Chapter 2
CDMA-Based Wireless Cellular Networks

Abstract In a CDMA-based cellular network, all radio cells share the same frequency bands, and users can transmit simultaneously. Transmissions from one user causes interference to other users. The more users are in the system and the higher power they transmit, the more interference they generate to one another. A CDMA-based system is typically interference-limited. The management of transmission power and mutual interference is directly related to system capacity and quality-of-service (QoS) to the users. In this chapter, we first briefly review the motivations of power control in cellular CDMA networks, then study the power allocation problem in a single-cell CDMA network. Based on the analysis, different aspects that affect the transmission power and system capacity are investigated. We then study the power allocation problem in a multi-cell CDMA network. The relationship between transmission power, network capacity, and QoS to the users is analyzed.
Keywords: CDMA, cellular network, power control, SINR, pole capacity, outage, soft handoff.

2.1 Motivations for Power Control

In a CDMA-based network, each transmitter uses a unique spreading code to generate the transmitted signals. The intended receiver can reproduce the spreading code used by the transmitter and recover the desired signals. The cross-correlation of different spreading codes is ideally zero, so that the desired signal can be recovered and interfering signals can be removed at the receiver. In a practical system, the radio channel can be non-linear, and the spreading codes may not be orthogonal to one another. Therefore, transmissions of the users can cause interference to one another. The signal-to-interference-plus-noise ratio (SINR), defined as the desired signal power divided by the total power of interference and noise, can be used to evaluate the received signal quality.

Power control is originally used to solve the near-far problems in the uplink of cellular CDMA networks, where homogeneous traffic (mainly voice) is supported.

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In the uplink, all the transmissions in the same cell share the same receiver, which is the base station (BS). If different users transmit at the same power, their signals arrive at the BS receiver with different strength. On average, signals from the users far away from the BS are weaker than that from the users close to the BS. Therefore, the SINR of the former can be much worse than that of the latter. That is, the signals from the users near the BS can block the transmissions of the signals from the users far away from the BS. This is the "near-far" effect. In order to balance the received SINRs for the signals from different users, power control is applied. The main purpose of power control is for the signals from different users to arrive at the BS with the same and acceptable SINR. The transmission power of each user is controlled and adjusted based on its channel gain to the BS. Users with poorer channel gain to the BS should transmit higher power.

The function of power control in the downlink is different. Since the signal and interference from the BS arrive at a given mobile station (MS) go through the same radio channel and undergo the same attenuation, power control is not needed to combat the near-far problem. Instead, it is used to provide more power to users located near the cell borders, where the users can suffer from high interference from the transmissions in nearby cells. In addition, the transmission power of the BS should be controlled in order to reduce the interference to nearby cells.

Consider N users transmitting to the same BS. Their signals are all power controlled to have the same power S at the BS receiver input. Each user's signal experiences interference from the transmissions of all the other $N-1$ users, the total interference power at the receiver is $I=(N-1)S$, and the SINR of the signal is $S/(I+\eta)$, where η is the background noise power. Let R be the information bit rate, and W be the spreading bandwidth. Then $E_b = S/R$ gives the energy per information bit, and $I_0 = (I+\eta)/W$ gives the interference-plus-noise power spectrum density. The ratio of energy per bit to interference-plus-noise power spectral density (E_b/I_0) is the SINR normalized to each transmission bit, and is commonly used for evaluating the receiving quality of the CDMA users. This is based on a fairly reasonable assumption that the bit-error-rate (BER) at a receiver is a monotonically decreasing function of E_b/I_0. When the noise power is zero, the expression for E_b/I_0 is given by

$$E_b/I_0 = \frac{S/R}{(N-1)S/W} = \frac{W/R}{N-1}, \tag{2.1}$$

where the quantity W/R is called the processing gain, which is a basic parameter for spread spectrum communications. If γ^* is the minimum required value for E_b/I_0 (corresponding to a maximum acceptable BER for a given physical layer design), then N can be solved as

$$N \leq 1 + \frac{W}{R\gamma^*} \triangleq N_p, \tag{2.2}$$

where N_p is referred to as the single cell pole capacity. From (2.2) we can find that for given spreading bandwidth W, a higher pole capacity is achieved if users require lower transmission rate and target γ^*. The pole capacity provides an important upper bound for the capacity, and is independent of the channel conditions of individual links. It is also the maximum capacity for each cell in a multi-cell network. The pole capacity can never be achieved in a real network, since it is obtained by assuming zero noise and interference power, because of which the pole capacity is independent of S. On the other hand, each receiver requires a minimum signal power in order to detect and decode the desired signals. When the link condition is poor, the required transmission power can be high in order to guarantee the minimum power at the receiver end. The actual capacity of the system is then limited by the maximum transmission power of the MSs. In addition, different users may require different transmission rates and target SINRs, which also affect the target receiving and transmission power of each user's signal. This scenario is studied in the next section.

In the remaining part of this book, all SINRs are normalized to per information bit. In another word, they are in fact energy per bit to interference-plus-noise power spectral density ratio.

2.2 Power Allocations in a Single Cell Network

We consider an FDD-based network, where different frequency bands are used for the uplink and the downlink transmissions, and therefore, there is no interference between the uplink and the downlink transmissions. This allows us to study the uplink and the downlink performance separately. We use R_i to denote the transmission rate and γ_i^* to denote the the minimum required SINR for user i, where $i = 1, 2, \ldots, N$, and N is the total number of active users.

Uplink Power Analysis

Each user may have a different rate and SINR requirement, and the target receiving power for different users may be different at the BS. Let $S_{u,i}$ be the target receiving power of user i's signal, and η be the noise power at the BS receiver. The relationship between the target receiving power and the required SINR is given by

$$\frac{W}{R_i} \frac{S_{u,i}}{\sum_{j=1, j\neq i}^{N} S_{u,j} + \eta} \geq \gamma_i^*, \tag{2.3}$$

for all $i = 1, 2, \ldots, N$. This formulation is equivalent to (1.2), if the processing gain (W/R_i) is equal to 1 in (2.3) and all the background noise powers are the same in (1.2). Based on the analysis for (1.2), we know that the minimum power is required for each user when equality holds in (2.3), i.e.,

$$\frac{W}{R_i} \frac{S_{u,i}}{\sum_{j=1, j\neq i}^{N} S_{u,j} + \eta} = \gamma_i^*. \tag{2.4}$$

