

Chapter 2

Academic Education: 1873–1879

In the fall of 1873, Henri, then 19 years old, travelled to Paris, accompanied by his mother and sister, to enroll in the École Polytechnique. While in Paris, Henri's mother and Aline stayed with the Rinck family, old friends from Lorraine, whose son, Élie Rinck, was of the same age as Henri. They remained in Paris a week, during which time they visited Henri and saw him for the first time in uniform (see Figure 2.1); they found it very difficult to say goodbye. The feeling was mutual, attested by the fact that during his first two years in Paris, Henri wrote hundreds of letters home, more than two a week.

2.1 A Difficult Year

As we shall see, life at a military school was not easy. In 1873, there were more than two hundred freshmen (indeed men; women were not admitted), and all of them had to attend lectures, perform military drill and learn to use weapons, stand guard, and take turns at orderly duty. In the relatively few free evenings, Henri visited the hospitable Rinck family or his relatives the Olleris, or else went to the theatre. Socially, his relations with his classmates were more or less as they had been in Nancy. He was much too absorbed in his own thoughts to have an active social life. During breaks, he often walked alone, but he was helpful when it was necessary, and he was respected because of his knowledge and insight. He followed the lectures with his arms crossed and often without taking notes. At times, he would suddenly become socially involved, and this, too, was accepted by his classmates. We will discuss some of these occasions later on. Appell [Appell 1925a] describes a typical discussion of a problem between Henri and a classmate:

If someone asked him a question, he took his arm confidentially and forced him to walk with him but in a very irregular way. Suddenly he plunged forward, halted all at once, and then walked backward. It was a little bit analogous to what the physicists call Brownian motion.

Fig. 2.1 A slightly bewildered-looking Henri Poincaré in the uniform of the École Polytechnique



Henri found some of the lectures interesting, in particular those of Hermite in analysis. In that year, his other professors were Résal (mechanics), Mannheim (geometry), Faye (astronomy), Cornu (physics), Frémy (chemistry), and Zeller (history and literature). Outside of lectures, the students were supervised in their studies by Bonnet and other lecturers, including Halphen. Pierre-Ossian Bonnet (1819–1892), see Figure 2.2, formulated and proved a number of important results in differential geometry. The Gauss–Bonnet theorem uses the notion of geometric curvature and is a well-known example of his achievements (Gauss formulated the theorem in terms of an example; Bonnet supplied the general theorem). George Henri Halphen (1844–1889) served as an officer in the French army during the Franco-Prussian war of 1870–1871, for which he was decorated. His mathematical career began after the war. His research was concerned with invariants of differential equations and differential geometry. He died at age 44.

In his letters home, Henri had little to say about his scholarly achievements. The following fragment from a letter is an exception:

One day Hermite was ill; Laguerre, who replaced him, discussed a certain problem during his lecture. Because the writing on the blackboard was not clear, I had made no notes. I paid no attention to it, but a few days later, a classmate asked me whether I could explain the problem to him. I answered that I had made no notes but that I would reconstruct Laguerre's proof. This I did, or I thought that I did, but meanwhile, I felt somewhat uncertain, for I had no use in the proof for the only remark that I had written down. That evening, the student was called up for a preliminary examination, and Halphen asks him exactly this problem. The student presents my proof. The examiner asks whether it is his own. The student looks me up, asks whether it is mine, and returns then to Halphen to tell him. Halphen says that he is not surprised. Halphen informs Laguerre, who sends for me and tells me that my proof is simpler than his own. It will replace his proof in the publications that, I believe, are adorned by the pompous name of *Archives of the École Polytechnique*.

Fig. 2.2 Pierre-Ossian Bonnet, director of studies at the École Polytechnique when Henri Poincaré was a student



At the beginning of 1873, the students were ranked as follows: 1. Poincaré, 2. Bonnefoy, 3. Petitdidier. The same three students continued as the highest ranked in their class until the final examination. In the spring, however, Henri's mood deteriorated, and he dropped from first place. He found the level of instruction too low, and he missed the challenge of difficult problems. In May 1874, he wrote dejectedly:

Here it is like a gigantic machine whose motion one has to follow on pain of being overrun; people do what twenty generations of the École before us did and what $2n + 1$ generations of recruits after us will do.

