

Contents

1	Introduction to Oscillation Theory	1
1.1	Introduction	1
1.2	Nonoscillation of Autonomous Delay Equations with Positive Coefficients	1
1.3	Nonlinear Equations of Mathematical Biology	9
1.3.1	Linearization of Nonlinear Delay Equations	9
1.3.2	Hutchinson's Equation	10
1.3.3	Lasota-Ważewska Equation	12
1.3.4	Nicholson's Blowflies Equation	13
1.3.5	Mackey-Glass Equations	15
1.4	Impulsive Equations	16
1.5	Some Other Classes of Equations	18
1.6	Discussion and Open Problems	20
2	Scalar Delay Differential Equations on Semiaxes	23
2.1	Introduction	23
2.2	Preliminaries	24
2.3	Nonoscillation Criteria	25
2.4	Comparison Theorems	28
2.5	Nonoscillation Conditions, Part 1	32
2.6	Nonoscillation Conditions, Part 2	37
2.7	Oscillation Conditions	43
2.8	Estimations of Solutions	46
2.9	Positivity of Solutions	49
2.10	Slowly Oscillating Solutions for Delay Differential Equations	51
2.11	Stability and Nonoscillation	52
2.12	Discussion and Open Problems	52
3	Scalar Delay Differential Equations on Semiaxis with Positive and Negative Coefficients	59
3.1	Introduction	59
3.2	Nonoscillation Criteria	59

3.3	Nonoscillation Conditions, Part 1	66
3.4	Nonoscillation Conditions, Part 2	71
3.5	Equations with an Oscillatory Coefficient	77
3.6	Discussion and Open Problems	79
4	Oscillation of Equations with Distributed Delays	83
4.1	Introduction	83
4.2	Preliminaries	84
4.3	Existence of a Positive Solution—General Results	86
4.4	Comparison Theorems	92
4.5	Nonoscillation Criteria for Some Autonomous Integrodifferential Equations	97
4.6	Explicit Nonoscillation and Oscillation Conditions	101
4.7	Slowly Oscillating Solutions	107
4.8	Equations with Positive and Negative Coefficients	108
4.9	Discussion and Open Problems	118
5	Scalar Advanced and Mixed Differential Equations on Semiaxes	123
5.1	Introduction	123
5.2	Advanced Equations	123
5.3	Mixed Equations with Positive Coefficients	132
5.4	Mixed Equation with Negative Coefficients	134
5.5	Positive Delay Term, Negative Advanced Term	135
5.6	Negative Delay Term, Positive Advanced Term	141
5.7	Discussion and Open Problems	144
6	Neutral Differential Equations	149
6.1	Introduction and Preliminaries	149
6.2	Nonoscillation Criteria	151
6.3	Efficient Nonoscillation Conditions	156
6.4	Explicit Oscillation Conditions	160
6.5	Positivity of Solutions	164
6.6	Slowly Oscillating Solutions	165
6.7	Neutral Equations with Positive and Negative Coefficients	166
6.8	Discussion and Open Problems	168
7	Second-Order Delay Differential Equations	171
7.1	Introduction	171
7.2	Preliminaries	171
7.3	Nonoscillation Criteria	172
7.4	Comparison Theorems	176
7.5	Explicit Nonoscillation and Oscillation Conditions	182
7.6	Slowly Oscillating Solutions	187
7.7	Existence of a Positive Solution	188
7.8	Discussion and Open Problems	190

8	Second-Order Delay Differential Equations with Damping Terms . .	193
8.1	Introduction	193
8.2	Preliminaries	193
8.3	Nonoscillation Criteria	195
8.4	Comparison Theorems	200
8.5	Explicit Nonoscillation Conditions	203
8.6	Discussion and Open Problems	204
9	Vector Delay Differential Equations	207
9.1	Introduction	207
9.2	Preliminaries	208
9.3	Main Results	210
9.4	Comparison Results	214
9.5	Higher-Order Scalar Delay Differential Equations	217
9.6	Positivity and Solution Estimates	220
9.7	Positive Solutions and Stability	223
9.8	Systems of Differential Equations with a Distributed Delay	229
9.8.1	Nonnegativity of Fundamental Matrices	229
9.8.2	Comparison Results and Positivity of Solutions	232
9.8.3	Solution Estimates	233
9.8.4	Nonoscillation and Stability	236
9.9	Discussion and Open Problems	238
10	Linearization Methods for Nonlinear Equations with a Distributed Delay	241
10.1	Introduction	241
10.2	Preliminaries	241
10.3	Linearized Oscillation	244
10.4	Applications	248
10.4.1	Logistic Equation	249
10.4.2	Lasota-Ważewska Equation	250
10.4.3	Nicholson's Blowflies Equation	253
10.5	"Mean Value Theorem" for Equations with a Distributed Delay .	258
10.6	Discussion and Open Problems	261
11	Nonlinear Models—Modifications of Delay Logistic Equations . . .	263
11.1	Introduction	263
11.2	Generalized Logistic Equation with Several Delays	265
11.2.1	Preliminaries	265
11.2.2	Sublinear Case $\alpha_k < 1$, $k = 1, \dots, m$	266
11.2.3	Superlinear Case $\alpha_k > 1$, $k = 1, \dots, m$	269
11.2.4	Mixed Cases	272
11.2.5	Generalized Logistic Equation—Main Results	275
11.3	Multiplicative Delay Logistic Equation	277
11.3.1	Preliminaries	277
11.3.2	Nonoscillation Criteria	278