With some simple manipulations, (2.4) can be rewritten as

$$\sum_{j=1}^{N} S_{u,j} + \eta = \left(\frac{W}{\gamma_i^* R_i} + 1 \right) S_{u,i} \tag{2.5}$$

for all $i = 1, 2, \ldots, N$. Define $Q_i = \frac{W}{\gamma_i^* R_i} + 1$, (2.5) becomes

$$\sum_{j=1}^{N} S_{u,j} + \eta = Q_i S_{u,i}. \tag{2.6}$$

Since the left-hand side of (2.6) does not depend on individual user's parameters, the right-hand side of the equation should be the same for different users. That is,

$$Q_i S_{u,i} = Q_j S_{u,j} \tag{2.7}$$

for all $i, j = 1, 2, \ldots, N$. Then $S_{u,j}$ can be written as

$$S_{u,j} = S_{u,i} \frac{Q_i}{Q_j}. \tag{2.8}$$

Replacing $S_{u,j}$ in (2.6) with the right-hand side in (2.8), we can solve $S_{u,i}$ as

$$S_{u,i} = \frac{\eta}{Q_i \left(1 - \sum_{j=1}^{N} \frac{1}{Q_j} \right)}. \tag{2.9}$$

Dividing $S_{u,i}$ by $g_{u,i}$, which is the link gain from MS i to the BS, we can find the required transmission power from user i as

$$P_{u,i} = \frac{\eta}{Q_i \left(1 - \sum_{j=1}^{N} \frac{1}{Q_j} \right) g_{u,i}}. \tag{2.10}$$

From the above derivations we have the following observations:

- Without considering the maximum transmission power limit, a feasible solution to the power allocation problem exists (i.e., all $S_{u,i}$'s are non-negative) if and only if

  $$\sum_{j=1}^{N} Q_j^{-1} < 1. \tag{2.11}$$

  If the total capacity of the system is normalized to 1, which is the right-hand side of (2.11), Q_j^{-1} can be considered as the normalized amount of resources consumed by user j. Because of this property, (2.11) can be used as a criterion for admission control in cellular CDMA networks.
- For homogeneous traffic, all the users require the same rate and SINR, $Q_i = N_p = W/(\gamma^* R) + 1$ for all $i = 1, 2, \ldots, N$, is the pole capacity derived in the previous section. In order to have a feasible solution for the receiving power, $N/N_p < 1$

should always hold, or $N < N_p$. The required target receiving power is then given by

$$S_{u,i} = \frac{\eta}{N_p - N}, \tag{2.12}$$

which indicates that the required power increases with the number of users supported in the network.

- The target receiving power for the ith user is inversely proportional to Q_i. This is straightforward, since higher transmission rate and larger SINR requirement (which results in smaller Q_i) requires the support of higher power.
- The target receiving power at the BS for each user is independent of the link gains. The power control manages the target receiving power, which directly affects the mutual interference among the links.
- The required transmission power from a user is inversely proportional to the link gain between itself and the BS. Users with worse link gains should transmit higher power. Furthermore, the required transmission power for each user does not depend on the link gain of any other links. That is, poor link gain of one link, although results in high transmission power from a particular user, does not affect the transmission power of other links in a single cell network. (This is not the case in a multi-cell network, where inter-cell interference exists.)
- By further looking at (2.12) we can find that $S_{u,i}$ can increase significantly with the number of users when the latter is close to the pole capacity. As a result, the required transmission power also increases, and may exceed the maximum transmission power limit of the MS. When the maximum transmission power is relatively small or the link conditions are poor, the actual capacity can be much smaller than the pole capacity.

Downlink Power Analysis

The power distribution in the downlink is similar to that in the uplink, except that orthogonal codes may be used in the downlink for users associated with the same BS in order to reduce co-channel interference within the cell. However, the orthogonality may not be maintained perfectly at the receiver end due to the non-linearity inherent in the radio channel propagation. A variable ξ is used to denote this effect. When $\xi = 0$, different transmissions are kept orthogonal at the receiver end; and when $\xi = 1$, all power for one user contributes to interference to others. In addition, different receivers may experience different noise power. Let $P_{d,i}$ be the required transmission power from the BS to the ith MS, $g_{d,i}$ be the link gain from the BS to the MS, and η_i be the background noise power at the receiver of MS i. The downlink transmission power should satisfy the following relationship,

$$\frac{W}{R_i} \frac{P_{d,i} g_{d,i}}{\sum_{j=1,j\neq i}^{N} \xi P_{d,j} g_{d,i} + \eta_i} \geq \gamma_i^*. \tag{2.13}$$

When equality holds in (2.13), the transmission power to each user is minimized. Using a similar approach as for the uplink, we can find the minimum value for $P_{d,i}$ as

$$P_{d,i} = \frac{\eta_i}{Q_{\xi,i}\left(1 - \sum_{j=1}^{N} \frac{1}{Q_{\xi,j}}\right) g_{d,i}}, \tag{2.14}$$

where $Q_{\xi,i} = 1 + \frac{W}{\xi \gamma_i^* R_i}$. Similar to the uplink, $1/Q_{\xi,i}$ can be considered as the normalized amount of resource consumed by user i in the downlink, if the total downlink capacity is normalized to 1. The target receiving power at user i is given by

$$S_{d,i} = P_{d,i} g_{d,i} = \frac{\eta_i}{Q_{\xi,i}\left(1 - \sum_{j=1}^{N} \frac{1}{Q_{\xi,j}}\right)}. \tag{2.15}$$

When $\gamma_i^* = \gamma^*$ and $R_i = R$ for all i, $Q_{\xi,i} = 1 + \frac{W}{\xi \gamma^* R} \triangleq N_{\xi,p}$ is the single cell pole capacity in the downlink for homogeneous traffic.

For homogeneous traffic, we can see that the target transmission power and receiving power in the downlink have similar properties as in the uplink. In addition, if the rates and SINRs in both the directions are the same (this may not be true in a practical system), then we have the following observations:

- When $\xi < 1$, the pole capacity in the downlink is higher than that in the uplink, and the normalized resource consumed by each user in the downlink is smaller than that in the uplink; furthermore, if the channel is reciprocal, i.e., $g_{u,i} = g_{d,i}$, the required transmission power in the downlink is lower than that in the uplink.
- When $\xi = 1$, the pole capacity and the normalized resource consumed by each user in the downlink are the same as in the uplink; furthermore, if the channel is reciprocal, the required transmission power in the uplink is exactly the same as that in the downlink.

