential equations.

In the beginning of Sect. 2.2, we studied the blown-ups of quadratic models. Now, let us look at the essence of singularities by investigating spatial dynamic transformations. To make this presentation easier to follow, let us first cite some relevant results from Sect. 2.2 on quadratic evolution models. For the evolution described by

$$\dot{u} = a_0 + a_1 u + u^2, \quad (2.37)$$

we have three possibilities:

1. When $a_1^2 - 4a_0 = 0$,

$$u = -\left(\frac{1}{t + A_0} - \frac{1}{2}a_0\right). \quad (2.38)$$

2. When $a_1^2 - 4a_0 > 0$, there exist the following two possibilities:

(a) If $|\frac{1}{2}a_1 + u| < \frac{1}{2}\sqrt{a_1^2 - 4a_0}$, then

$$u = -\frac{1}{2}\sqrt{a_1^2 - 4a_0} \tanh\left(\frac{1}{2}\sqrt{a_1^2 - 4a_0}t + \frac{1}{2}A_0\right) - \frac{1}{2}a_1. \quad (2.39)$$

(b) If $|\frac{1}{2}a_1 + u| > \frac{1}{2}\sqrt{a_1^2 - 4a_0}$, then

$$u = \frac{1}{2}\sqrt{a_1^2 - 4a_0} \coth\left(-\frac{1}{2}\sqrt{a_1^2 - 4a_0}t - \frac{1}{2}A_0\right) - \frac{1}{2}a_1. \quad (2.40)$$

3. When $a_1^2 - 4a_0 < 0$, then

$$u = \frac{1}{2}\sqrt{4a_0 - a_1^2} \tan\left(\frac{1}{2}\sqrt{4a_0 - a_1^2}t + \frac{1}{2}A_0\right) - \frac{1}{2}a_1. \quad (2.41)$$

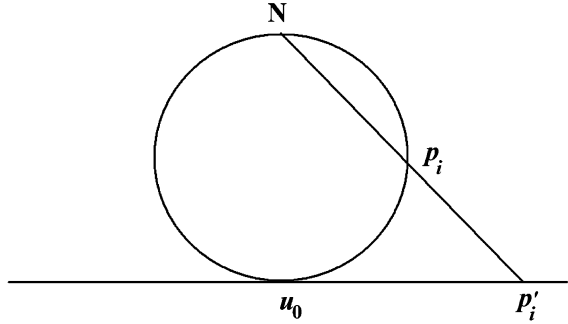
These results imply that other than Eq. 2.40, which represents a local smooth continuity, all other cases, as shown in Eqs. 2.38, 2.39 and 2.41, experience discontinuous blown-ups in their whole evolutions. Now, let us explain how these results on evolutions of quadratic evolutions correspond to the dynamic implicit transformations of the projection mapping between a planar circle and a straight line.

1. Implicit transformation between a planar circle and a line tangent to the circle.

For this situation, Fig. 2.7 depicts the dynamic relationship between the point p_i on the circle and the projection point p'_i on the tangent line. Here, the set of all points p_i is one-to-one corresponding to the set of all points p'_i . Now, let the point N on the circle correspond to the singular point $+\infty$ on the tangent line. So, when the point p'_i travels directly from the singular point $+\infty$ to the other singular point $-\infty$ on the straight line, it simply reflects the traveling of the point p_i on the circle through the polar point N with a change in direction. Combining this explanation with the situation of two equal real roots of the quadratic form in Eq. 2.38, the movement of the point p_i is limited by u_0 (a real root or an equilibrium state). When the point p_i travels upward on the right-hand side of the circle, the corresponding point p'_i on the tangent line travels to $+\infty$. When the point p_i goes across the polar point N, the corresponding point p'_i leaps from $+\infty$ to $-\infty$ directly. That is, a direction change on the circle stands for a discontinuous singularity on the line, which is shown as a blown-up in Eq. 2.38.

