Chapter 2
Thermodynamic Ideal Cycle Analysis

In studying an ideal cycle relevant to aircraft propulsion, one can have a good insight into the relevant important parameters that give the performance parameters of a given system. For such a study, the following assumptions are usually taken:

1. Compression and expansion processes are isotropic.
2. The working fluid is a perfect gas with a constant specific heat and a constant adiabatic exponent.
3. The fuel mass added to air is neglected, and heat is added as if it is from an external source.

Under the above assumptions, we will first study piston engines followed by jet engines, including gas turbines.

2.1 Propeller-Driven Engines

Among the propeller-driven engines, we discuss first the piston engines and later the turboprops. A schematic sketch of a piston engine is shown in Fig. 2.1, in which TDC and BDC refer to the top dead center and bottom dead center, respectively, as per the convention to describe extreme positions of a vertically operating piston. Further, V_c is the *clearance volume*, V_d is the *displacement volume*, and the sum of the two, $V_t = V_c + V_d$, is the *total volume*. The diameter of the cylinder is D, known as the *bore*; the length of the cylinder that is traversed by the piston is the stroke length, or *stroke*. Thus, the displacement volume is the product of the stroke length and cross section of the cylinder.

In the operation of piston engines, we have to differentiate between *two-stroke* and *four-stroke* engines. In the four-stroke engine, air is allowed inside the cylinder by opening the valve and withdrawing the piston, creating a vacuum inside the cylinder. Then the valve is closed and air is compressed by the piston's pushing forward. The fuel is ignited with the resultant withdrawal of the piston, giving the

T. Bose, *Airbreathing Propulsion: An Introduction*,
Springer Aerospace Technology, DOI 10.1007/978-1-4614-3532-7_2,

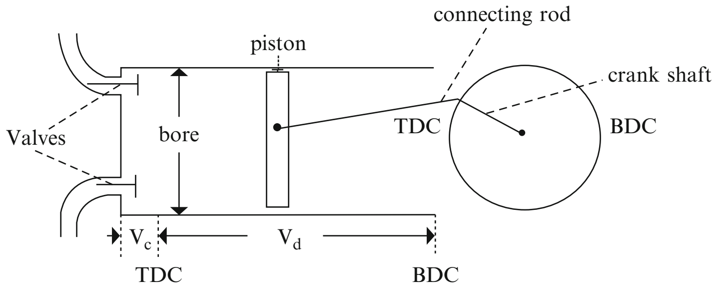

Fig. 2.1 Schematic sketch of a piston engine

effective work (*power stroke*), and, finally, the valve opens and the exhaust gas is pushed out. Thus, in a four-stroke engine, there are four different strokes in two complete cycles of motion of the crank shaft for each power stroke. For ideal cycle analysis, however, it has been found more expedient to consider the two-stroke engine, in which there are no separate sections and exhaust strokes; they are combined as well as possible in the design, near the bottom dead center, which ideally is a constant-volume process. Incidentally, in piston engines, a *volume compression ratio* is defined as $\varepsilon = V_t/V_c$, the ratio of the total volume to the clearance volume, whereas for gas turbines, usually the compression ratio means the ratio of pressure across the compression. The volume compression ratio for aircraft piston engines with Otto cycles is between 5.9 and 10.5 and with fiesel cycles is between 15 and 22; the aircraft diesel engine in the databank with this book has a volume compression ratio of 27.0. The pressure in a diesel engine rises to about 40.0 bar, while in a gasoline engine, after the compression stroke, is between 8 and 14 bar.

With simple algebraic manipulation, we can get the following relations among the three volumes (see Fig. 2.2):

$$V_c = V_t/\varepsilon = V_2; \; V_d = V_t\frac{\varepsilon - 1}{\varepsilon}; \; V_t = V_1$$

We now separately consider the ideal *Otto cycle* and the ideal *diesel cycle*. *Ideal cycle* means the isentropic process is without any heat exchange with the wall, the *isobar* process is completely at constant temperature, and the *isochore* (constant-volume) process involves instantaneous change in the pressure at a constant volume. The gas behaves like an ideal gas, and the ideal change of the isentropic process is through the isentropic coefficient for ideal air, $\gamma = 1.4$.