One needs here only two aspects of one's intelligence: memory and eloquence. To understand a course you have only to work, and that is why everybody can overtake me if he really wants to by grinding away. The examiners never ask anything exciting.

He was not always so dejected, quite the contrary, but he continually saw his two years at the École Polytechnique as a necessary phase on the road to his future instead of as an experience that was interesting in itself. Would he have been better placed at the École Normale? Perhaps. When visiting his friend Paul Appell at the École Normale, he regularly met some of the school's lecturers. For example, he once dined with the mathematician Briot. Those meetings gave him a great deal of pleasure. The École Polytechnique did not lack brilliant and stimulating lecturers, such as Hermite, Laguerre, and Halphen. However, they were not very prominent in a teaching environment geared to the education of officers and engineers.

Nonetheless, Henri's creativity in his first year could not be stopped. In October 1874, his first research paper appeared, and of course its topic was geometry: "Démonstration nouvelle des propriétés de l'indicatrice d'une surface" appeared

in *Annales de Mathématiques*, 2^e série, vol. XIII. It is an interesting exercise in curvature and osculating surfaces. It certainly should have made an impression that a student had written such a paper on his own and had it published, but what did the professor of geometry think about it? That will become clear indirectly.

We look first at an incident that occurred early in 1874. For each class meeting, a student was appointed to summarize the preceding lecture. Preparation for this was a joint effort, with the best students helping out. One day, the students' preparations were not going well, and they appealed to Henri. He said, "stop looking; the problem is wrong." Since their summary was an officially assigned task, the students informed the principal for teaching, Ossian Bonnet. The lecturer concerned, professor Mannheim, was asked for an explanation. Mannheim insisted that the problem formulation was correct, but much later, the problem associated with that lecture was removed from the curriculum.

At the end of Henri's first year, the geometry examination was supervised by Jules de la Gournerie, called Gournard, who was a friend of Mannheim. Henri received poor marks for geometry, and he came in second on the examination overall, after Bonnefoy but ahead of Petitdidier. It was clear that Gournard had penalized Henri severely for the quality of his drawings: drawing had always been his weak spot. The whole school was in turmoil over this injustice. Madame Rinck, who always welcomed Henri warmly during his student years, wanted to lodge a formal complaint, but Henri discouraged her. It was enough that those in positions of importance, such as the principal Bonnet, shared in the general indignation.

Finally, the summer holidays of 1874 began. As Aline noted, "We lived as if at a permanent party. We undertook everything Henri liked, everything that gave him pleasure" [Boutroux 1912].

2.2 Second Year at the École Polytechnique

The second year started well, with the publication of Henri's first scientific paper, but Henri's mood was not always cheerful. He suffered from a lack of intellectual stimulation and the feeling that the fight for first place in his class did not suit him. The struggle to be first in one's class was more than a competition for superiority over one's classmates; in France, achieving first place in a school ranking opens the next step in one's career. Henri, however, was interested in matters far removed from such trivial school affairs. Fortunately, he got on well with his classmates, and furthermore, it helped that in the second year, he could welcome some of his former comrades from Nancy who arrived at the École Polytechnique, among them Hartmann. He got on well with Bonnefoy, his most serious competitor in the rankings.

His many letters home continued, including letters written separately to Aline, but they became more carelessly written and contained less and less information about his daily life in Paris. On several occasions he was admitted to the infirmary

of the École for a few days, but his illnesses were not serious. The worst of it was that the topic he enjoyed most, geometry, was in the hands of Mannheim and Gournard.

In 1875 came the examination at the end of the second year. Henri lost points in topography, drawing, and architecture. The class ranking became 1. Bonnefoy, 2. Poincaré, 3. Petididier.

The talented students Bonnefoy and Petididier would die young in mining accidents. Henri's difficulties with Mannheim had a sequel—or better, a settling of accounts, one might say. Following the death of Laguerre in 1886, a new member of the Académie des Sciences was elected the next year. Both Poincaré and Mannheim were on the list of nominees. Poincaré, 32 years old, was elected with 34 votes, while Mannheim, who could look back on a long and productive career in geometry, received 24 votes.