2. Implicit transformation between a planar circle and a line secant to the circle.

Fig. 2.7 The implicit transformation between a circle and a tangent line



If we focus on locally bounded movements, such as the point p_{i1} in Fig. 2.8, whose movement is limited by the distinct real roots u_1 and u_2 , then the point p_{i1} cannot go across the boundaries u_1 and u_2 and would not be able to travel over the polar point N. So, the corresponding projection point on p'_{i1} the secant line will not approach either $+\infty$ or $-\infty$. That is, the point p'_{i1} does not experience any singularity. This fact corresponds exactly to the smooth continuous solution Eq. 2.39. If we do not limit ourselves to bounded movements, such as the movement of the point p_{i2} in Fig. 2.8, which can go across the polar point N, then the corresponding projection point p'_{i2} can approach $+\infty$ and be transformed into $-\infty$ when the point p_{i2} goes across the point N. This situation corresponds to the blown-up solution in Eq. 2.40.

3. Implicit transformation between a planar circle and a line disjoint from the circle.

This situation corresponds to the periodic blown-ups of Eq. 2.41 with $a_1^2 - 4a_0 < 0$ and $\sqrt{4a_0 - a_1^2}t + \frac{1}{2}A_0 = (\frac{1}{2} + 2k)\pi$, $k = 0, \pm 1, \pm 2, \dots$. Since the movement of the point p_{i3} is not limited by any condition, it can go across the polar point N repeatedly, the corresponding projection p'_{i3} point on the line detached from the circle experiences periodic transformations from $+\infty$ to $-\infty$ or from $-\infty$ to $+\infty$, Fig. 2.9.

Our discussion here indicates that the traditional view of singularities as meaningless indeterminacies has not only revealed the obstacles of the thinking logic of the narrow observ-control, but also the careless omissions of spatial or dynamic implicit transformations.

Summarizing what has been discussed in this chapter, we can see that non-linearity, speaking mathematically, stands (mostly) for singularities in the Euclidean spaces, the imaginary plane or the straight line discussed above. In terms of physics, nonlinearity represents eddy motions, the movements on the curvature spaces, the Riemann ball or the circle above. Such motions are a problem about structural evolutions, which are a natural consequence of uneven

Fig. 2.8 The implicit transformation between a circle and a secant line

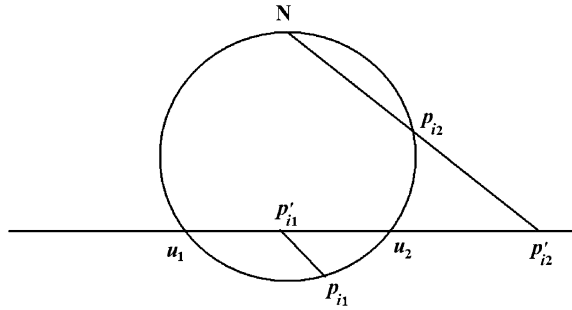
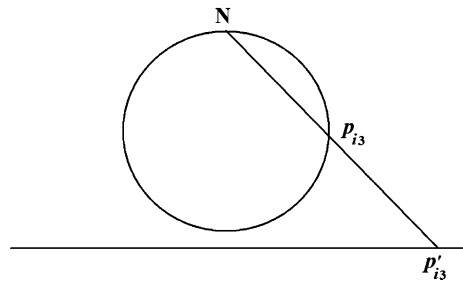


Fig. 2.9 The implicit transformation between a circle and a disjoint line



evolutions of materials. So, nonlinearity accidentally describes discontinuous singular evolutionary characteristics of eddy motions (in curvature spaces) from the angle of a special, narrow observ-control system, the Euclidean spaces. To support this end, please go to [Sect. 2.3.1: Bjerknes' Circulation Theorem](#), for details.