2.1.1 Ideal Otto Cycle

In an Otto cycle, a fuel–air mixture is introduced during the changing of the cylinder, and near the top dead center (TDC), the fuel–air mixture is ignited

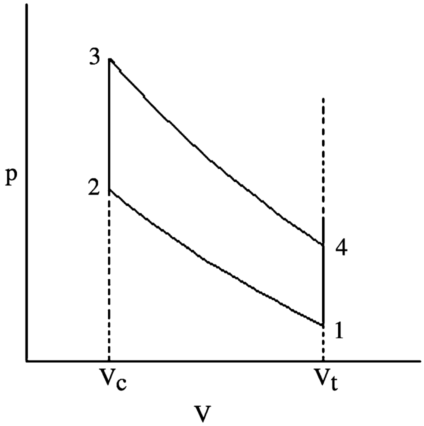

Fig. 2.2 Thermodynamic process in Otto cycle of a piston engine

electrically with the help of a spark-ignition system. Ideally, the combustion is instantaneous and can be considered a constant-volume process. It is therefore also called a *spark-ignition (SI)* engine. Hence, the thermodynamic cycle consists of a four-part process, shown schematically in Fig. 2.2, as follows: Air is brought into the cylinder at state 1 (*intake stroke*); it is compressed isentropically (with constant entropy) to state 2 (*compression stroke*); combustion takes place at a constant volume to reach state 3; it is expanded isentropically to state 4 (*power stroke*); the exhaust gas is expelled (*exhaust stroke*), and simultaneously a fresh change in the fuel–air mixture is introduced at a constant volume (*isochore*). This is when all four processes are completed within one cycle in two half-cycles (*two-stroke engines*), as in the present ideal case, or in four half-cycles; the last two are separate processes for the suction of air and expelling the hot gas (*four-stroke engine*).

In addition to the *compression ratio* $\varepsilon = V_t/V_c$, let the other operational parameter be $\Theta_3 = T_3/T_1$, which depends on the *fuel–air ratio*. Now, for the two *isentropic processes*,

$$p_1 V_t^\gamma = p_2 V_t^\gamma \tag{2.1a}$$

and

$$p_4 V_t^\gamma = p_3 V_c^\gamma \tag{2.1b}$$

and we get

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = \left(\frac{V_t}{V_c}\right)^\gamma = \varepsilon^\gamma = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{T_3}{T_4}\right)^{\gamma/(\gamma-1)} \tag{2.2}$$

Thus,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \varepsilon^{(\gamma-1)} \tag{2.3}$$

and

$$\frac{p_3}{p_2} = \frac{p_4}{p_1} = \frac{T_3}{T_2} = \left(\frac{T_3}{T_1}\right)\left(\frac{T_1}{T_2}\right) = \frac{\Theta}{\varepsilon^{(\gamma-1)}} \tag{2.4}$$

Similarly,

$$\frac{T_4}{T_1} = \left(\frac{T_4}{T_3}\right)\left(\frac{T_3}{T_1}\right) = \frac{\Theta}{\varepsilon^{(\gamma-1)}} \tag{2.5}$$

Now, heat added to the process at a constant volume is $q_{\mathrm{a}} = q_{23} = c_{\mathrm{v}}(T_3 - T_2)$, and heat rejected at a constant volume is $q_{\mathrm{r}} = q_{41} = c_{\mathrm{v}}(T_1 - T_4)$. Therefore,

$$\frac{q_{\mathrm{a}}}{c_{\mathrm{v}}T_1} = \frac{T_3}{T_1} - \frac{T_2}{T_1} = \Theta - \varepsilon^{\gamma-1} \tag{2.6a}$$

and

$$\left|\frac{q_{\mathrm{r}}}{c_{\mathrm{v}}T_1}\right| = \frac{T_4}{T_1} = -1 = \frac{\Theta - \varepsilon^{(\gamma-1)}}{\varepsilon^{\gamma-1}} \tag{2.6b}$$

