Chapter 2
Algorithms and Computational Complexity

As we saw in the last chapter, for some special graphs the computation of the rainbow connection numbers can be done without much difficulty. However, computing the rainbow connection number for a general graph is a hard problem. For the introduction of complexity theory, see [43]. In [11], Caro, Lev, Roditty, Tuza, and Yuster gave two conjectures (see Conjectures 4.1 and 4.2 in [11]) on the complexity of determining the rainbow connection numbers of graphs. Later, Chakraborty, Fischer, Matsliah and Yuster [12] verified these two conjectures.

Theorem 2.0.1 ([12]). *Given a graph G, deciding if $rc(G) = 2$ is NP-complete. In particular, computing $rc(G)$ is NP-hard.*

Proof. The proof will be divided into three steps: the first step (Claim 1) shows the computational equivalence of the problem of *rainbow connection number* 2, that asks for a red–blue edge coloring in which all vertex pairs have a rainbow path connecting them, to the problem of *subset rainbow connection number* 2, that asks for a red–blue coloring in which every pair of vertices in a given subset of pairs has a rainbow path connecting them. In the second step (Claim 2), we reduce the problem of *extending to rainbow connection number* 2, asking whether a given partial red–blue coloring can be completed to obtain a rainbow connected graph, to the problem of subset rainbow connection number 2. Finally, the proof of the theorem is completed by reducing the problem 3-SAT to the problem of extending to rainbow connection number 2.

Claim 1. The following problems are polynomial-time equivalent:

1. Given a graph G decide whether $rc(G) = 2$.
2. Given a graph G and a set of pairs $P \subseteq V(G) \times V(G)$, decide whether there is an edge-coloring of G with two colors such that all pairs $(u, v) \in P$ are rainbow connected.

Proof of Claim 1. It is enough to describe a reduction from Problem 2 to Problem 1. Given a graph $G = (V, E)$ and a set of pairs $P \subseteq V \times V$, we construct a graph $G' = (V', E')$ as follows.

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For every vertex $v \in V$, we introduce a new vertex x_v, and for every pair $(u,v) \in (V \times V) \setminus P$ we introduce a new vertex $x_{(u,v)}$. We set

$$V' = V \cup \{x_v : v \in V\} \cup \{x_{(u,v)} : (u,v) \in (V \times V) \setminus P\}$$

and

$$\begin{aligned} E' = E &\cup \{\{v,x_v\} : v \in V\} \cup \{\{u,x_{(u,v)}\},\{v,x_{(u,v)}\} : (u,v) \in (V \times V) \setminus P\} \\ &\cup \{\{x,x'\} : x,x' \in V' \setminus V\}. \end{aligned}$$

It is not hard to show that G' is 2-rainbow connected if and only if there is an edge coloring of G with two colors such that all pairs $(u,v) \in P$ are rainbow connected.

Claim 2. The first problem defined below is polynomially reducible to the second one:

1. Given a graph G and a partial 2-edge-coloring $\hat{\chi} : \hat{E} \to \{0,1\}$ for $\hat{E} \subset E$, decide whether $\hat{\chi}$ can be extended to a complete 2-edge-coloring $\chi : E \to \{0,1\}$ that makes G rainbow connected.
2. Given a graph G and a set of pairs $P \subseteq V(G) \times V(G)$, decide whether there is an edge-coloring of G with two colors such that all pairs $(u,v) \in P$ are rainbow connected.

Proof of Claim 2. Since the identity of the colors does not matter, it is more convenient that instead of a coloring $\chi : E \to \{0,1\}$ we consider the corresponding partition $\pi_\chi = (E_1,E_2)$ of E. Similarly, in the case of a partial coloring $\hat{\chi}$, the pair $\pi_{\hat{\chi}} = (\hat{E}_1,\hat{E}_2)$ will contain the corresponding disjoint subsets of E (which may not cover E). Now, given such a partial coloring $\hat{\chi}$, we extend the original graph $G = (V,E)$ to a graph $G' = (V',E')$, and define a set P of pairs of vertices.