2.3.2 Eddy Motions of the General Dynamic System

In this subsection, let us look at the general dynamic system and how it is related to eddy motions. The following is Newton's second law of motion

$$m \frac{d\vec{v}}{dt} = \vec{F}. \quad (2.42)$$

Based on Einstein's concept of uneven time and space of materials' evolutions, we can assume

$$\vec{F} = -\nabla S(t, x, y, z), \quad (2.43)$$

where $S = S(t, x, y, z)$ stands for the time–space distribution of the external acting object. Let $\rho = \rho(t, x, y, z)$ be the density of the object being acted upon. Then, the kinematic Eq. 2.42 for a unit mass of the object being acted upon can be written as

$$\frac{d\vec{u}}{dt} = -\frac{1}{\rho(t, x, y, z)} \nabla S(t, x, y, z), \quad (2.44)$$

where \vec{u} is used to replace the original \vec{v} in order to represent the meaning that each movement of some materials is a consequence of mutual reactions of materials' structures. Evidently, if ρ is not a constant, then Eq. 2.44 becomes

$$\frac{d(\nabla x \times \vec{u})}{dt} = -\nabla x \times \left[\frac{1}{\rho} \nabla S \right] \neq 0, \quad (2.45)$$

which stands for an eddy motion because of the nonlinearity involved. In other words, a nonlinear mutual reaction between materials' uneven structures and the unevenness of the external forcing object will definitely produce eddy motions.

On the other hand, since nonlinearity stands for eddy sourcess, it represents a problem about structural evolutions. This end has essentially resolved the problem of how to understand nonlinearity and also ended the particle assumption of Newtonian mechanics and the methodological point of view of melting shapes into numbers, which has been formed since the time of Newton in natural sciences. What is more important is that the concept of uneven eddy evolutions reveals the fact that forces exist in the structures of evolving objects, and does not exist independently out of objects, according to what Aristotle and Newton believed so that the movement of all things had to be pushed first by God. Based on such reasoning, the concept of second stir of materials' movements is introduced. At this junction, we need to point out that as early as over 2,500 years ago, Lao Tzu of China once said: "Tao is about physical materials. Even though nothing can be seen clearly, there exist figurative structures in the fuzziness." [Chap. 21, Tao Te Ching, (English and Feng, 1972)]. It is no doubt a scientific and epistemological progress that in our modern time, Einstein proposed that gravitations are originated from the unevenness of time and space, which had essentially ended the era of Aristotelian concept of forces existing independently outside of materials. However, due to the fact that Einstein did not notice the unevenness of time and space of the object being acted upon, he did not successfully reveal the essence of nonlinear mutual reactions.

For example, in terms of the problem of the universe's evolution, Newton needed the hands of God. The first push of God has been vividly seen in the second law of mechanics. Einstein had realized the materialism of forces. However, based on his general relativity theory, it is concluded that the universe is originated from a big bang out of a singular point. Obviously, the singular point represents the transition of materials' evolution, which possesses more realisticity of materials than Newton's hands of God. Therefore, the scientific community has accepted such a big bang theory. However, the acceptance does not mean that such a big bang theory has revealed the true evolution of the universe. It is because the theory of big bang is established on the basis of the universe's background radiation (3 K, where K is the absolute temperature index), which is assumed to be uniform in all directions. Next, should there be only one singular point? Since the so-called background radiation should also be originated from the unevenness of materials,

can the universe's background radiation be calm? Even though a calm state could be reached temporarily, there should be a pre-singular point universe. Since unevenness is the fundamental property of spinning materials and the origin of all multiple levels of materials' eddies, the corresponding singular point explosions should also be multiple. So, the big bang theory, as a scientific theory, is still not plausible.