Outage

The uplink channel is an access channel, where all the transmitters are distributed in different places and share the same receiver; while the downlink channel is a broadcast channel, where all the links share the same transmitter. Let $P_{\max,MS}$ and $P_{\max,BS}$, respectively, represent the maximum transmission power of an MS and a BS. In the uplink, communication outage occurs if $P_{u,i} > P_{\max,MS}$; and in the downlink, outage occurs if $\sum_{i=1}^{N} P_{d,i} > P_{\max,BS}$. Normally, $P_{\max,MS} < P_{\max,BS}$, and the capacity in the uplink is lower than that in the downlink, or outage in the uplink is higher than that in the downlink, if the users require the same rate and SINR in both directions.

For the uplink, when there are one or multiple users having $P_{u,i} > P_{\max,MS}$, removing all these users (i.e. putting these users in outage) would make the power allocation problem feasible for the remaining users. However, it may be not necessary to remove all these users. Based on the analysis for the uplink we can find that i) both high traffic load and poor link condition can result in high transmission power from a user, and ii) removing any user can decrease the target receiving power (and therefore reduce the required transmission power) of all the remaining users. When $P_{u,i} > P_{\max,MS}$ for one or multiple users, it depends on specific objectives to make decisions regarding how many and which existing users should be removed in order to make the power allocation feasible for the remaining users. The situation is similar for the downlink when $\sum_{i=1}^{N} P_{d,i} > P_{\max,BS}$.

We consider several simple criteria for user removal. For the uplink, when there is at least one user having $P_{u,i} > P_{\max,MS}$, we consider two criteria. In the first criterion, all the users with $P_{u,i} > P_{\max,MS}$ are removed, and after this the transmission power for the remaining users should be all below $P_{\max,MS}$. In the second criterion, the user with the largest $P_{u,i}$ is first removed, and the transmission power for the remaining users is recalculated. If $P_{u,i} > P_{\max,MS}$ for any of the remaining users, another user is removed based on the same criterion. This process is repeated until $P_{u,i} \le P_{\max,MS}$ for all the remaining users. For the downlink, the user with the largest $P_{u,i}$ is removed first. After this the transmission power for the remaining users is recalculated. If $\sum_{i=1}^{N} P_{d,i} \le P_{\max,BS}$, the removal process ends. Otherwise, another user is removed based on the same criterion. Outage occurs to the removed users. Figs. 2.1 and 2.2 show the outage probabilities for the uplink and downlink, respectively. These results are generated based on a link gain model that includes both path loss and log-normally distributed channel fading, and the channel is reciprocal in the uplink and the downlink, i.e., $g_{ui} = g_{di} = Ad_{ib}^{-\alpha} e^{-\beta X_{ib}}$, where A is the link gain at a reference distance and assumed to be 1, d_{ib} is the distance between MS i and the BS (normalized to the reference distance), α is the path loss constant, X_{ib} is a Gaussian distributed random variable with zero mean and a standard deviation of σ, and $\beta = \ln(10)/10$ is a constant. The outage probability is collected as the number of users in outage divided by the total number of users. It can be seen from the figures that when the number of users (N) is relatively small, the outage probability increases relatively slowly with N; and when N if close to the pole capacity, the outage probability increases significantly with N. From Fig. 2.1 we can see that using criterion 2 results in lower outage probability than using criterion 1. The difference between the outage performance using the two criteria increases as the number of MSs increases.

Note that in a practical cellular system, distributed power control algorithms may be used in both the uplink and the downlink to achieve the target SINRs, such as the distributed and constrained power control introduced in reference [1] (which is introduced in Subsection 1.2.2) and in reference [2]. When performing such algorithms and the exact link gains between the MSs and the BS are unknown, making decisions about which links should be removed during the iterations can be difficult. Several heuristic criteria can be found in [3, 4].

In addition to the maximum transmission power limit, other aspects can also cause outages. In the following two sections, we will look at outages caused by imperfect power control and bursty traffic, and study the required transmission power in each of these cases.

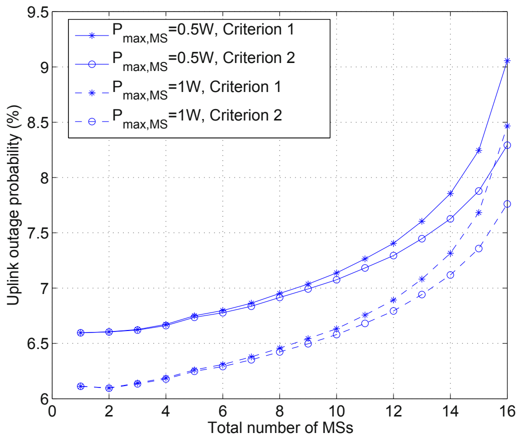

Fig. 2.1 Uplink outage probability for a single cell ($N_p = 16.6$, $\alpha = 4$, $\sigma = 8$dB, $\eta = 10^{-14}$W)

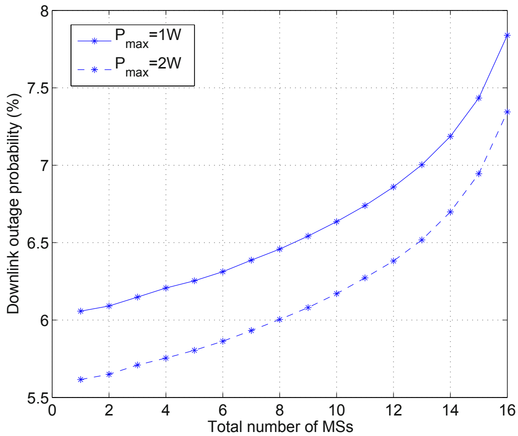

Fig. 2.2 Downlink outage probability for a single cell ($N_p = 16.6$, $\alpha = 4$, $\sigma = 8$dB, $\eta = 10^{-14}$W, $\xi = 1$)

2.3 Effect of Imperfect Power Control

We have introduced an iterative power control scheme in Subsection 1.2.2. Implementing such an iterative power control scheme requires the transmitter and receiver to exchange related information, so that the transmitter knows whether it should increase or decrease the transmission power in the next iteration, and how much the transmission power should be adjusted. In a practical system, the measurement at the receiver may not be accurate, and the transmitted signaling from the receiver to the transmitter can be corrupted by interference and noise. As a result, the transmitter may not adjust its transmission power towards the desired target value, resulting in imperfect power control. In this case, the actual transmission power can be larger or smaller than the desired power. When the transmission power is lower than the desired value, the target SINR of the user cannot be satisfied, causing outage to the communications. In order to protect the user's receiving quality, the target receiving power should be increased, compared to the perfect power control case. The high transmission power increases the co-channel interference in the network, and reduces the system capacity.