Let $\ell : V \to [|V|]$ be arbitrary linear ordering of the vertices, and let **high**: $E \to V$ be a mapping that maps an edge $e = \{u,v\}$ to u if $\ell(u) > \ell(v)$, and to v otherwise. Similarly, let **low**: $E \to V$ be a mapping that maps an edge $e = \{u,v\}$ to u if $\ell(u) < \ell(v)$, and to v otherwise.

We construct G' as follows. We add $3 + |\hat{E}_1| + |\hat{E}_2|$ new vertices

$$\{b_1,c,b_2\} \cup \{c_e : e \in (\hat{E}_1 \cup \hat{E}_2)\}$$

and add the edges

$$\{\{b_1,c\},\{c,b_2\}\} \cup \{\{b_i,c_e\} : i \in \{1,2\}, e \in \hat{E}_i\} \cup \{\{c_e,\mathbf{low}(e)\} : e \in (\hat{E}_1 \cup \hat{E}_2)\}.$$

Now we define the set P of pairs of vertices that have to be 2-rainbow connected:

$$\begin{aligned} P = \{\{b_1,b_2\}\} &\cup \{\{u,v\} : u,v \in V\} \cup \{\{c,c_e\} : e \in (\hat{E}_1 \cup \hat{E}_2)\} \\ &\cup \{\{b_i,\mathbf{low}(e)\} : i \in \{1,2\}, e \in \hat{E}_i\} \cup \{\{c_e,\mathbf{high}(e)\} : e \in (\hat{E}_1 \cup \hat{E}_2)\}. \end{aligned}$$

It is not hard to show that the answer for Problem 2 for the resulting graph G' is yes if and only if the answer for Problem 1 for the original graph G is yes.

We show that Problem 1 of Claim 2 is NP-hard, and then deduce that 2-rainbow-colorability is NP-complete by applying Claims 1 and 2 while observing that it clearly belongs to NP. We reduce the problem 3-SAT to Problem 1 of Claim 2. Given a 3CNF formula $\phi = \wedge_{i=1}^{m} c_i$ over variables $x_1, x_2, \cdots, x_n$, we construct a graph G_ϕ and a partial 2-edge coloring $\chi' : E(G_\phi) \to \{0,1\}$ such that there is an extension χ of χ' that makes G_ϕ rainbow connected if and only if ϕ is satisfiable.

We define G_ϕ as follows:

$$V(G_\phi) = \{c_i : i \in [m]\} \cup \{x_i : i \in [n]\} \cup \{a\}$$

$$\begin{aligned} E(G_\phi) = &\{\{c_i, x_j\} : x_j \in c_i \ in\ \phi\} \cup \{\{x_i, a\} : i \in [n]\} \cup \{\{c_i, c_j\} : i, j \in [m]\} \\ &\cup \{\{x_i, x_j\} : i, j \in [n]\} \end{aligned}$$

and we define the partial coloring χ' as follows:

$$\forall\, i, j \in [m]\ \chi'(\{c_i, c_j\}) = 0, \quad \forall i, j \in [n]\ \chi'(\{x_i, x_j\}) = 0,$$

$$\forall\, \{x_i, c_j\} \in E(G_\phi)\ \chi'(\{x_i, c_j\}) = 0 \text{ if } x_i \text{ is positive } in\ c_j,\ \ 1 \text{ otherwise}$$

while all the edges in $\{\{x_i, a\} : i \in [n]\}$ (and only they) are left uncolored.

Assuming without loss of generality that all variables in ϕ appear both as positive and as negative, one can verify that a 2-rainbow-coloring of the uncolored edges corresponds to a satisfying assignment of ϕ and vice versa. □

Chakraborty et al. [12] also proposed a problem. Suppose that we are given a graph G for which we are told that $rc(G) = 2$. Can we rainbow-color it in polynomial time with $o(n)$ colors? For the usual coloring problem, this version has been well studied. It is known that if a graph is 3-colorable (in the usual sense), then there is a polynomial time algorithm that colors it with $\tilde{O}(n^{3/14})$ colors [7]. Dong and Li [32] solved this problem by showing a stronger result.