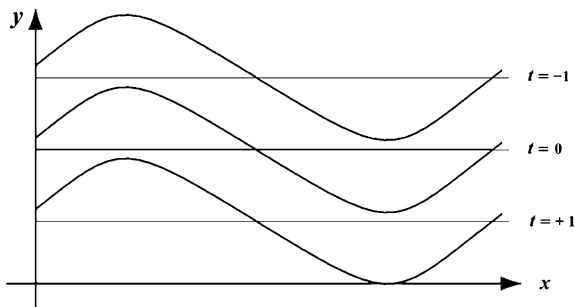
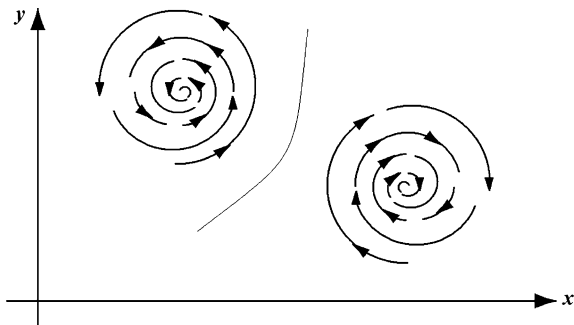
As a matter of fact, the concept of second stir can also be extended into a theory about the evolution of the universe. Since the concept of the second stir assigns forces with materials' structures, it naturally ends Newton's God. The duality of rotations must lead to differences in spinning directions. Such differences surely lead to singular points of singular zones. Through sub-eddies and sub-sub-eddies, breakages (or big bangs) are represented so that evolutionary transitions are accomplished. Evidently, according to the concept of second stir, the number of singular points would be greater than one, and the big bang explosions would also have their individual multiplicities. Therefore, the concept of second stir will be a thought left behind from the twentieth century, concerning the physical essence of materials' evolutions, worthy of further and deepened thinking.

2.3.3 Wave Motions and Eddy Motions

The concept of wave motions originated from the morphological changes of materials in motion, such as vibrating solids, water, and sound waves caused by local distributions in fluids, etc., caused by reciprocating movements. Later, this concept was extended to studies of other physical phenomena and to suit mathematical needs to such an extent that all absolute concepts about disturbances are treated as wave motions. That is how this concept has been widely employed in a great many branches of natural and social sciences. Because the concept of wave motions, current employed in a wide array of studies, is no longer the same as that in physics, we will refer to this greatly generalized concept of wave motions as extended wave motions for the convenience of our communication.

The extended wave motions, on the other hand, define eddy and wave motions on the basis of physics of the root cause and properties of the relevant disturbances instead of the need to suit some detailed mathematical requirements.

If a disturbance is the morphological change caused by reciprocating (linear) movements of materials, it is called a wave motion. If the disturbance is the morphological changes caused by spinning (curvilinear) movements of materials, it would be referred to as an eddy motion. Therefore, wave motions and eddy motions are two different concepts of physics. To this end, each wave motion is at least two directional, while each eddy motion is unidirectional. Unidirectionality is the characteristic of flows. So, eddy motions can also be named eddy currents or spinning currents (Figs. 2.10, 2.11). In terms of mathematics, wave motions are linear problems, while eddy motions are nonlinear problems. The current widely employed concept of extended wave motions is essentially developed by

Fig. 2.10 A wave motion**Fig. 2.11** An eddy motion

generalizing the original concept of wave motions to satisfy the requirements of relevant mathematical theories in order to establish the desired outcomes. As analyzed here, such generalization has caused confusion of concepts in physics. Typical examples of such confusion are extrapolations of the thinking logic of particle's continuity. Even if we assume such a generalization is OK, the continuity of the generalized wave motions is still conditional.

Speaking differently, even based on the traditional formal analysis, we can see the phenomenon of “break-offs” appearing in wave motions. To confirm this end, we will in the following use nonlinear dispersive “wave motions” as our platform to show this phenomenon.

Let the vibrating wave of an uneven material be written as follows:

$$\varphi(t, x) = A \cos \xi, \quad (2.46)$$

where $A = A(t, x)$ and $\xi = \xi(t, x)$ with t being the time variable and x the one-dimensional space. Then, the local wave number and frequency are given by

$$R(t, x) = \frac{\partial \xi}{\partial x} \text{ and } \omega(t, x) = -\frac{\partial \xi}{\partial t}. \quad (2.47)$$

Now, by eliminating ξ from Eq. 2.47, we produce

$$R_t + \omega_x = 0. \quad (2.48)$$

Let us define the group speed as

$$C(R) = \frac{\partial \omega}{\partial R} \quad (2.49)$$

so that Eq. 2.48 becomes

$$R_t + \frac{\partial \omega}{\partial R} R_x = 0. \quad (2.50)$$

According to Eq. 2.49 it follows that the local wave number $R(t, x)$ satisfies the following quasi-linear hyperbolic equation

$$R_t + C(R)R_x = 0. \quad (2.51)$$

Now, Eq. 2.51 stands for a typical blown-up problem. If t_b is a point of discontinuity in the solution of Eq. 2.51, then when $t < t_b$, R evolves continuously and the corresponding wave motion is continuous. When $t = t_b$, R approaches ∞ or $L = \frac{2\pi}{R} \rightarrow 0$ experiences a blown-up with “broken” wave length. So, nonlinear dispersive wave motions can evolve through blown-up(s) so that long waves are transformed into “short waves” or “broken waves.”