When the power control is imperfect, the actual receiving power is the target receiving power multiplied by a random error. Based on [5], the error due to imperfect control is log-normally distributed. Let $\tilde{S}_i$ denote the target receiving power of user i, then the actual receiving power is $\tilde{S}_i e^{\beta Y_i}$, where $\beta = \ln(10)/10$, and Y_i is a normally distributed random variable with zero mean and variance σ_y^2. Larger σ_y indicates larger variations between the target and the actual power. We consider that all the Y_i's are independent and identically distributed. When the actual SINR is below the SINR threshold for user i's transmission, outage occurs to the user. The outage probability is given by

$$p_{out,i} = \Pr.\left\{ \frac{W}{R} \frac{\tilde{S}_i e^{\beta Y_i}}{\sum_{j=1,j\neq i}^{N} \tilde{S}_j e^{\beta Y_j} + \eta} < \gamma^* \right\} \tag{2.16}$$

for all $i = 1, 2, \ldots, N$. Mathematical analysis and comparison between the required power for perfect and imperfect power control can be found in [6]. Here we use computer simulation results to demonstrate this effect. Fig. 2.3 shows the relationship between the outage probability and the standard deviation of imperfect power control, where the target receiving power is kept the same as that in the prefect power control case. The figure shows that imperfect power control causes communication outages, and the outage probability increases with σ_y. Furthermore, as σ_y increases, the outage probability can increase very significantly towards an unacceptable level, especially when the pole capacity is small.

Fig. 2.4 shows that increasing the target receiving power can effectively reduce the outage probability, but only for a certain range of the outage probability. Beyond this range, increasing the target receiving power has very minor effect on the outage probability. The standard deviation of imperfect power control determines the best outage performance that can be achieved by increasing the target receiving power.

2.4 Adaptive Power and Adaptive Rate

User's traffic often exhibits random active and inactive periods. During an active period, a user generates traffic and transmits to the destination; and during a silent period, the user has no active traffic and does not transmit. Within a network, the number of active users changes randomly, causing random changes in co-channel

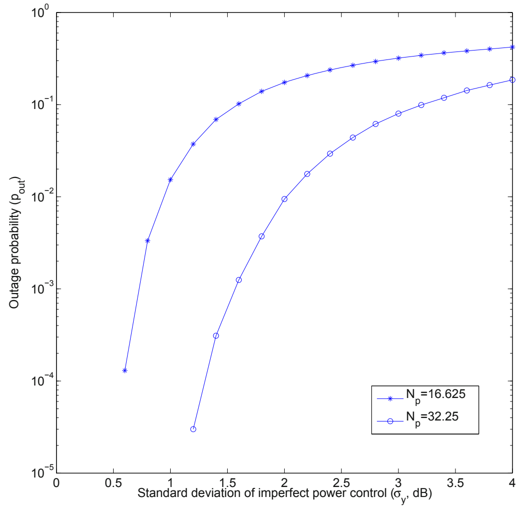

Fig. 2.3 Outage performance vs. standard deviation of imperfect power control ($\eta = 10^{-14}$W and $N = 10$)

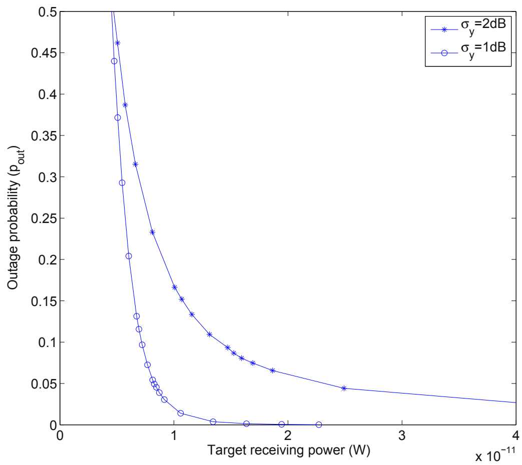

Fig. 2.4 Outage performance vs. target receiving power with imperfect power control ($N_p = 16.625$ and $\eta = 10^{-14}$W)

interference, which affects the resource allocations and system capacity. In this section, we follow some analysis in [7] to look at a simple system with bursty traffic, and study the effect of traffic burstiness on outage, transmission rates, and power allocations.

Consider N_{tot} users, indexed by $i = 1, 2, \ldots, N_{tot}$, all communicating with a common receiver, which, for example, can be the BS in a cellular network. Each user generates bursty traffic. Define a set of binary variables χ_i's. When user i transmits, $\chi_i = 1$; otherwise, $\chi_i = 0$. The transmission rate for an active user i is R_i, and the SINR of its transmission at the BS receiver should satisfy the following condition

$$\frac{W}{R_i} \frac{S}{\sum_{j=1, j\neq i}^{N_{tot}} \chi_j S + \eta} \geq \gamma_i^*. \tag{2.17}$$

Consider homogeneous traffic with $R_i = R$ and $\gamma_i^* = \gamma^*$ for all i. Given R and γ^*, the pole capacity N_p is fixed. Assume there is no maximum transmission power limit. If $N_{tot} \leq N_p$, i.e., the total number of users is less than the pole capacity, there is no outage, since all users can be supported even when they are all active. Let $N_a = \sum_{i=1}^{N_{tot}} \chi_i$ be the random variable representing the number of active users. When $N_a > N_p$, $N_a - N_p$ active users are in outage. As the number of active users changes, the minimum target receiving power changes. Given R and the number of active users, the target receiving power $S = S^*$ for an active user (not in outage) is given by

$$S^*|_{N_a=j} = \frac{\eta R \gamma^*}{W - (\min\{j, N_p\} - 1) R \gamma^*}, \tag{2.18}$$

where $\min\{j, N_p\}$ is due to the fact that when $j > N_p$, only N_p users can be supported. This is referred to as power adaptation, which is to adjust the target receiving power (through transmission power control) so that the required SINR is satisfied for the given transmission rate. The average target receiving power is given by

$$\begin{aligned} \mathrm{E}[S^*] &= \sum_{j=1}^{N_{tot}} S^*|_{N_a=j} \min\{N_p, j\} \Pr.\{N_a = j\} \\ &= \sum_{j=1}^{N_{tot}} \frac{\eta R \gamma^*}{W - (\min\{j, N_p\} - 1) R \gamma^*} \min\{N_p, j\} \Pr.\{N_a = j\}. \end{aligned} \tag{2.19}$$