Theorem 2.0.2 ([32]). *Suppose that we are given a graph G for which we are told that $rc(G) = 2$. We can rainbow-color it in polynomial time with no more than five colors.*

For general k, it has been shown in [56] that for every $k \geq 2$, deciding if $rc(G) = k$ is NP-complete. Ananth and Nasre in [4] derived the following results.

Theorem 2.0.3 ([4]). *For every $k \geq 3$, deciding whether $rc(G) \leq k$ is NP-hard.*

Proof. We only describe the outline of our reduction. Let $(G = (V, E), P)$ be the input to the k-subset rainbow connected problem where $P \subseteq V \times V$. We construct a graph $G' = (V', E')$ such that G is a subgraph of G' and $rc(G') \leq k$ if and only if G is k-subset rainbow connected. To construct G', we first construct a graph $H_k = (W_k, E_k)$ such that $V \subset W_k$ and $V' = W_k$. Corresponding to the set P of pairs of vertices we associate a set of pairs of vertices P_k with respect to the graph H_k (The set P_k is only a relabeling of pairs of vertices in the set P). Then, the edge disjoint union of the two graphs G and H_k will yield the graph G'. The graph G' is constructed such that it satisfies the following two properties:

(1) There exists an edge-coloring χ of H_k with k colors such that all pairs of vertices in G' except those in P, i. e., all pairs in $(V' \times V') \setminus P$ are rainbow connected. Thus, if G is k-subset rainbow connected then there exists a k-edge-coloring of G such that all pairs in P will be rainbow connected. From this, we prove that G' can be rainbow colored using k colors if G is k-subset rainbow connected.

(2) All paths of length k or less between any pair of vertices in P are contained entirely in G (as a subgraph of G') itself. This ensures that for a rainbow coloring of G' with k colors, any pair of vertices in P should have all its rainbow paths inside G itself. Hence, if G' can be rainbow colored with k colors, then G can be edge-colored with k colors such that it is subset rainbow connected with respect to P.

We construct the family of graphs H_k inductively. For the base cases $k = 2$ and $k = 3$ we give explicit constructions, then show our inductive step. Finally, we describe our graph G'.

Construction of H_2: The construction of the graph H_2 is derived from the reduction of Chakraborty et al. [12] used to prove that the 2-subset rainbow connected problem is NP-hard (see the proof of Theorem 2.0.1). Let $H_2 = (W_2, E_2)$ where the vertex set W_2 is defined as follows:

$$W_2 = W_2^{(1)} \cup W_2^{(2)}, \quad W_2^{(1)} = \{v_{i,2} : i \in \{1, \cdots, n\}\},$$
$$W_2^{(2)} = \{u_i : i \in \{1, \cdots, n\}\} \cup \{w_{i,j} : (v_i, v_j) \in (V \times V) \setminus P\}.$$

The edge set E_2 is defined as:

$$E_2 = E_2^{(1)} \cup E_2^{(2)} \cup E_2^{(3)}, \quad E_2^{(1)} = \{(v_{i,2}, u_i) : i \in \{1, \cdots, n\}\},$$
$$E_2^{(2)} = \{(v_{i,2}, w_{i,j}), (v_{j,2}, w_{i,j}) : (v_i, v_j) \in (V \times V) \setminus P\}, \quad E_2^{(3)} = \left\{(x, y) : x, y \in W_2^{(2)}\right\}.$$

The set of vertices in $W_2^{(1)}$ are referred to as base vertices of H_2. Let $P_2 = \{(v_{i,2}, v_{j,2}) : (v_i, v_j) \in P\}$. The graph H_2 has the property that for all $(v_{i,2}, v_{j,2}) \in P_2$ there is no path of length ≤ 2 between $v_{i,2}$ and $v_{j,2}$. Also, if $(v_{i,2}, v_{j,2}) \notin P_2$ the shortest path between $v_{i,2}$ and $v_{j,2}$ is of length 2.