If when $t \rightarrow t_b$, $R_x = \frac{\partial R}{\partial x} \rightarrow \infty$, that is, the first order derivative of the wave number with respect to space experiences a blown-up, then from $R = \frac{2\pi}{L}$, it follows that

$$R_x = \left(\frac{2\pi}{L} \right)_x = -2\pi \left(\frac{L_x}{L^2} \right) \text{ or } \frac{L^2}{L_x} = -\frac{2\pi}{R_x},$$

where $R_x \rightarrow \infty$ and $L \rightarrow 0$. Therefore, uneven materials’ nonlinear dispersive waves are a blown-up problem. This is fundamentally different from the continuous movement of linear dispersive waves of even materials.

Of course, the discussion above is about nonlinear “waves” in their mathematical forms. So, what has been said is only about the difference in the mathematical properties of linearity and nonlinearity. That difference should not and could not be employed to illustrate the corresponding physical morphologies and properties. So, nonlinear “waves” of the mathematical form are not equivalent to any physical existence of mutual reacting waves. Because wave motions are originated from the diverging or converging currents of materials, and because both diverging and converging currents of materials in objective and realistic processes always contain unevenness that causes twistings so that subeddies are formed and rotating eddy motions are resulted. The morphology of broken waves of materials must be spinning “spindriffs” instead of waves. Thus, wave motions are only local and relative physical phenomenon, while eddy motions, on the other hand, are the commonly existing physical morphology. Because the mutual reaction of uneven structures constitutes twisting forces, which is “twisting motions,” spinning motions are inevitable results of mutual reactions of uneven materials, forming the origin from which eddy currents are from.

2.4 Equal Quantitative Effects

Another important concept studied in the blown-up theory is that of equal quantitative effects. Although this concept was initially proposed in the study of fluid motions, it essentially represents the fundamental and universal characteristics of all movements of materials. What is more important is that this concept reveals the fact that nonlinearity is originated from the figurative structures of materials instead of non-structural quantities of the materials.

The so-called equal quantitative effects stand for the eddy effects with non-uniform vortical vectorities existing naturally in systems of equal quantitative movements due to the unevenness of materials. And, by equal quantitative movements, it is meant the movements with quasi-equal acting and reacting objects or under two or more quasi-equal mutual constraints. For example, the relative movements of two or more planets of approximately equal masses are considered equal quantitative movements. In the microcosmic world, an often seen equal quantitative movement is the mutual interference between the particles to be measured and the equipment used to make the measurement. Many phenomena in daily lives can also be considered equal quantitative effects, including such events as wars, politics, economies, chess games, races, plays, etc.

Comparing to the concept of equal quantitative effects, the Aristotelian and Newtonian framework of separate objects and forces is about unequal quantitative movements established on the assumption of particles. On the other hand, equal quantitative movements are mainly characterized by the kind of measurement uncertainty that when I observe an object, the object is constrained by me. When an object is observed by another object, the two objects cannot really be separated apart. At this juncture, it can be seen that the Su-Shi Principle of Xuemou Wu's panrelativity theory (1990), Bohr (Bohr 1885–1962) principle and the relativity principle about microcosmic motions, von Neumann's Principle of Program Storage, etc., all fall into the uncertainty model of equal quantitative movements with separate objects and forces.