Instead of having some users in outage when $N_a > N_p$, the pole capacity can be dynamically changed according to the current number of active users, so that all the active users can be supported. The pole capacity can be changed by adjusting R or γ^*. Assume γ^* is fixed. Reducing R can increase the pole capacity and allow the system to accommodate more users; while larger R is possible when fewer users are active. The channel rate can be changed by varying the spreading factor [8] or using multi-code CDMA technique [9]. Given $N_a = j$, the maximum achieved rate R can be found as

$$R^*|_{N_a=j} = \frac{W}{\gamma^*} \frac{S}{S(j-1) + \eta}. \tag{2.20}$$

As N_a changes, the transmission rate of the active users is adaptively changed. This is referred to as rate adaptation. That is, the target SINR is fixed, and the transmission rate is adjusted so that it is maximized based on the current traffic load and the target receiving power. Given that each user has the same probability of being active, the mean of the transmission rate for a user is given by

$$\begin{aligned}
\mathrm{E}[R^*] &= \sum_{j=1}^{N_{tot}} R^*|_{N_a=j} \times j \times \Pr.\{N_a = j\} \\
&= \sum_{j=1}^{N_{tot}} \frac{W}{\gamma^*} \frac{S}{S(j-1)+\eta} \times j \times \Pr.\{N_a = j\}.
\end{aligned} \tag{2.21}$$

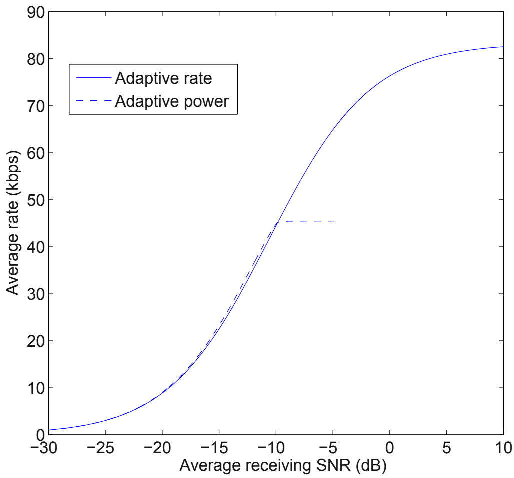

Fig. 2.5 Comparison between adaptive rate and adaptive power ($N_{tot} = 12$, $p_{on} = 0.5$, and $\eta = 10^{-10}$W)

Fig. 2.5 shows the average rate and average receiving SNR (ratio of the target receiving power to the noise power) for both adaptive rate and adaptive power allocations. It is seen that the two systems are approximately the same in low SNR and low rate region, but the adaptive rate system can achieve much higher rate in high SNR region.

- As the number of active users changes randomly, the adaptive rate system allows the users to adjust their transmission rates (and therefore change the pole capacity) based on the current traffic load. When a smaller number of users are active, higher rate can be achieved for each active user; and when a larger number of users are active, a lower rate is served for the users, but none is in outage.
- On the other hand, the adaptive power system fixes the transmission rate, which is independent of the current traffic load. When the number of active users is small, the available network resource is wasted; when the number of active users is large, it has to force some users in outage. In addition, the required power in the power adaptive system can change significantly as the number of active users

changes, making the power adaptive system more likely to have outages than the rate adaptive system when the transmission power is upper bounded.

2.5 Power Allocations in a Multi-cell Network

In this section we study power allocations in a cellular network with multiple cells. Based on the results, we discuss possible approaches to improving the system performance. We use B to represent the total number of BSs, g_{ib} to denote the link gain between BS b and the MS carrying connection i, and $i \in \mathscr{B}_c$ to indicate that connection i is associated to BS c. The main difference between a multi-cell network and a single cell network is that each transmission in a multi-cell network experiences not only interference from the transmissions within the same cell (intra-cell interference), but also that from other cells (inter-cell interference). We consider homogeneous traffic with all the users requiring the same transmission rate R and target SINR γ^*.

Uplink Analysis

In the uplink, power control ensures that all homogeneous connections associated to the same BS have the same power level at the BS receiver input. Let S_b be the target power at the BS receiver input for a connection associated to BS b, and N_b be the total number of connections currently associated to BS b. Then for a given connection associated to BS b, the experienced intra-cell interference for its signal at the BS receiver input is $(N_b - 1)S_b$. The inter-cell interference is from all other cells. For connection i associated to BS c, $c \neq b$, its transmission power is given by S_c/g_{ic}, and the interference level that its transmission causes at the BS b receiver is $S_c g_{ib}/g_{ic}$. Therefore, the total interference that a connection associated to BS b experiences is given by

$$I_b = (N_b - 1)S_b + \sum_{c=1, c\neq b}^{B} S_c \sum_{i \in \mathscr{B}_c} \frac{g_{ib}}{g_{ic}}. \tag{2.22}$$

The SINR at the BS receiver for the connection associated to BS b is given by

$$\gamma_b = \frac{W}{R} \frac{S_b}{I_b + \eta_b}, \tag{2.23}$$

where η_b is the background noise power at the receiver of BS b. When the power control is perfect and all the users transmit at the lowest power, $\gamma_b = \gamma^*$, and

$$\frac{W}{R} \frac{S_b}{I_b + \eta_b} = \gamma^*. \tag{2.24}$$

Replacing I_b in (2.24) with the right-hand side of (2.22) and manipulating, we have the following relationship:

$$S_b - \frac{1}{\frac{W}{R\gamma^*} + 1 - N_b} \sum_{c=1,c\neq b}^{B} S_c \sum_{i\in\mathscr{B}_c} \frac{g_{ib}}{g_{ic}} = \frac{\eta_b}{\frac{W}{R\gamma^*} + 1 - N_b}, \tag{2.25}$$

where $b = 1,2,\ldots,B$. Define

$$\Delta_b = \frac{W}{R\gamma^*} + 1 - N_b = N_p - N_b, \tag{2.26}$$

where $N_p = \frac{W}{R\gamma^*} + 1$ is the pole capacity of a single cell. We can rewrite (2.25) as

$$S_b - \Delta_b^{-1} \sum_{c=1,c\neq b}^{B} S_c \sum_{i\in\mathscr{B}_c} \frac{g_{ib}}{g_{ic}} = \eta_b \Delta_b^{-1}. \tag{2.27}$$