Furthermore, $q_{12} = q_{34}$. Now the compression work is $w_{12} = c_{\mathrm{v}}(T_2 - T_1)$, and the expansion work is $w_{34} = c_{\mathrm{v}}(T_4 - T_3)$. For the other two-part processes, $w_{23} = w_{41} = 0$ since both are constant-volume processes. Thus,

$$\left|\frac{w_{12}}{c_{\mathrm{v}}T_1}\right| = \frac{T_2}{T_1} - 1 = \varepsilon^{(\gamma-1)} - 1 \tag{2.7a}$$

and

$$\left|\frac{w_{12}}{c_{\mathrm{v}}T_1}\right| = \left(\frac{T_3}{T_4}\right)/T_1 = \Theta\,\frac{\left(\varepsilon^{(\gamma-1)} - 1\right)}{\varepsilon^{(\gamma-1)}} \tag{2.7b}$$

Now, in a cyclic process, the overall work gained must be the same as the overall heat exchanged, and thus,

$$\frac{w}{c_{\mathrm{v}}T_1} = \left|\frac{w_{12}}{c_{\mathrm{v}}T_1}\right| - \left|\frac{w_{12}}{c_{\mathrm{v}}T_1}\right| = \frac{\left(\varepsilon^{(\gamma-1)} - 1\right)\left(\Theta - \varepsilon^{(\gamma-1)}\right)}{\varepsilon^{(\gamma-1)}} \tag{2.8a}$$

$$\frac{q}{c_{\mathrm{v}}T_1} = \left|\frac{q_{\mathrm{a}}}{c_{\mathrm{v}}T_1}\right| - \frac{q_{\mathrm{r}}}{c_{\mathrm{v}}T_1} = \frac{\left(\Theta - \varepsilon^{(\gamma-1)}\right)\left(\varepsilon^{(\gamma-1)} - 1\right)}{\varepsilon^{(\gamma-1)}} \tag{2.8b}$$

Obviously, in view of the *first law of thermodynamics*, the *overall work* and *overall heat* must be the same. However, the thermodynamic efficiency is defined as the overall work divided by the added heat. Thus, the *thermodynamic efficiency* is

$$\eta_{\mathrm{th}} = \frac{w}{q_{\mathrm{a}}} = \frac{(\varepsilon^{\gamma-1} - 1)}{\varepsilon^{\gamma-1}} \tag{2.8}$$

and the two limiting values of the thermodynamic efficiency are

$$\varepsilon \to 1 : \eta_{\mathrm{th}} \to 0 \quad \text{and} \quad \varepsilon \to \infty : \eta_{\mathrm{th}} \to 1$$

However, the compression ratio for Otto cycles with gasoline as fuel for a typical aircraft engine is within the range 6.3–10.5, and any higher value is restricted due to autoignition.

Although Θ as a *temperature ratio* is introduced here as an independent parameter, it depends on the fuel–air ratio, the maximum inverse value of which can be taken from Table 1.3. Analogous to (1.19), we may write

$$\dot{m}_{\mathrm{f}} \Delta H_{\mathrm{v}} = \dot{m}_{\mathrm{a}} c_{\mathrm{v}} (T_3 - T_2) \tag{2.9a}$$

and hence,

$$f = \frac{\dot{m}_{\mathrm{f}}}{\dot{m}_{\mathrm{a}}} = \frac{c_{\mathrm{v}} T_1}{\Delta \mathrm{H}_{\mathrm{v}}} (\Theta - \varepsilon^{\gamma-1}) \tag{2.9b}$$

In the above equation, ΔH_{v} is the heat of reaction, the *lower heat of reaction*, at a constant volume. Noting further that the relation between the heat of reaction for constant-pressure combustion and for constant-volume combustion is given by the relation

$$\Delta H_{\mathrm{p}} = \Delta H_{\mathrm{v}} + v \Delta p$$

it follows that

$$\Delta H_{\mathrm{v}} = \Delta H_{\mathrm{p}} - \frac{V_{\mathrm{c}}}{M_1 \varepsilon} \left(\frac{p_3}{p_1} - \frac{p_2}{p_1} \right)$$

where M_1 is the mass of air at point 1.