What is practically important and theoretically significant is that eddy motions are confirmed not only by daily observations of surrounding natural phenomena, but also by laboratory studies from as small as atomic structures to as huge as nebular structures of the universe. At the same time, eddy motions show up in mathematics as nonlinear evolutions. The corresponding linear models can only describe straight-line-like spraying currents and wave motions of the morphological changes of reciprocating currents. What is interesting here is that wave motions and spraying currents are local characteristics of eddy movements. This fact is very well shown by the fact that linearities are special cases of nonlinearities. Please note that we do not mean that linearities are approximations of nonlinearities.

Since 99% of all materials in the universe are fluids, and since under certain conditions solids can be converted to fluids, OuYang (1998) pointed out that "when fluids are not truly known, the amount of human knowledge is nearly zero.

And, the epistemology of the Western civilization, developed in the past 300 plus years, is still under the constraints of solids.” This statement no doubt points to the central weakness of the current theoretical studies and also located the reason why “mathematics met difficulties in the studies of fluids,” as said by Engels (1939). It also well represents Lao Tzu’s teaching (English and Feng 1972): “If crooked, it will be straight. Only when it curves, it will be complete.” As a matter of fact, mankind exists in a spinning universe with small eddies nested in bigger eddies or multiple eddies. Through sub-eddies and sub-sub-eddies, heat-kinetic energy transformations are completed. These nested eddies and energy transformations vividly represent the forever generation changes of blown-up evolutions of all things in the universe.

The birth–death exchanges and the non-uniformity of vortical vectorities of eddy evolutions naturally explain where and how quantitative irregularities, complexities, and multiplicities of materials’ evolutions, when seen from the current narrow observ-control system, come from. Evidently, if the irregularity of eddies comes from the unevenness of materials’ internal structures, and if the world is seen at the height of structural evolutions of materials, then the world is simple. And, it is so simple that there are only two forms of motions. One is clockwise rotation, and the other counterclockwise rotation. The vortical vectority in the structures of materials has very intelligently resolved the Tao of Yin and Yang of the “*Book of Changes*” of the eastern mystery (Wilhalm and Baynes 1967), and has been very practically implemented in the common form of motion of all materials in the universe. That is where the concept of invisible organizations of the blown-up system comes from.

The concept of equal quantitative effects not only possesses a wide range of applications, but also represents an important omission of modern science, developed in the past 300 plus years. Evidently, not only are equal quantitative effects more general than the mechanic system of particles with far-reaching significance, but also they have directly pointed to some of the fundamental problems existing in modern science. For instance,

1. Equal quantitative effects can throw calculations of equations into computational uncertainty. Evidently, if $x \approx y$, then $x-y$ becomes a mathematical problem of computational uncertainty, involving large quantities with infinitesimal increments. To this end, please consult with the second crisis in the foundations of mathematics (Kline 1972; Lin 2008a). Although this end has been well known, in practical applications, people are still often misguided into such uncertainties unconsciously. For example, in meteorological science, one situation involves

$$2\vec{\Omega} \times \vec{V}_h \approx -\frac{1}{\rho} \nabla p h$$

where the left-hand side stands for the deviation force caused by the earth’s rotation and the right-hand side the stirring force of the atmospheric density

pressure. However, scholars have tried for many decades to compute $\frac{d\vec{V}_h}{dt}$ under the influence of such quasi-equal computational uncertainty. As a matter of fact, the concept of equal quantitative effects has computationally declared that equations are not eternal, or there does not exist any equation under equal quantities. That is why OuYang introduced the methodology of abstracting numbers (quantities) back into shapes (figurative structures). The purpose of abstracting numbers back into shapes is to describe the formalization of eddy irregularities, which is different from regularized mathematical quantification of structures. That is why we should very well see that if 300 years ago, it was human wisdom to abstract numbers out of everything, then it would be human stupidity to continue to do so today.