Define vector $\mathbf{S}_u = (S_1, S_2, \ldots, S_B)^T$. The B equations defined by (2.27) (for $b = 1,2,\ldots,B$) can then be rewritten in a matrix form as

$$(\mathbf{I} - \boldsymbol{\Delta}_u \mathbf{G}_u)\mathbf{S}_u = \boldsymbol{\eta}_u, \tag{2.28}$$

where $\mathbf{I}$ is a $B \times B$ identity matrix, $\boldsymbol{\Delta}_u = \mathrm{diag}(\Delta_1^{-1}, \Delta_2^{-1}, \ldots, \Delta_B^{-1})$, $\mathbf{G}_u$ is a $B \times B$ matrix whose bth row and cth column is given by

$$G_{u,bc} = \begin{cases} 0, & \text{when } b = c \\ \sum_{i\in\mathscr{B}_c} \frac{g_{ib}}{g_{ic}}, & \text{when } b \neq c \end{cases} \tag{2.29}$$

and $\boldsymbol{\eta}_u$ is a column vector whose bth element is given by $\eta_{u,b} = \frac{\eta_b}{\Delta_b}$.

Downlink Analysis

For the downlink transmissions, all the connections associated with the same BS share the BS transmission power. Denote the total transmission power from BS c as P_c, and let P_{ci} be the transmission power from BS c to user i in cell c.

For user i associated with BS c, the signal level at the user's receiver input is $P_{ci} g_{ic}$, the received interference from transmissions for other users associated with the same BS is $\xi(P_c - P_{ci}) g_{ic}$, and the interference from a neighboring cell b is $P_b g_{ib}$. Then the SINR of the received signal for the MS is given by

$$\begin{aligned} \gamma_i &= \frac{W}{R} \frac{P_{ci} g_{ic}}{\xi(P_c - P_{ci}) g_{ic} + \sum_{b=1,b\neq c}^{B} P_b g_{ib} + \eta_i}, \\ &= \frac{W}{R} \frac{P_{ci}}{\xi(P_c - P_{ci}) + \sum_{b=1,b\neq c}^{B} P_b \frac{g_{ib}}{g_{ic}} + \frac{\eta_i}{g_{ic}}}. \end{aligned} \tag{2.30}$$

Letting $\gamma_i = \gamma^*$, the allocated transmission power for each connection can be found as

$$P_{ci} = \frac{1}{W/(\xi\gamma^* R)+1}\left(P_c + \frac{1}{\xi}\sum_{b=1,b\neq c}^{B} P_b \frac{g_{ib}}{g_{ic}} + \frac{\eta_i}{\xi g_{ic}}\right). \tag{2.31}$$

The total transmission power from BS c for all connections in the cell is given by

$$P_c = \textstyle\sum_{i\in\mathscr{B}_c} P_{ci}. \tag{2.32}$$

Replacing P_{ci} in (2.32) by the right-hand side of (2.31) and manipulating we have

$$P_c - \frac{\sum_{b=1,b\neq c}^{B}\sum_{i\in\mathscr{B}_c}\frac{g_{ib}}{g_{ic}}}{\Delta_{\xi,c}}P_b = \frac{\sum_{i\in\mathscr{B}_c}\frac{\eta_i}{g_{ic}}}{\Delta_{\xi,c}}, \tag{2.33}$$

where

$$\Delta_{\xi,c} = \xi\left(\frac{W}{\xi\gamma^* R} + 1 - N_c\right) = \xi(N_{\xi,p} - N_c), \tag{2.34}$$

and $N_{\xi,p}$ is the downlink pole capacity of a single cell. The expression in (2.33) gives a set of B linear equations for $c = 1,2,\ldots,B$.

Define a column vector $\mathbf{P}_d = (P_1, P_2, \ldots, P_B)^T$, a $B\times B$ matrix $\mathbf{G}_d$ whose cth row and bth column element is given by

$$G_{d,cb} = \begin{cases} 0, & \text{if } c = b \\ \sum_{i\in\mathscr{B}_c}\frac{g_{ib}}{g_{ic}}, & \text{if } c\neq b, \end{cases} \tag{2.35}$$

a diagonal matrix $\boldsymbol{\Delta}_d = \mathrm{diag}(\Delta_{\xi,1}^{-1}, \Delta_{\xi,2}^{-1}, \ldots, \Delta_{\xi,B}^{-1})$, and a column vector $\boldsymbol{\eta}_d$ with the cth element given by

$$\eta_{d,c} = \frac{\sum_{i\in\mathscr{B}_c}\frac{\eta_i}{g_{ic}}}{\Delta_{\xi,c}}. \tag{2.36}$$

Then (2.33) can be rewritten as

$$(\mathbf{I} - \boldsymbol{\Delta}_d\mathbf{G}_d)\mathbf{P}_d = \boldsymbol{\eta}_d. \tag{2.37}$$

Discussions

The solution to $\mathbf{S}_u$ in (2.28) is the minimum receiving power that satisfies the required SINR of all users in the uplink, and the solution to $\mathbf{P}_d$ in (2.37) is the minimum BS transmission power in order to support all users in the downlink. These two equations can be combined in a common form as

$$(\mathbf{I} - \boldsymbol{\Delta}\mathbf{G})\mathbf{P} = \boldsymbol{\eta}, \tag{2.38}$$

where $\mathbf{G} = \mathbf{G}_{\mathrm{u}}$, $\mathbf{P} = \mathbf{S}_u$, $\boldsymbol{\Delta} = \boldsymbol{\Delta}_u$, and $\boldsymbol{\eta} = \boldsymbol{\eta}_u$ for the uplink, and $\mathbf{G} = \mathbf{G}_d$, $\mathbf{P} = \mathbf{P}_d$, $\boldsymbol{\Delta} = \boldsymbol{\Delta}_d$, and $\boldsymbol{\eta} = \boldsymbol{\eta}_d$ for the downlink. When there is a feasible solution to $\mathbf{P}$, i.e., all elements in $\mathbf{P}$ are non-negative (assume there is no maximum transmission power limit), the required SINR is achievable, and all the users can be simultaneously supported. From the analysis in previous sections we know that the power distribution is feasible if and only if the dominant eigenvalue of $\boldsymbol{\Delta}\mathbf{G}$, denoted as $\rho(\boldsymbol{\Delta}\mathbf{G})$, is less than 1. When any element of $\mathbf{P}$ is less than 0, the target SINR or required transmission rate should be reduced, if the same number of users are to be supported. Otherwise, some users should be in outage so that the remaining users can be served with the required rate and SINR.