2.3 L'École des Mines

In November 1875, Henri, who was then 21, continued his higher education at the École des Mines in Paris, together with his École Polytechnique classmates Bonnefoy and Petididier. Today, in the twenty-first century, with practically unlimited educational possibilities, this choice seems a rather unlikely one. But there was not so much choice in 1875. The École des Mines was (and is) an excellent engineering school that provides an education for a useful and socially important profession: mining engineer. Another possibility for Henri would have been to attend the University of Paris (the Sorbonne), but that would have given him a more general education, not one that would directly qualify him for a profession. Henri would finish the mining school successfully, but his interest was in a few topics only, particularly mineralogy. With no particular effort, he remained one of the best students, but he lost his interest in the rankings.

Becoming a Mining Engineer

In his first year at the École des Mines, Henri's courses included mine management and machinery, metallurgy, mineralogy, geology, palaeontology, assaying, drawing, and English.

The director, Daubrée, who was a distant relative of Henri, maintained that Henri should not pay attention to mathematics while at the École des Mines. In December, Henri wrote home:

This morning I have been to see M. Daubrée. He was very nice and told me, as had Uncle Antoni, that he advised me not to do mathematics before I had finished school. It appears that Bonnet had asked that I be given dispensation for certain hours so that I could attend lectures at the Sorbonne. He told me he had refused this. I answered him that I understood perfectly.

The point was that Henri did not need the lectures at the Sorbonne to prepare for their mathematics examination. Bonnet provided him with exercises, and in August 1876, he passed the Sorbonne's examination in mathematics.

During this first year, he visited Bonnet regularly at the École Polytechnique. Thus at the beginning of 1876, Henri wrote, "On Wednesday I was again visiting Bonnet, who was very nice. Also I saw Bouquet, who was as nice as was possible for him. He lent me an old book that I needed."

Studying at the École des Mines with parallel study at the Sorbonne did not interfere with Henri's social life. He kept up his stream of letters home, although at a pace slightly slower than that of his first two years in Paris. The letters describe visits to the theatre, visits to relatives, and political questions, and often they contain humorous verse. After the experiences of the Franco-Prussian War and the annexation of Alsace, there remained considerable apprehension in Nancy about the military might of Germany and in particular about the question of the possible annexation of Lorraine, Nancy included. Henri wrote to his family that such a turn of events seemed highly improbable to him, since it would require a permanent military occupation of the hostile French population of Lorraine. He added this:

What seems more probable is the Prussian annexation of Belgium and Holland. This would be very unfortunate for us, for it would double the length of the border with Germany, and it would double the German navy; it would present Germany with rich colonies, not to mention the industrial richness of Holland and the abominable military position that would arise for us.

Henri went on to suggest that after Belgium and Holland, Bohemia and the countries to the east of Germany would be next in line. Roughly sixty years later, all of Henri's fears were realized.

Now and then, the name of Émile Boutroux appeared in letters from home. In 1876, Boutroux was appointed to the university of Nancy to lecture in philosophy. He became a regular visitor to the Poincaré family. Émile Boutroux married Aline in October 1878.

The young philosopher was interested in graphology, which led to frequent discussions with Henri's cousin Raymond Poincaré, who was studying philosophy in Nancy, and with Henri in Paris. In the fall of 1877, Raymond moved to Paris to study law. He took a room adjacent to Henri's in the Boulevard St. Michel. The philosophical discussions between the cousins that had been carried on at a distance now continued much more intensively.

As mentioned above, graphology, the study of the relationship between character and handwriting, interested Henri. In a letter home, he expressed surprise that women, who in his experience often think in a haphazard way and without much logic, often have neat and well-ordered handwriting. This caused him to reflect on his own writing:

As far as I am concerned, I find that my main characteristics can be recognized in my handwriting. The way I make the last letter of each word very small illustrates that I am bad at waiting. My pliability shows in the softness of rounding off, corrected only by the first influence. Consider my n's and my u's, which look like Greek ω 's and not like the German w's as in your case. Consider, on the other hand, the way my lines are positioned