2. It is because the current variable mathematics is entirely about regularized computational schemes where there must be the problem of disagreement between the variable mathematics and irregularities of objective materials' evolutions. The corresponding quantified comparability can only be relative. And, at the same time, there exists a problem with quantification where distinct properties cannot be distinguished because the relevant quantifications produce the same indistinguishable numbers. Because of this, it is both incomplete and inaccurate to employ quantitative comparability as the only standard for judging scientificity. For example, in terms of weather forecasting, it should be clear that difficulties we have been facing are consequences of the incapability of the existing theories in handling equal quantitative effects. It can be also said that the quantitative science, developed in the past 300 plus years, is incomplete and incapable of resolving problems about figurative structures.
3. The introduction of the concept of equal quantitative effects has not only made the epistemology of natural sciences gone from solids to fluids, but also completed the unification of natural and social sciences. That is because many social phenomena, such as military conflicts, political struggles, economic competitions, chess games, races, plays, etc., can all be analyzed on the basis of figurative structures of equal quantitative effects. However, what needs to be pointed out is that the current system of natural sciences is basically extensions of the research under unequal quantitative effects. At the same time, in some areas of research, there has been the problem of misusing unequal quantitative effects. These areas include, but are not limited to, the Rossby's long waves in meteorology, topographic leeward wave theory, chaos theory, etc.

In order for us to intuitively see why equal quantitative effects are so difficult for modern science to handle by using the theories established in the past 300 plus years, let us first look at why all materials in the universe are in rotational movements. According to Einstein's uneven space and time, we can assume that all materials have uneven structures. Out of these uneven structures, there naturally exist gradients. With gradients, there will appear forces. Combined with uneven arms of forces, the carrying materials will have to rotate in the form of moments of

forces. That is exactly what the ancient Chinese Lao Tzu, (English and Feng 1972) said: “Under the heaven, there is nothing more than the Tao of images,” instead of Newtonian doctrine of particles (under the heaven, there is such a Tao that is not about images but sizeless and volumeless particles). The former stands for an evolution problem of rotational movements under stirring forces. Since structural unevenness is an innate character of materials, that is why it is named second stir, considering that the phrase of first push was used first in history (OuYang et al. 2000). What needs to be noted is that the phrases of first push and second stir do not mean that the first push is prior to the second stir.

Now, we can imagine that the natural world and/or the universe is composed of entirely with eddy currents, where eddies exist in different sizes and scales and interact with each other. That is, the universe is a huge ocean of eddies, which change and evolve constantly. One of the most important characteristics of spinning fluids, including spinning solids, is the difference between the structural properties of inwardly and outwardly spinning pools and the discontinuity between these pools. Due to the stirs in the form of moments of forces, in the discontinuous zones, there exist subeddies and sub-subeddies (Fig. 2.12, where subeddies are created naturally by the large eddies M and N). Their twist-ups (the subeddies) contain highly condensed amounts of materials and energies. Or in other words, the traditional frontal lines and surfaces (in meteorology) are not simply expansions of particles without any structure. Instead, they represent twist-up zones concentrated with irregularly structured materials and energies [this is where the so-called small probability events appear and small probability information is observed and collected, so such information (event) should also be called irregular information (and event)]. In terms of basic energies, these twist-up zones cannot be formed by only the pushes of external forces and cannot be adequately described by using mathematical forms of separate objects and forces. Since evolution is about changes in materials’ structures, it cannot be simply represented by different speeds of movements. Instead, it is mainly about transformations of rotations in the form of moments of forces ignited by irregularities. The enclosed areas in Fig. 2.13 stand for the potential places for equal quantitative effects to appear, where the combined pushing or pulling is small in absolute terms. However, it is generally difficult to predict what will come out of the power struggles. In general, what comes out of the power struggle tends to be drastic and unpredictable by using the theories and methodologies of modern science.

Summarizing what has been discussed above, when a mathematical model, which truthfully and adequately describe the physical situation of our concern, blows up at a specific time moment or a specific spatial location or both and the underlying physical system also goes through a transitional change, then the solution of the model is called a blown-up solution and the relevant physical movement expresses a blown-up. Blown-up phenomena appear in life all the time, especially in studies involving evolutions.