When each BS is supporting at least one connection, we can see that $\boldsymbol{\Delta}\mathbf{G}$ is non-negative (element-wise) for both the uplink and the downlink. Furthermore, since i) $\boldsymbol{\Delta}$ is a diagonal matrix with all the diagonal elements larger than zero, and ii) the diagonal elements of $\mathbf{G}$ are all zero and all other elements in $\mathbf{G}$ are greater than zero, we can conclude that the matrix given by $\boldsymbol{\Delta}\mathbf{G}$ is also irreducible. For such matrices, the Perron-Frobenius Theorem [10] indicates that increasing any entry of $\boldsymbol{\Delta}\mathbf{G}$ may increase $\rho(\boldsymbol{\Delta}\mathbf{G})$. Therefore, larger elements of $\boldsymbol{\Delta}\mathbf{G}$ lead to a higher possibility that $\rho(\boldsymbol{\Delta}\mathbf{G}) > 1$, and a higher chance that $\mathbf{P}$ is infeasible. When the transmission rate and SINR requirements of the connections are given, large values in $\boldsymbol{\Delta}\mathbf{G}$ may be due to large values in these two matrices. A large element in $\boldsymbol{\Delta}$ may be due to a large number of users in a cell (large N_c) or small single cell pole capacity (large $\gamma^* R$), and a large element in $\mathbf{G}$ means high normalized link gain (g_{ib}/g_{ic} for $i \in \mathscr{B}_c$), which is resulted from poor transmission conditions — weak desired link and/or strong interfering link. If it is possible to control the elements in $\boldsymbol{\Delta}$ and $\mathbf{G}$ through different techniques so that $\rho(\boldsymbol{\Delta}\mathbf{G})$ can be reduced, then the power allocation problem may be changed from infeasible to feasible. In the next section we introduce the method of using soft handoff to achieve this objective, and in Section 4.1 the technique of using multihop relaying is introduced to improve the relative link gains so that to improve the power allocation feasibility as well as the network performance.

2.6 Soft Handoff and Power Allocations

When an MS is moving across the boundary of two cells, it should handoff from one BS to another BS. In a wireless cellular network where neighboring radio cells use different frequency channels, the MS has to disconnect from the previous BS before connecting to the new BS. This is called hard handoff (HHO). During the time period when the handoff is performed, communication outage occurs and packets may be lost. The CDMA-based cellular networks allow soft handoff (SHO). Since all the cells share the same spectrum band, there is no need to break the connection from the previous BS before establishing a connection to the new BS. During the SHO period, an MS can simultaneously connect to multiple BSs. Performing SHO can make data transmissions smoother than using HHO. The multiple BSs that an MS can be connected to during SHO form a set, referred to as the active BS set of the

MS. In the uplink, the transmitted signal of an MS can be received simultaneously by all BSs in the active set; and in the downlink, an MS can simultaneously receive the same signals from all the BSs in the active set. Compared with using HHO, using SHO allows the MS to take advantage of the diversity provided by multiple BSs in the active set and always connect to the "best" BS [11]. By taking advantage of this property, transmission power of the MSs and BSs may be reduced. As the CDMA network is interference limited, minimizing the transmission power is directly related to improving the network capacity.

In the uplink, if multiple BSs send power control commands to an MS, the decision for increasing the transmission power at the MS is made only if all the BSs in the active set require it to increase the power. That is, the transmission power from the MS is only required to guarantee sufficient SINR at one BS in the active set. This ensures that the transmission power of the MS is always the minimum. Let P_m represent the transmission power of MS m, then

$$P_m = \min_{b \in \mathscr{S}_b} S_b / g_{mb}, \tag{2.39}$$

where $\mathscr{S}_b$ is the active BS set of MS m, and S_b is the target receiving power at BS b. Assuming S_b's are known for all b, P_m can be found. With the objective to minimize the MS transmission power, the "best" BS for MS m is given by

$$b^* = \arg \min_{b \in \mathscr{S}_b} S_b / g_{mb}. \tag{2.40}$$

As an example, consider that an MS is located in the middle of BSs 1 and 2, and is connected to BS 1 if HHO is performed. If SHO is performed, then both BSs 1 and 2 are in its active set. The transmission power from the MS using HHO is S_1/g_{m1}, and using SHO is $\min\{S_1/g_{m1}, S_2/g_{m2}\}$. When $S_1 = S_2$, the ratio of the transmission power using HHO to that using SHO is

$$\frac{S_1/g_{m1}}{\min\{S_1/g_{m1}, S_2/g_{m2}\}} = \frac{1/g_{m1}}{\min\{1/g_{m1}, 1/g_{m2}\}}. \tag{2.41}$$

Using the same channel model with path loss and log-normally distributed shadowing as in Sections 2.2, and assuming $d_{m1} = d_{m2}$ and independent shadowing effect between the MS to the two BSs, Fig. 2.6 shows the average of the power ratio, where the x-axis is the standard deviation of the shadowing. From the figure we can see that the transmission power can be largely reduced by using SHO. Furthermore, the effect of this becomes more significant when there are larger variations in channel fading. That means, using SHO as defined above can get more benefit in highly fading channels.

From (2.39) we can see that whether a BS in the active set can become the "best" BS of MS m depends on both the target receiving power of the connection at the BS and the link gain between MS m and the BS. From the analysis in previous sections we know that given the transmission rate and SINR requirements, the value of S_b depends on the number of MSs associated to BS b, or N_b. The values of N_b count

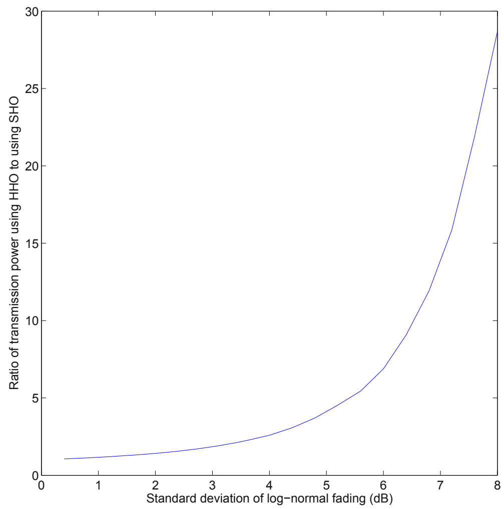

Fig. 2.6 Ratio of transmission power using HHO to transmission power using SHO in uplink: single MS case

both the HHO MSs associated to BS b and the SHO MSs having BS b as their "best" BS. In a practical system, performing optimum SHO and power control in order to minimize the transmission power of all MSs can be difficult, since this requires the knowledge of the "best" BSs of all SHO MSs, and the power allocations of the MSs in all the cells should be jointly performed.