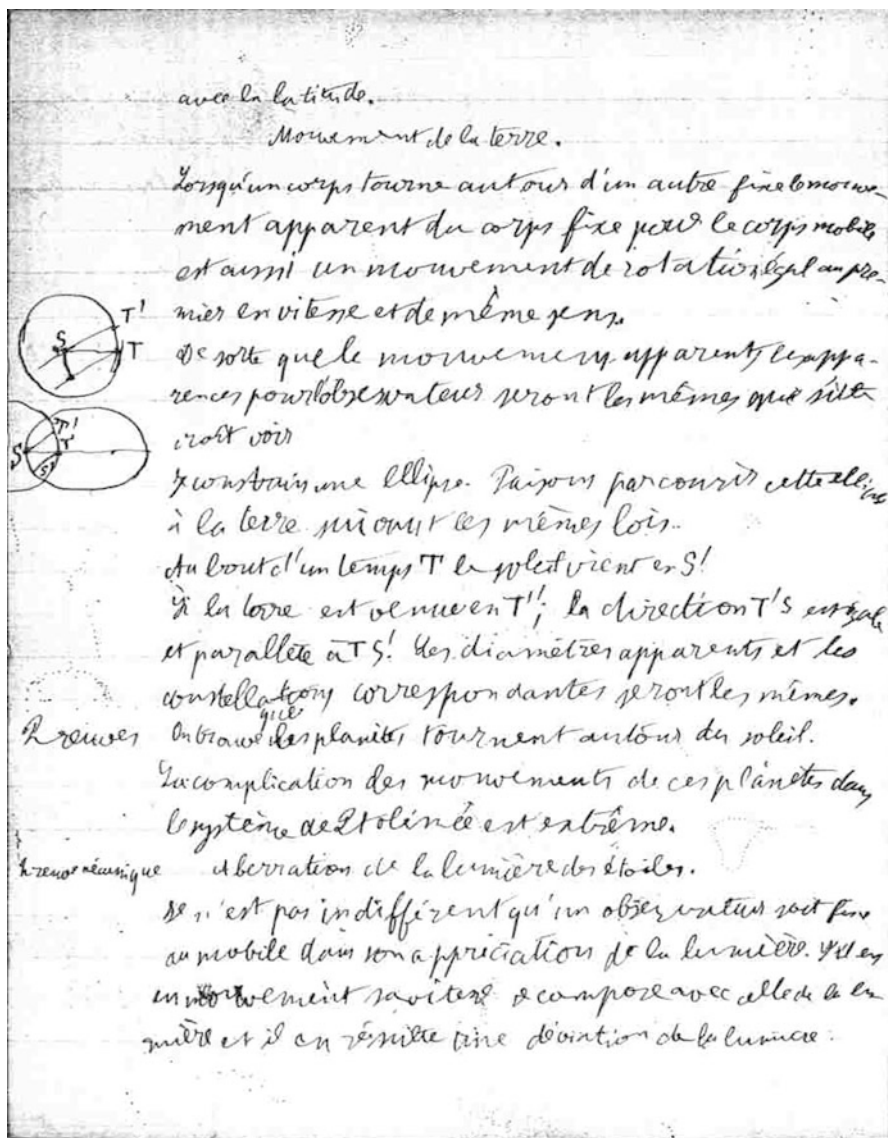


Fig. 2.3 Henri Poincaré on the Earth's motion

like vehicles emerging fresh from their village (what a difference compared to the Prussian coordination of the lines of Barrois). It is this particular thing that gives away my total absence of the bureaucratic feelings that are so widespread among the French people.

(For Barrois, read Raymond Poincaré.) For a sample of Henri's handwriting, see, for instance, the page depicted in Figure 2.3.

Fig. 2.4 Cousin Raymond Poincaré (1860–1934) shown in his position as a leading politician. He became prime minister and president of the republic



Raymond Poincaré (1860–1934)

Raymond Poincaré, see Figure 2.4, was one of the sons of Henri's uncle Antoni. He was born in Bar-le-Duc and studied law at the University of Paris. As a lawyer, he would later defend Jules Verne against a libel suit. He became a government minister in 1893 at the age of 33, and was prime minister during five government periods. Raymond was president of the republic in the critical period 1913–1920, which covered the First World War and the post-war treaty negotiations (Treaty of Versailles). He was a hardliner regarding the relationship with Germany and the exaction of war reparations.