Construction of H_3: We now describe the construction of the graph H_3. Let $H_3 = (W_3, E_3)$ be a graph where the vertex set W_3 is defined as follows:

$$W_3 = W_3^{(1)} \cup W_3^{(2)} \cup W_3^{(3)}, \ W_3^{(1)} = \{v_{i,3} : i \in \{1, \cdots, n\}\},$$
$$W_3^{(2)} = \{u_i : v_i \in V\} \cup \{a_{i,j}, b_{i,j} : (v_i, v_j) \in (V \times V) \setminus P\},$$
$$W_3^{(3)} = \{u_i' : v_i \in V\} \cup \{a_{i,j}', b_{i,j}' : (v_i, v_j) \in (V \times V) \setminus P\}.$$

The edge set E_3 is defined as:

$$E_3 = E_3^{(1)} \cup E_3^{(2)} \cup E_3^{(3)} \cup E_3^{(4)}, \quad E_3^{(1)} = \{(v_{i,3}, u_i) : v_i \in V\},$$
$$E_3^{(2)} = \{(v_{i,3}, a_{i,j}), (v_{j,3}, b_{i,j}) : (v_i, v_j) \in (V \times V) \setminus P\},$$
$$E_3^{(3)} = \{(x, x') : x \in W_3^{(2)}, x' \in W_3^{(3)}\}, \quad E_3^{(4)} = \{(a_{i,j}, b_{i.j}) : (v_i, v_j) \in (V \times V) \setminus P\}.$$

The set of vertices in $W_3^{(1)}$ are referred to as base vertices of H_3. Let $P_3 = \{(v_{i,3}, v_{j,3}) : (v_i, v_j) \in P\}$. The graph H_3 has the property that for all $(v_{i,3}, v_{j,3}) \in P_3$ there is no path of length ≤ 3 between $v_{i,3}$ and $v_{j,3}$. Also, if $(v_{i,3}, v_{j,3}) \notin P_3$, then the shortest path between $v_{i,3}$ and $v_{j,3}$ is of length 3.

Inductive Step: Assume that we have constructed $H_{l-2} = (W_{l-2}, E_{l-2})$. From H_{l-2}, we first construct an intermediate graph H'_{l-2} from which we construct H_l.

The graph H'_{l-2} is constructed as follows: Each base vertex $v_{i,l-2}$ in W_{l-2} is split into three vertices $v_{i,l-2}^{(1)}, v_{i,l-2}^{(2)}, v_{i,l-2}^{(3)}$ and edges are added between them, i. e., the vertices $v_{i,l-2}^{(1)}$, $v_{i,l-2}^{(2)}$, $v_{i,l-2}^{(3)}$ form a triangle. Any edge of the form $(w, v_{i,l-2})$ is replaced by three edges $(w, v_{i,l-2}^{(1)})$, $(w, v_{i,l-2}^{(2)})$, $(w, v_{i,l-2}^{(3)})$. Formally, the graph $H'_{l-2} = (W'_{l-2}, E'_{l-2})$ is defined as follows. The set of vertices is $W'_{l-2} = W_{l-2}^{(1)} \cup W_{l-2}^{(2)}$ where $W_{l-2}^{(1)} = \{v_{i,l-2}^{(1)}, v_{i,l-2}^{(2)}, v_{i,l-2}^{(3)} : i \in \{1, \cdots, n\}\}$, $W_{l-2}^{(2)} = W_{l-2} \setminus \{v_{i,l-2} : i \in \{1, \cdots, n\}\}$ and the edge set is $E'_{l-2} = E_{l-2}^{(1)} \cup E_{l-2}^{(2)} \cup E_{l-2}^{(3)}$ where

$E_{l-2}^{(1)} = \{(v_{i,l-2}^{(j_1)}, w) : (v_{i,l-2}, w) \in E_{l-2}, j_1 \in \{1,2,3\}\}$,
$E_{l-2}^{(2)} = \{(v_{i,l-2}^{(j_1)}, v_{i,l-2}^{(j_2)}) : j_1, j_2 \in \{1,2,3\}\}$, $E_{l-2}^{(3)} = E_{l-2} \setminus \{(v_{i,l-2}, w) : w \in W_{l-2}\}$.