The main points of our discussion above can be described as follows. For the situation that blown-ups appear only with time, by analyzing nonlinear equations and more general nonlinear models, it is found that in most cases, nonlinearity

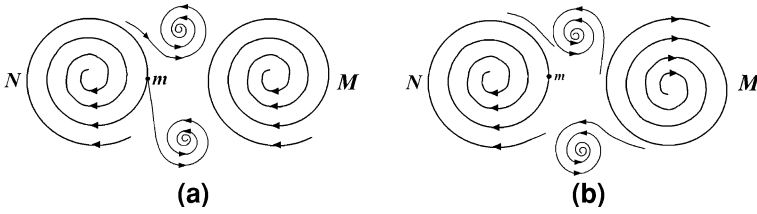
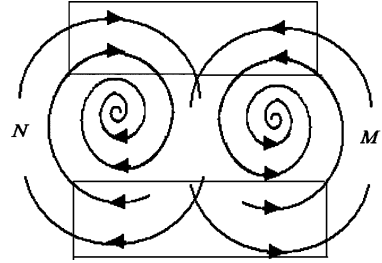


Fig. 2.12 Appearance of sub-eddies

Fig. 2.13 Structural representation of equal quantitative effects



implies blown-ups and the requirements for well-posedness of modern science (existence, uniqueness, and stability) do not hold true, meaning that most of the methods and thinking logic in modern science cannot be employed to resolve the relevant problems involving nonlinearity. Only under very special local conditions, there will not be any blown-up. That is when the available methods and theories in modern science can be applied. When blown-ups appear in both time and space, it is shown that if a system, when seen as a rotational entity, is initially divergent, then over time the whole evolution of the system is continuous and the divergent development of the system will eventually disappear smoothly and quietly. However, if the initial state of the system is convergent, then it is guaranteed that a moment of blown-up will appear at a definite time moment in the foreseeable future. After the system goes through a transitional change (blown-up), it will restart as a divergent system.

By introducing the concept of implicit transformations between a Euclidean space and a curvature space, it is shown that blown-ups in the Euclidean space are simply some transitional changes in the curvature space. And, periodic blown-ups of the Euclidean space and rotational movements in the curvature space correspond very well, illustrating a method on how to resolve the problems of the quantitative ∞ , numerical instability, and computational spills, by reconsidering the situations in curvature spaces. By doing so, all these problems, which seem unsolvable in modern science, are avoided.

It is concluded that the physical characteristics of blown-ups are spinning currents, which will be shown theoretically and empirically in the next chapter. One of the very significant outcomes of this discovery is that discontinuities and singularities existing in calculus-based models can actually be applied as a tool to

predict forthcoming transitional changes. By looking at the general dynamic system, it is shown that Newton's second law of mechanics actually indicates that as long as the acting and reacting objects have uneven structures, their mutual reaction will be nonlinear and a rotational movement. So, the concept of second stir is introduced. On the basis of this new concept, new explanations and a generalization of the big bang theory are provided in terms of a new theory on the evolution of the universe.

With the theoretical background established for rotations to be the common form of movements in the universe, the concepts of equal quantitative effects and equal quantitative movements are introduced. Through using these concepts, it is analyzed that mathematical nonlinearity is created by uneven internal structures of the materials involved instead of the non-structural quantities of the materials. It is found that equal quantitative effects lead to computational uncertainties and have the ability to unify natural and social sciences, since in both situations, figurative structural analyses can be employed. By looking at the geometry of equal quantitative effects, one can easily locate where small probability events would appear and where small probability information can be observed.

Let us conclude this chapter with a note on the references. Other than at specific locations we have directly pointed to the source of information, the presentation of this chapter is mainly based on Glassey (1977), Guo (1995), Hess (1959), Holton (1979), Kuchemann (1965), Lin (2000a), Lin and Fan (1997), Lin and OuYang (1998), Lin and Wu (1998), Lorenz (1993), OuYang (1994; 1995; 1998; 1998a), OuYang, Miao, Wu, Lin, Peng, and Xiao (2000), OuYang, Wu, Lin and Li (1998), Saltzman (1962), Wu (1990), Wu (1998), and Wu and Lin (2002). For more details and references, please consult with these works.

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Systemic Structure Behind Human Organizations
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Lin, Y.; Forrest, B.

2012, XVIII, 422 p., Hardcover

ISBN: 978-1-4614-2310-2