# Preface

The concept of rainbow connection of a graph was first introduced by G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang in 2006. Let $G$ be a nontrivial connected graph on which an edge-coloring $c : E(G) \to \{1,2,\cdots,n\}$, $n \in \mathbb{N}$, is defined, where adjacent edges may be colored the same. A path is *rainbow* if no two edges of it are colored the same. An edge-colored graph $G$ is *rainbow connected* if every two distinct vertices are connected by a rainbow path. An edge-coloring under which $G$ is rainbow connected is called a *rainbow coloring*. Clearly, if a graph is rainbow connected, it must be connected. Conversely, every connected graph has a trivial edge-coloring that makes it rainbow connected by coloring edges with distinct colors. Thus, we define the *rainbow connection number* of a connected graph $G$, denoted by $rc(G)$, as the smallest number of colors that are needed in order to make $G$ rainbow connected. A rainbow coloring using $rc(G)$ colors is called a *minimum rainbow coloring*. Obviously, the rainbow connection number can be viewed as a new kind of chromatic index.

The rainbow connection number is not only a natural combinatorial measure, but it also has applications to the secure transfer of classified information between agencies. In addition, the rainbow connection number can also be motivated by its interesting interpretation in the area of networking. Suppose that $G$ represents a network (e.g., a cellular network). We wish to route messages between any two vertices in a pipeline, and require that each link on the route between the vertices (namely, each edge on the path) is assigned a distinct channel (e.g., a distinct frequency). Clearly, we want to minimize the number of distinct channels that we use in our network. This number is precisely $rc(G)$.

There is a vertex version of the rainbow connection, called the rainbow vertex-connection number $rvc(G)$, which was introduced by M. Krivelevich and R. Yuster. There are also the concepts of strong rainbow connection or rainbow diameter, the rainbow connectivity, and the rainbow index. For details, we refer to Sect. 1.4 of this book.

The rainbow connections of graphs are very new concepts. Recently, there has been great interest in these concepts and a lot of results have been published. The goal of this book is to bring together most of the results that deal with rainbow connections of graphs. We begin with an introductory chapter. In Chap. 2, we address the computing complexity of the rainbow connections. In general, it is NP-hard. Many upper bounds have been obtained in the literature, which appear in Chap. 3. In Chaps. 4 and 5, dense and sparse graphs and some graph classes are studied. Chapter 6 concerns graph products, such as the Cartesian product, the direct product, the strong product, and the composition or lexicographic product of graphs. Chapter 7 is on the rainbow connectivity, which actually includes the rainbow $k$-connectivity, the $k$-rainbow index, and the $(k,\ell)$-rainbow index. In the final chapter, results of the vertex version, the rainbow vertex-connection number, are reported. In each chapter we list conjectures, open problems, or questions at appropriate places. We hope that this can motivate more young graph theorists and graduate students to do further study in this subject. We do not give proofs for all results. Instead, we only select some of them for which we give their proofs because we feel that these proofs employed some typical techniques, and these proof techniques are popular in the study of rainbow connections. New results are still appearing. There must be some or even many of them for which we have not realized their existence, and therefore have not included them in this book.

The readers of the book are expected to have some background in graph theory and some related knowledge in combinatorics, probability, algorithms, and complexity analysis. All relevant notions from graph theory are properly defined in Chap. 1, but also elsewhere where needed.

The anticipated readers of the book are mathematicians and students of mathematics, whose fields of interest are graph theory, combinatorial optimization as well as communication network design. Consequently, the present book will be found suitable for courses in these fields. The exposition of the details of the proofs of some main results will enable students to understand and eventually master a good part of graph theory and combinatorial optimization.

People working on communication networks may also be interested in some aspects of the book. They will find it useful for designing networks that can securely transfer classified information.

The material presented in this book was used in graph theory seminars, held three times at Nankai University, in 2009, 2010, and 2011. We thank all the members of our group for their help in the preparation of this book. Without their help, we would have not finished writing it in such a short period of time. We also thank the Natural Science Foundation of China (NSFC) for financial support to our research project on rainbow connections. Last, but not least, we are very grateful to the editor for algebraic combinatorics and graph theory of this new series of books of Springer Briefs, Professor Ping Zhang, for inviting us to write this book. Without her encouragement, this book may not exist.

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