Chapter 2
Commentary: Likelihood-Free Inference in Cosmology: Potential for the Estimation of Luminosity Functions

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Abstract The identification, diagnosis and removal of systematic biases, due to e.g. measurement errors and observational selection effects, has become a key challenge for the so-called 'era of precision cosmology'. In this commentary I will describe some specific examples of where and how this challenge may arise in the analysis of astronomical surveys, thus illustrating ways in which the construction of an explicit likelihood function is rendered complicated in this field. These various examples therefore provide further motivation for the potential usefulness of the likelihood-free inference approach which Schafer has proposed.

2.1 Introduction

The 20 years since the first SCMA conference have seen rapid growth in the reach and impact of astrostatistics—particularly in the field of cosmology. The application of physically well-motivated cosmological probes such as Type Ia supernovae (SNIe) and the cosmic microwave background radiation has placed strong constraints on the parameters which define our cosmological model, leading to the emergence of the so-called "Concordance Cosmology", supported by observations across a range of astrophysical phenomena. While there remain serious unresolved issues with the Concordance model, the quantity and quality of the data that emerged in the late 1990s prompted the label "the era of precision cosmology" to enter common use [1].

The appropriateness of this label is undermined, however, by the potential impact of systematic errors. These may arise for a variety of reasons, including instrumental or atmospheric effects, measurement errors and observational selection

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due to e.g. truncation or censoring, and may be strongly correlated, non-Gaussian, non-stationary or otherwise problematic. Their identification and diagnosis can present significant challenges for the analysis of astronomical surveys via traditional likelihood-bases methods. In this brief commentary I will describe some specific examples of where and how these challenges may arise—thus providing further motivation for the potential usefulness of the likelihood-free inference approach which Schafer has proposed.

2.2 Systematic Effects in Astronomical Surveys

The surveying of astronomical populations is commonplace across a wide range of scales, from the statistics of nanoflares on the Sun to the demographics of distant quasars. As Schafer has noted in the preceding article, the approach adopted to date in studying astronomical populations has generally been likelihood based. For instance in estimating the galaxy luminosity function (LF) a range of maximum likelihood methods—both parametric and robust—has been developed, many of which explicitly account for the impact of observational selection (see [2] for a recent and comprehensive review) and the semi-parametric method of [3] is a powerful recent addition to these techniques.

In this context however, and as the preceding article also discusses, a significant complication in this field is the growing prevalence in very large survey datasets of photometric redshifts. These have hugely increased the volume and size of redshift surveys and the efficiency with which they may be carried out but at the cost of introducing a significant measurement error on the redshift of each source. The trend towards extremely large photometric redshift surveys is firmly set to continue as we approach the era of 'petascale' datasets promised by the Large Synoptic survey Telescope [4]. Consequently the impact of photometric redshift errors on likelihood-based approaches to survey analysis, and the exploration of alternative methodologies, appears to be an important future research direction—a conclusion which was also reached at SCMA4 in the context of the report presented there on the work of the astronomical surveys group within the 2006 Astrostatistics program at SAMSI [5]. This conclusion would appear to be equally relevant, if not more so, today.

A common feature shared by likelihood-based methods to probe survey luminosity functions is the adoption of a simple, approximate form for the sample selection function—for example a step function to describe the flux limit(s) of the survey [6]. While these approximations may be necessary to make the problem analytically tractable, the reality may be considerably more complicated, particularly when objects (such as distant SNIe or high redshift galaxies) are being detected in crowded fields, where issues of blended sources and source misclassification can be important [7]. These effects can render the flux limit of selected sources strongly dependent on environment, sky direction and 'seeing' conditions at the time of observation—all of which may not easily be reducible to a simple step function of flux alone.

Another common problem with flux limited surveys is where the sources are originally selected in the optical—based on a historical catalogue of e.g. B-band galaxy apparent magnitudes—but the survey involves observations made in another waveband, for example I-band photometry for the purpose of estimating galaxy distances and peculiar velocities via the Tully-Fisher relation [8]. In this situation the intrinsic correlation between galaxy luminosity and colour means that the B-band selection to which the original catalogue was subject will translate into an I-band selection function in the Tully-Fisher survey. However, since the correlation between B-band luminosity and $B - I$ colour is not perfect but has an appreciable scatter, the I-band selection function will be blurred even if the original B-band selection is well described by a sharp apparent magnitude limit [9].