Next we consider a simple SHO decision. Each active MS can communicate with the two nearest BSs, and always chooses to connect to the one to which it has better link gain. If BSs b and c are the two nearest BSs for MS m, then $i \in \mathscr{B}_c$ when $g_{ic} \geq g_{ib}$, and $i \in \mathscr{B}_b$ otherwise. Using this criterion, the association between the MSs and the BSs does not depend on the traffic load in each cell, and the analysis in Section 2.5 can be used to find the required transmission power for each MS. We consider that all the users transmit at the same rate, and find the maximum transmission rate that can be supported to the users. For comparison, we also consider a systems with HHO only, where all MSs communicate directly with their nearest BSs. Note that when the channels experience random fading, the MSs performing SHO can switch between different BSs more dynamically. A simple algorithm as shown in Algorithm 1 is used to find the maximum transmission rate for given link gains. We start from a small rate, and find the transmission power using the analytical model developed in Section 2.5. A variable UP is initially reset to zero. If the power allocation is feasible, the rate is doubled. This is repeated until the power allocation is infeasible, when UP is set to 1. Record this rate as $2R_1$. The maximum rate is then between R_1 and $2R_1$. The transmission rate is then returned to R_1, and a step size is initialized to $R_1/2$. Starting from this point, the rate is either increased or decreased by a step size after each iteration, depending on whether the power allocation is feasible in the current iteration, and then the step size is halved.This process is repeated until the step size is very small and the change that it causes to the rate can be neglected. In the algorithm, $R_{\min}$ is the minimum step size, which is set to 1bps in generating the numerical results. We simulate a two-dimensional cellular network, which consists of 19 hexagonal cells as shown in Fig. 2.7, where cell 1 is the center cell, cells 2 to 7 are the six first tier cells, and cells 8 to 19 are the second tier cells. A large set of the rates are obtained based on randomly generated link gains and then averaged. We consider distance related path loss and independent log-normal fading. The link gain model between the MSs and the BSs is the same as in Section 2.2. The CDMA bandwidth $W = 5$MHz, the path loss exponent is $\alpha = 4$, the standard deviation for shadowing is 8dB, the channel orthogonality factor for the downlink is $\xi = 0.5$, and the target SINR for the users' traffic is 6.8dB. Figs. 2.8 and 2.9 show the transmission rate that can be supported when the number of MSs changes in the uplink and the downlink, respectively. From these figures we can observe that using SHO can provide higher transmission rate than using HHO, and this is consistent for both the uplink and the downlink.

Algorithm 1 Finding the maximum transmission rate

```
F = 0, UP=0, and R = 1bps.
while F = 0 do
    Find target receiving power and transmission power
    if UP=0 then
        if Solution is feasible then
            R = 2R;
        else
            R = R/2, UP=1, and δ_R = R/2.
        end if
    else
        if Solution is feasible then
            R = R + δ_R;
        else
            R = R − δ_R;
        end if
    end if
    if δ_R < R_min then
        F = 1;
        Record the value of R;
    else
        δ_R = δ_R/2;
    end if
end while
```

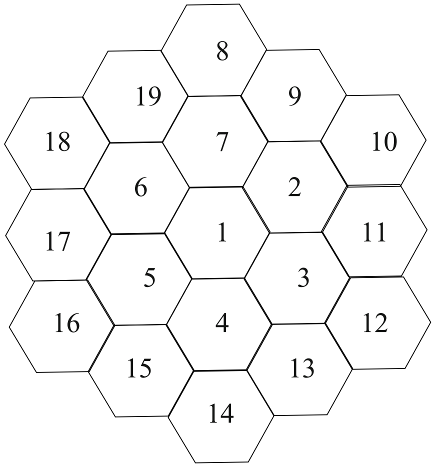

Fig. 2.7 Cell layout

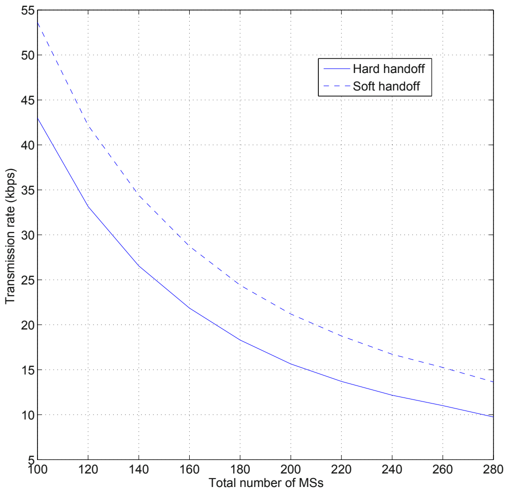

Fig. 2.8 Transmission rates for the uplink using SHO and HHO

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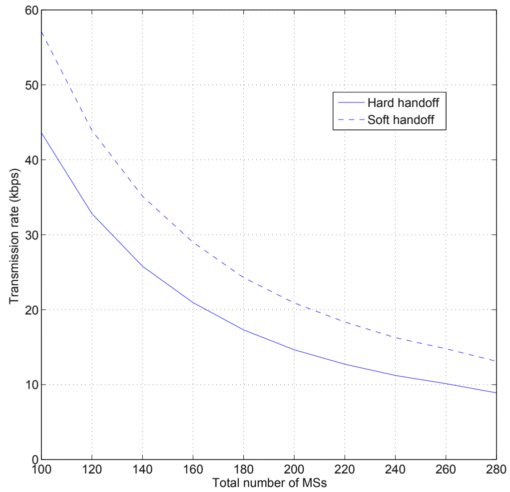

Fig. 2.9 Transmission rates for the downlink using SHO and HHO

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