Noting further that

$$M_1 = \frac{p_1 V_{\mathrm{t}}}{RT}; \frac{p_2}{p_1} = \varepsilon^{\gamma}; \ \frac{p_3}{p_1} = \Theta \varepsilon$$

we get

$$\frac{\Delta H_\mathrm{v}}{c_\mathrm{v}T_1} = \frac{\Delta H_\mathrm{p}}{c_\mathrm{v}T_1} - (\gamma - 1)\left(\Theta - \varepsilon^{\gamma-1}\right)$$

By rearrangement, we therefore get

$$\Theta - \varepsilon^{\gamma-1} = \gamma f \frac{\Delta H_\mathrm{p}}{c_\mathrm{p}T_1[1 + f(\gamma - 1)]} \approx \frac{\Delta H_\mathrm{p}}{c_\mathrm{p}T_1}$$

It is worth mentioning again that in the above, $M_1 = p_1 V_\mathrm{t}/(RT_1)$ is the mass of the medium (air) in the cylinder. Further, the *mean indicated pressure*, which is an important design parameter, is defined as the work done in a cycle, divided by the displacement volume. Thus, $p_\mathrm{m} = wM_1/V_\mathrm{d}$, from which it follows that

$$\frac{p_\mathrm{m}}{p_1} = (\gamma - 1)\frac{\varepsilon}{\varepsilon - 1}\left(\Theta - \varepsilon^{\gamma-1}\right)\left(\frac{\varepsilon^{\gamma-1} - 1}{\varepsilon^{\gamma-1}}\right) \tag{2.10}$$

Now, for piston engines, the power developed is given by the relation

$$p_\mathrm{E} = p_\mathrm{m}V_\mathrm{d}\left(\frac{n}{60}\right)\left(\frac{2}{N}\right) \tag{2.11}$$

where n is the rpm, and $N = 2$ for a two-stroke and $N = 4$ for a four-stroke engine. In (2.11), if p_m is in N/m^2 and V_d is in m^3, then P_E is in watts. For a practical aircraft piston engine, $p_\mathrm{m} = 7.8$–18.7 bar, but generally it is 12–13.5 bar.

We may recall that the *specific fuel consumption* is given by the relation

$$\mathrm{SFC} = \frac{\dot{m}_\mathrm{f}}{\dot{m}_\mathrm{a}w} = \frac{f}{c_\mathrm{v}T_1}.\frac{c_\mathrm{v}T_1}{w} = \frac{\varepsilon^{\gamma-1}}{[\Delta H_\mathrm{v}(\varepsilon^{\gamma-1} - 1)]} = \frac{1}{(\Delta H_\mathrm{v}\eta_\mathrm{th})}$$

The above definition of the specific fuel consumption is dependent on the heat content of the fuel, and hence is dimensional. Thus, we define a *nondimensional specific fuel consumption*

$$\mathrm{SFC}^* = \mathrm{SFC}.\Delta H_\mathrm{v} = 1/\eta_\mathrm{th}$$

which is inversely proportional to the thermodynamic efficiency, which depends on the compression ratio only.

We will now make an estimate of the compression ratio based on the performance parameters of the engine. We take gasoline as the fuel, for which, according to Tables 1.2 and 1.3, $\Delta H_\mathrm{p} = 4.2707 \times 10^4$ kJ/kg and *stoichiometric*

Table 2.1 Some nondimensional results for different compression ratios in Otto cycles