His brother Lucien Poincaré (1862–1920) was a physicist who became inspector-general of public instruction. Raymond and Henri always stayed in contact, even in their later careers, discussing, for instance, appointments and the awarding of medals to distinguished people. Both of them were members of the prestigious Académie Française.

Completion of Studies at the École des Mines

The École des Mines organized excursions of several days for its students, but as a rule, there were two longer stays in a foreign country during the course of study. In the summer of 1877, at the end of the second year, Henri travelled, together with his classmate Lecornu, to Austria and Hungary. Following that trip, he wrote two reports, one on the coal mines of Hungary and the other on the pewter industry. Lecornu later wrote to friends how cheerful Henri was during their trip and with how much pleasure he received and read the long letters that Aline wrote to him [Appell 1925a].

A similar educational tour was made at the end of the third year, in 1878, this time together with Bonnefoy. They travelled to Sweden and Norway, and again following this trip, Henri wrote two reports on mining operations.

In June 1878, Henri's studies at the École des Mines came to an end. The final ranking had Henri third, after Bonnefoy and Petitdidier, but the result did not seem to interest him. The following year, in March 1879, Henri Poincaré, 24 years old, was appointed to the post of mining engineer in Vesoul, relatively close to Nancy. The appointment was made by the National Inspection of Mines, with the formulation "appointment to ordinary mining engineer of the third class charged with the mineralogical subdistrict Vesoul and in addition the supervision of the railways in the east." As we will see, the job was not without danger. Bonnefoy and Petitdidier also became mining engineers. They died in their late twenties from accidents that occurred in the course of their duties.

Henri's activities in Vesoul were short-lived. In December 1879, he was appointed to a lectureship of mathematics at the Faculté des Sciences of Caen, in Normandy. Formally, he remained his whole life a member of the corps of mining engineers. On June 16, 1910, he was appointed inspector-general of mines, in this case most likely an honorary title.

2.4 Dissertation in Mathematics

During all the activities of these years, Henri's mathematical discussions and research had never been interrupted. It seems that Henri almost casually wrote his dissertation during his second and third years at the École des Mines; for mathematical details, see Section 9.1. Its inspiration was from a paper by Briot and Bouquet in the *Journal de l'École Polytechnique* [Briot and Bouquet 1856] dealing with solutions of differential equations. As a first result, Henri wrote a short paper, which he submitted to that same journal [Poincaré 1878]. The dissertation, titled *Les propriétés des fonctions définies par des équations aux dérivées partielles* [Poincaré 1916, Vol. 1], was submitted at the turn of the year 1877–1878 to his supervisors Darboux, Laguerre, and Bonnet. It took some effort for Henri to get their comments. In 1878 he wrote:

Darboux resides at number 36
 In the same house as the good cousin.
 His advice I received with great pleasure
 And a short time after that a long sermon
 Filling ten large pages;
 Off to Laguerre where I was not so lucky,
 I wanted to be counselled, but alas,
 I found the door closed, and infuriated,
 I headed for Ossian, and there a wooden door as well.
 But I will find him some day, thank God.