The graph $H_l = (W_l, E_l)$ where $W_l = W'_{l-2} \cup V_l$ and $E_l = E'_{l-2} \cup E'$ such that

$$V_l = \{v_{1,l}, \cdots, v_{n,l}\}, \quad E' = \{(v_{i,l-2}^{(1)}, v_{i,l}), (v_{i,l-2}^{(2)}, v_{i,l}), (v_{i,l-2}^{(3)}, v_{i,l}) : 1 \leq i \leq n\}.$$

The vertices in V_l are the base vertices of H_l (see Fig. 2.1 for the construction of H_l).

Reduction: Let the instance for k-subset rainbow connected problem be $(G = (V, E), P)$. We construct a graph G' as an instance for k-rainbow connected problem as follows:

(1) Construct $H_k = (V_k, E_k)$. (2) Let $V' = W_k$, $E' = E_k \cup \{(v_{i,k}, v_{j,k}) : (v_i, v_j) \in E\}$.

Then $G' = (V', E')$ is the required graph. We call the induced subgraph of G' containing the base vertices as base graph G_k. It can be seen that G_k is isomorphic to G.

Then, it is not difficult to check that G is k-subset rainbow connected if and only if G' is k-rainbow connected. The details are omitted. □

For the strong rainbow connection number, they obtained a similar result.

Theorem 2.0.4 ([4]). *For every $k \geq 3$, deciding whether $src(G) \leq k$, is NP-hard even when G is bipartite.*

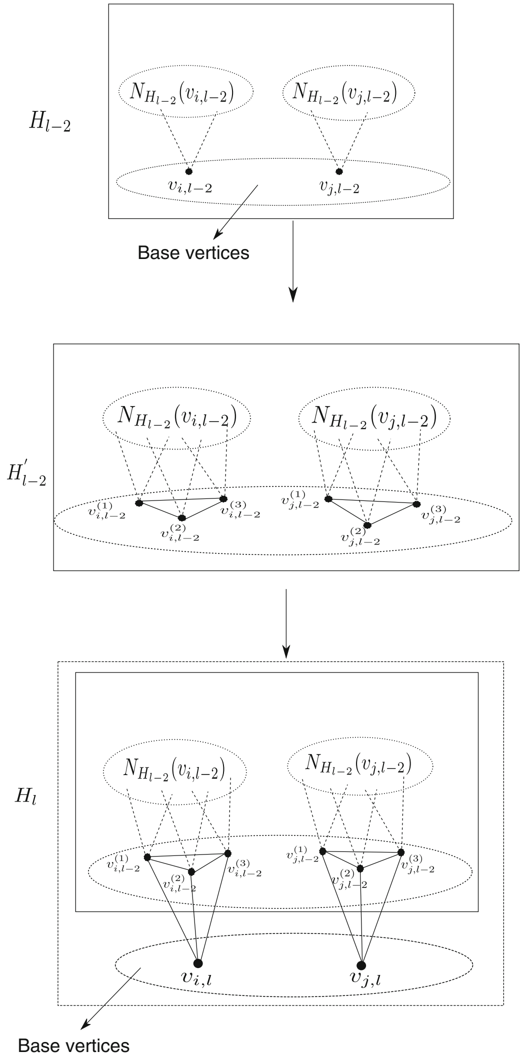

Fig. 2.1 Construction of H_l

Proof. We will first consider an intermediate problem called the *k-subset strong rainbow connected problem* which is the decision version of the *subset strong rainbow connected problem*. The input to the k-subset strong rainbow connected problem is a graph G along with a set of pairs $P = \{(u,v) : (u;v) \subseteq V \times V\}$ and an integer k. Our goal is to answer whether there exists an edge-coloring of G with at most k colors such that every pair $(u,v) \in P$ has a geodesic rainbow path.