A further complication when observing the very distant Universe is that surveys of e.g. quasars or high redshift galaxies may be subject to complex and poorly understood evolutionary effects (indeed probing this source evolution is often the main object of the survey in the first place!). In addition the application of so-called 'k-corrections' is required because the spectral energy distribution emitted by a high redshift source in its rest frame will be observed redshifted towards longer wavelengths by the expansion of the Universe [10, 11].

Other surveyed sources such as radio pulsars, gamma ray bursts or active galactic nuclei may be affected by geometrical selection effects, where the emitted radiation is strongly anisotropic [12]. These effects can impact significantly on the detectability of sources and influence their apparent brightness due to e.g. relativistic beaming, as well as introducing strong degeneracies between source parameters such as inferred distance and inclination to the line of sight. Similar issues are now being confronted in the nascent field of gravitational-wave astronomy [13], where the selection function of e.g. observed inspiralling binary neutron star sources will be the result of a complex interplay between the underlying cosmological model, the intrinsic star formation rate and a sky sensitivity pattern which is strongly dependent on direction, source orientation and frequency of the emitted gravitational waves [14].

Another very common and important source of systematic error in survey data is the effect of extinction: the wavelength dependent absorption of light by dust either in the environs of the source itself or within our own Milky Way galaxy. Extinction effects are often dealt with by carrying out multi-wavelength observations and correcting for their impact by fitting a (usually parametric) extinction law as a function of wavelength. This technique has been used extensively for example to infer extinction-free estimates of the distance to Cepheid variable stars in external galaxies observed by the Hubble Space Telescope [15].

Multiwavelength observations are also a key feature of the methodology used to harness SNIe as cosmological distance indicators. The multiwavelength approach is employed both to diagnose and correct for extinction and to improve the precision of the distance indicator itself by exploiting empirical correlations between the shape of the SNIe light curves and their intrinsic luminosity at different wavelengths. For more than 15 years advanced Bayesian methods have been applied for calibrating these relations to derive SNIe distance estimates [16]. Recently Mandel [17] has presented a sophisticated multilevel Bayesian model that addresses simultaneously extinction, intrinsic light curve shape, possible source evolution and cosmological parameter extraction. While this treatment is certainly ‘state of the art’ it shares with many of the other survey examples listed here the requirement of a complicated likelihood function, perhaps featuring a significant number of nuisance parameters, to fully capture the intrinsic characteristics of the source population and the observational selection effects to which they are subject.

2.3 The Case for a Likelihood-Free Approach

All of the complicating factors listed in the previous section—crowded fields, colour correlations, evolutionary effects, k-corrections, source orientation and beaming, extinction—are relatively straightforward to *simulate*, i.e. to model numerically via Monte Carlo simulation, but are not so easy to explicitly include in a likelihood model without potentially rendering that model unwieldy. In contrast, therefore, to the traditional methodology whereby adopts a likelihood function model that is as simple as possible and estimates the parameters of that model (see e.g. the VELMOD approach of [18] as a good archetype, in the area of peculiar velocity reconstruction), one can envisage instead a “forward modelling” approach in which one constructs sophisticated “mock” datasets that can simulate faithfully some or all of the above factors that would influence the journey of a real photon (or graviton!) from source to detectors. As described in the preceding article, one would draw inferences about the source population by comparing these mock datasets with the real survey data—analogous to the approach that has been adopted for many years in generating mock galaxy catalogues from high resolution n-body simulations of large scale structure [19].

As the preceding article has recognised, the key challenge in this approach is identifying a suitable metric for comparing the mock and real datasets, or some appropriate summary statistic constructed therefrom. The ABC algorithms which Schafer presents appear to offer a useful and practical solution to this challenge—particularly the sequential Monte Carlo algorithm which largely overcomes the problem of inefficient sampling of the Rejection algorithm. This is a crucial improvement since, as we have seen in Sect. 67.2, the complexity of simulations required to capture adequately the details of many future cosmological data sets may be considerable.

In a similar vein the preceding article underlines the importance of identifying and constructing useful summary statistics that *efficiently* measure the degree of similarity between the observed and simulated datasets. He proposes, for example, fitting a low-dimensional smoother through the real and simulated supernovae redshift and magnitude data to represent the luminosity distance-redshift relation. This is an approach that has already been explored—using a variety of different basis functions [20–22]—as an efficient method for representing non-parametrically the luminosity distance-redshift relation and its integral relationship to the cosmic equation of state. An approach of this form, applied to a variety of other cosmological datasets, would appear to hold promise for the efficient implementation of likelihood-free inference methods in the future.

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