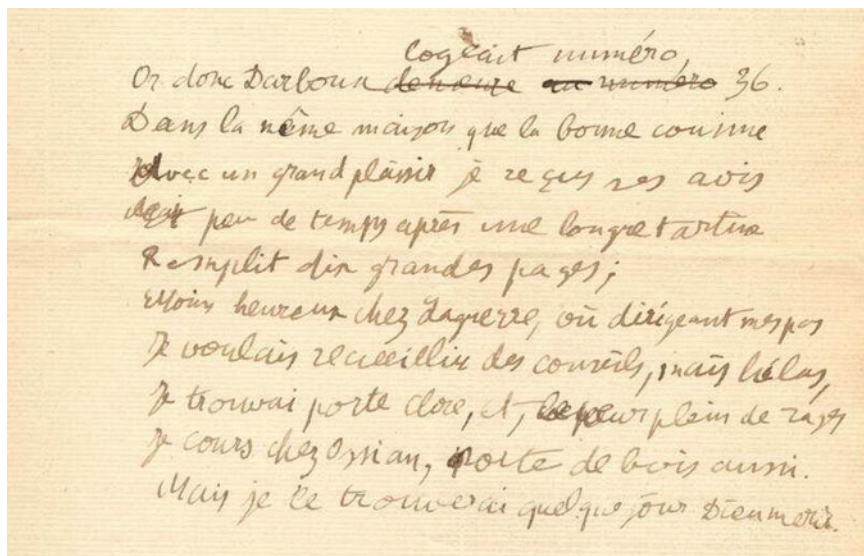


Fig. 2.5 The original text of Poincaré’s poetic discontent (courtesy Archive Poincaré Nancy)

The “good cousin” was the daughter of his relatives the Olleris. The French original is reproduced in Figure 2.5; the text reads as follows:

Or donc Darboux logeait numéro 36
 Dans la même maison que la bonne cousine.
 Avec un grand plaisir je reçus ses avis
 et peu de temps après une longue tartine.
 Remplit dix grand pages;
 Moins heureux chez Laguerre où dirigeant mes pas
 Je voulais recueillir des conseils, mais hélas,
 Je trouvais porte close et, le cœur plein de rage,
 Je cours chez Ossian, porte de bois aussi.
 Mais je le trouverai quelque jour, Dieu merci.

The dissertation was accepted on August 1, 1879, one and a half years later. It had been written hurriedly, and the supervisors made many critical remarks. Much later, Gaston Darboux wrote this about his role as supervisor [Poincaré 1916, Vol. 2]:

Joseph Bertrand used to say that the article in which Briot and Bouquet explained their results had brought the greatest advance in this part of analysis since Euler. Henri Poincaré made his first appearance by studying and perfecting that great work. In the dissertation that he submitted in 1878, he threw himself at a still more difficult question, the integration of partial differential equations with an arbitrary number of independent variables. At first glance, it was clear to me that the manuscript went beyond the usual and contained enough material for several good dissertations. But to give a precise idea of the way Poincaré worked, we must not shrink from stating that there were many points begging for correction or explanation. Poincaré thought intuitively. . . . It was easy for him to make the corrections

and cleaning up that I found necessary. But he explained to me later that at the time I asked him for it, he was occupied with completely different concepts. Whatever the case, his dissertation is valuable because of a number of new and important ideas.

The new ideas to which Poincaré was referring in his later conversation with Darboux are in large part contained in his revolutionary memoir [Poincaré 1881] and his articles on quadratic and cubic forms and their invariants and the so-called Fuchsian functions, published in 1880 and 1881 (and still later); see [Poincaré 1916]. The Fuchsian, or automorphic, functions are discussed in the next chapter and in Chapter 8. The memoir is discussed in Section 9.2. It presents a completely new approach to the theory of nonlinear second-order differential equations. It gives a classification of singular points, the index theorem for closed curves, the idea of “consequents,” or the Poincaré map for plane systems, and the basic ideas of the Poincaré–Bendixson theorem for limit cycles. It is now part of the general theory of ordinary differential equations, a topic on which Poincaré would publish a great deal in the years to follow. The dissertation can therefore not be separated from the memoir [Poincaré 1881] on the *global* qualitative and quantitative analysis of differential equations in the plane.

In the dissertation itself, the treatment is local, with first-order partial differential equations analysed with characteristic equations that may contain weak singularities. This leads to a technically complicated analysis with many different cases. It is understandable that the supervisors needed time to digest the material and also that they asked for examples to illustrate the theory. Unfortunately, there are not many examples presented. Of great interest in the thesis are the new concepts introduced by Poincaré. We mention the algebroid functions, the concept of what is today called a Poincaré domain, and the concept of resonance of eigenvalues. The last two ideas will return often in Poincaré’s work on dynamical systems, for instance for the equations of the solar system and even more so in general approximation methods using normal forms, the so-called Poincaré–Dulac normalization.



<http://www.springer.com/978-1-4614-2406-2>

Henri Poincaré

Impatient Genius

Verhulst, F.

2012, XII, 260 p., Hardcover

ISBN: 978-1-4614-2406-2