11.3.3	Multiplicative Logistic Equation—Main Results	281
11.4	Discussion and Open Problems	282
12	First-Order Linear Delay Impulsive Differential Equations	285
12.1	Introduction	285
12.2	Preliminaries	286
12.3	Nonoscillation Criteria for Impulsive Equations	287
12.4	Explicit Nonoscillation Tests and Comparison Theorems	292
12.5	Reduction to Equations Without Impulses	295
12.6	Impulsive Equations with a Distributed Delay	296
12.7	Discussion and Open Problems	299
13	Second-Order Linear Delay Impulsive Differential Equations	301
13.1	Introduction	301
13.2	Preliminaries	302
13.3	Nonoscillation Criteria	303
13.4	Comparison Theorems	308
13.5	Explicit Nonoscillation and Oscillation Conditions	309
13.6	Impulsive Equations with Damping Terms	315
13.7	Discussion and Open Problems	318
14	Linearized Oscillation Theory for Nonlinear Delay Impulsive Equations	319
14.1	Introduction	319
14.2	Preliminaries	319
14.3	Oscillation and Nonoscillation	321
14.4	Applications to Equations of Mathematical Biology	329
14.4.1	Logistic Equation: Theoretical Results	329
14.4.2	Logistic Equation: Numerical Simulations	333
14.4.3	Generalized Lasota-Ważewska Equation	334
14.5	Discussion and Open Problems	336
15	Maximum Principles and Nonoscillation Intervals	339
15.1	Introduction	339
15.2	Preliminaries	340
15.3	Maximum Principles in the Case of Positive Volterra Operator $(-B)$	346
15.4	Nonoscillation and Positivity of Green's Functions for Positive Volterra Operator B	350
15.5	Nonoscillation on the Semiaxis	356
15.6	Positivity Tests for Green's Functions Through Choice of $v(t)$	357
15.7	The Generalized Periodic Problem for Positive Volterra Operator B	360
15.8	Regular Behavior of the Green's Function to a One-Point Boundary Value Problem	362
15.9	Positivity of Green's Functions for Equations Including Difference of Positive Operators	363

15.10	Positivity of the Cauchy and Green's Functions	367
15.11	Equations with an Oscillating Coefficient	376
15.12	Positivity of the Cauchy Function and Exponential Stability . . .	384
15.13	General Boundary Value Problems	388
15.14	Discussion and Open Problems	391
16	Systems of Functional Differential Equations on Finite Intervals . .	399
16.1	Introduction	399
16.2	Nonnegativity and Nonpositivity of Green's Matrices	401
16.3	Positivity of the n -th Row of the Cauchy Matrix	408
16.4	Positivity of the Fixed n -th Row of Green's Matrices	417
16.5	Nonpositivity Conditions for the n -th Row of Green's Matrices	420
16.6	Discussion and Open Problems	426
17	Nonoscillation Intervals for n-th-Order Equations	429
17.1	Introduction	429
17.2	Homogeneous Functional Differential Equations of the n -th Order	430
17.3	Wronskian of the Fundamental System	432
17.4	Nonoscillation of Functional Differential Equations	434
17.5	Nonoscillation and Regular Behavior of Green's Functions . . .	437
17.6	Tests for Differential Equations with Deviating Arguments . . .	444
17.7	Discussion and Open Problems	451
Appendix A	Useful Theorems from Analysis	455
A.1	Vector Spaces	455
A.2	Functional Spaces	456
A.3	Sets in Functional Spaces	458
A.4	Linear Operators in Functional Spaces	458
A.5	Nonlinear Operators	462
A.6	Gronwall-Bellman and Coppel Inequalities	463
Appendix B	Existence and Uniqueness Theorems, Solution Representations	465
B.1	Linear Functional Differential Equations	465
B.1.1	Differential Equations with Several Concentrated Delays	465
B.1.2	Mixed Equations with an Infinite Number of Delays . . .	468
B.1.3	Equations with a Distributed Delay	472
B.1.4	Equations of Neutral Type	475
B.1.5	Higher-Order Scalar Delay Differential Equations	477
B.2	Estimations of the Fundamental Matrix	480
B.3	Nonlinear Delay Differential Equations	482
B.4	Linear Delay Impulsive Differential Equations	490
B.4.1	First-Order Impulsive Equations	490
B.4.2	Second-Order Impulsive Equations	494
B.5	Bohl-Perron Theorems	499
References	503
Index	517

Nonoscillation Theory of Functional Differential
Equations with Applications

Agarwal, R.P.; Berezansky, L.; Braverman, E.;
Domoshnitsky, A.

2012, XVI, 520 p., Hardcover

ISBN: 978-1-4614-3454-2