Let $G = (V,E)$ be an instance of the k-vertex-coloring problem. We say that G can be vertex-colored using k colors if there exists an assignment of at most k colors to the vertices of G such that no pair of adjacent vertices are colored using the same color. This problem is NP-hard for $k \geq 3$. Given an instance $G = (V,E)$ of the k-vertex-coloring problem, we construct an instance $(G' = (V',E'),P)$ of the k-subset strong rainbow connected problem below:

$$V' = \{a\} \cup V; E' = \{(a,v) : v \in V\}, \quad P = \{(u,v) : (u,v) \in E\}.$$

It is not hard to show the following claim which establishes the hardness of the k-subset strong rainbow connected problem.

Claim 3. The graph $G = (V,E)$ is vertex colorable using $k(\geq 3)$ colors if and only if the graph $G' = (V',E')$ can be colored using k colors such that for every pair $(u,v) \in P$ there is a geodesic rainbow path between u and v.

Note that our graph G' construed in the above reduction is a tree, in fact a star and hence between every pair of vertices there is exactly one path. Thus, all the above arguments apply for the k-subset rainbow connected problem as well. As a consequence we can conclude the following: For every $k \geq 3$, the k-subset strong rainbow connected problem and the k-subset rainbow connected problem are NP-hard even when the input graph G is a star (which is a bipartite graph).

Next, we will show that the problem of deciding whether a graph G can be strongly rainbow colored using k colors is polynomial time equivalent to the k-subset strong rainbow connected problem.

Claim 4. The following two problems are reducible to each other in polynomial time:

(1) Given a graph $G = (V,E)$ and an integer k, decide whether the edges of G can be colored using k colors such that between every pair of vertices in G there is a geodesic rainbow path.
(2) Given a graph $G = (V,E)$, an integer k and a set of pairs $P \subseteq V \times V$, decide whether the edges of G can be colored using k colors such that between every pair $(u,v) \in P$ there is a geodesic rainbow path.

Proof of Claim 4. It suffices to show that problem (2) reduces to problem (1). Let $(G = (V,E),P)$ be an instance of the k-subset strong rainbow connected problem. As noted above, we know that the k-subset strong rainbow connected problem is NP-hard even when G is a star as well as the pairs $(u,v) \in P$ are such that both u and v are leaves of the star. We assume both these properties on the input (G,P) and use it crucially in our reduction. Let us denote the central vertex of the star G by a and the leaves by $L = \{v_1, v_2, \cdots, v_n\}$, that is, $V = \{a\} \cup L$. Using the graph G and the pairs P, we construct a new graph $G' = (V',E')$ as follows: For every leaf $v_i \in L$ we introduce two new vertices u_i and u_i', and for every pair of leaves $(v_i,v_j) \in (L \times L) \setminus P$, we introduce two new vertices $w_{i,j}$ and $w'_{i,j}$. Then the vertex set V' is defined as

$$V' = V \cup V_1 \cup V_2, \quad V_1 = \{u_i : i \in \{1,\cdots,n\}\} \cup \{w_{i,j} : (v_i,v_j) \in (L \times L) \setminus P\},$$
$$V_2 = \{u_i' : i \in \{1,\cdots,n\}\} \cup \{w'_{i,j} : (v_i,v_j) \in (L \times L) \setminus P\},$$

and the edge set E' is defined as

$$E' = E \cup E_1 \cup E_2 \cup E_3,$$

$$E_1 = \{(v_i, u_i) : v_i \in L, u_i \in V_1\} \cup \{(v_i, w_{i,j}), (v_j, w_{i,j}) : (v_i, v_j) \in (L \times L) \setminus P\},$$

$$E_2 = \{(x, x') : x \in V_1, x' \in V_2\}, E_3 = \{(a, x') : x' \in V_2\}.$$

It is not hard to show that G' is strongly rainbow colorable using k colors if and only if there is an edge-coloring of G (using k colors) such that every pair $(u, v) \in P$ has a strong rainbow path. The details are omitted. Note that the graph $G' = (V', E')$ constructed in the proof of Claim 4 is in fact bipartite, i.e., the vertex set V' can be partitioned into two sets A and B, where $A = \{a\} \cup V_1$ and $B = L \cup V_2$ such that there are no edges between vertices in the same partition. The result is thus proved. □

From the same construction above, one can see that the following corollary is immediate.

Corollary 2.0.5 ([4]). *Deciding if the rainbow connection number of a graph G is at most 3 is NP-hard even when G is bipartite.*

In [64] Li and Li showed that for any fixed integer k, the problems in Theorems 2.0.3 and 2.0.4 as well as Corollary 2.0.5 are all in *NP* class, and therefore, "*NP*-hard" can be replaced by "*NP*-complete" in the three results if we replace "For every k" by "For any fixed k."

Corollary 2.0.5 suggests us the following problem.

Problem 2.1. For every (any fixed) integer $k \geq 2$, deciding if a given bipartite graph G has $rc(G) \leq k$ is *NP*-hard (complete).

In [12], Chakraborty, Fischer, Matsliah and Yuster also derived some positive algorithmic results. Parts of the following two results will be shown in Theorems 3.1.9 and 4.1.9.

Theorem 2.0.6 ([12]). *For every $\varepsilon > 0$ there is a constant $C = C(\varepsilon)$ such that if G is a connected graph with n vertices and minimum degree at least εn, then $rc(G) \leq C$. Furthermore, there is a polynomial time algorithm that constructs a corresponding coloring for a fixed ε.*

As mentioned above, Theorem 2.0.6 is based upon a modified degree-form version of Szemerédi Regularity Lemma that they proved and that may be useful in other applications. From their algorithm, it is also not hard to find a probabilistic polynomial time algorithm for finding this coloring with high probability (using on the way the algorithmic version of the Regularity Lemma from [2] or [39]).

Theorem 2.0.7 ([12]). *If G is an n-vertex graph with diameter 2 and minimum degree at least $8\log n$, then $rc(G) \leq 3$. Furthermore, such a coloring is given with high probability by a uniformly random 3-edge-coloring of the graph G, and can also be found by a polynomial time deterministic algorithm.*

Because computing $rc(G)$ is *NP*-hard, it was tried to give approximate algorithms to compute the rainbow connection number. Basavaraju, Chandran, Rajendraprasad and Ramaswamy in [5] presented an $(r+3)$-factor approximation algorithm which runs in $O(nm)$ time and a $(d+3)$-factor approximation algorithm which runs in $O(dm)$ time to rainbow color any connected graph G on n vertices, with m edges, diameter d and radius r.

Suppose we are given an edge-coloring of a graph. Is it then easier to verify whether the colored graph is rainbow connected? Clearly, if the number of colors is constant, then this problem becomes easy. However, if the coloring is arbitrary (with an unbounded number of colors), the problem becomes NP-complete.

Theorem 2.0.8 ([12]). *The following problem is NP-complete: Given an edge-colored graph G, check whether the given coloring makes G rainbow connected.*

For the proof of Theorem 2.0.8, Chakraborty, Fischer, Matsliah and Yuster first showed that the $s-t$ version of the problem is NP-complete, that is, given two vertices s and t of an edge-colored graph, decide whether there is a rainbow path connecting them. Then they reduced the problem of Theorem 2.0.8 from it.

By subdividing each edge of a given edge-colored graph G exactly once, one can get a bipartite graph G'. Then color the edges of G' as follows: Let e' and e'' be the two edges of G' produced by subdividing at the edge e of G. Then color the edge e' with the same color of e and color the edge e'' with a new color, such that all the new colors of the edges e'' are distinct. In this way, Li and Li [64] reduced from Theorem 2.0.8 a result: Given an edge-colored bipartite graph G, checking whether the given coloring makes G rainbow connected is NP-complete. Reducing also from Theorem 2.0.8 in a clever way, Huang et al. in [49] first obtained a result: Given an edge-colored planar graph G, checking whether the given coloring makes G rainbow connected is NP-complete. Then by using the same subdividing technique as above, they got the following stronger result.

Theorem 2.0.9 ([49]). *Given an edge-colored bipartite planar graph G, checking whether the given coloring makes G rainbow connected is NP-complete.*

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