ε	4	5	6	8	10	15
T_2/T_2	1.741	1.903	2.048	2.297	2.511	2.954
Θ	11.710	11.880	12.020	12.270	12.490	12.930
$w/(c_v T_1)$	4.247	4.736	5.104	5.634	6.005	6.600
p_m/p_1	14.150	14.800	15.310	16.090	16.680	17.680
η_{th}	0.426	0.475	0.510	0.564	0.602	0.661
SFC*	1.954	1.852	1.771	1.710	1.662	1.621
$\eta_{th,actual}$	0.310	0.360	0.370	0.420	0.460	0.510

fuel–air ratio $f = 1/20.03 \approx 0.05$. Assuming further that the specific heat of air is $c_p = 1.005$ kJ/(kgK), ambient temperature $T_1 = 298.15$ K, and *specific heat ratio* $\gamma = 1.4$, we get for the *temperature ratio*

$$\Theta = \varepsilon^{0.4} + \left[\frac{1.4 \times 0.05 \times 4.2707 \times 10^4}{1.005 \times 298.15}\right] = \varepsilon^{0.4} + 9.976914$$

Some of these results are computed and given in Table 2.1.

It is evident that for an Otto motor to have better thermodynamic efficiency and better specific work, the compression ratio must be increased. This is not possible because of the autoignition of the fuel–air mixture if the temperature after the compression, T_2, is too high. It is therefore necessary to introduce the fuel just around the top dead center, but in order to keep the already high pressure within limits, fuel is introduced at a controlled rate. This leads us to a compression-ignition engine, better known as a diesel engine.

2.1.2 Ideal Diesel Cycle

In a diesel cycle, fuel is introduced at a controlled rate to produce a constant-pressure combustion, while the piston is withdrawn to a cylinder volume V_3, and under ideal conditions, further expansion takes place isentropically. The process is shown schematically in Fig. 2.3.

Let's reintroduce first the three nondimensional ratios: the *volume compression ratio,* $\varepsilon = V_t/V_c$, the *combustion volume ratio,* $\Omega = V_3/V_c$, and the *temperature ratio*, $\Theta = T_3/T_1$ Obviously, $\Omega < \varepsilon$, and the value of Ω depends on both Θ and ε. Since

$$\frac{T_3}{T_2} = \frac{V_3}{V_c} = \Omega = \left(\frac{T_3}{T_1}\right)\left(\frac{T_1}{T_2}\right) = \frac{\Theta}{\varepsilon^{\gamma-1}}$$

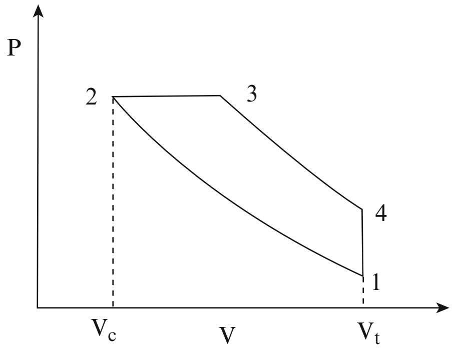

Fig. 2.3 Schematic sketch of a compression ignition engine

we get

$$\Omega = \frac{\Theta}{\varepsilon^{\gamma-1}} \tag{2.12}$$

Further, since $p_2 = p_3$ and $p_1 V_t^\gamma = p_2 V_c^\gamma$, we get

$$\frac{p_2}{p_1} = \frac{p_3}{p_1} = \varepsilon^\gamma \tag{2.13a}$$

Now,

$$\frac{p_3}{p_4} = \left(\frac{V_t}{V_c}\right)^\gamma = \left[\left(\frac{V_t}{V_c}\right)\left(\frac{V_c}{V_c}\right)\right]^\gamma = \left(\frac{\varepsilon}{\Omega}\right)^\gamma$$

and thus,

$$\frac{p_4}{p_1} = \frac{p_3}{p_1} . \frac{p_4}{p_3} = \Omega^\gamma \tag{2.13b}$$

Further,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{2.13c}$$

and

$$\frac{T_4}{T_1} = \left(\frac{T_4}{T_3}\right)\left(\frac{T_3}{T_1}\right) = \left(\frac{T_3}{T_1}\right)\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \Theta \left(\frac{\Omega}{\varepsilon}\right)^{\gamma-1} \Omega^\gamma \frac{p_4}{p_1} \tag{2.13d}$$

Thus, the specific work done (per unit mass) for various part processes are

$$\left|\frac{w_{12}}{c_v T_1}\right| = \left(\frac{T_2}{T_1}\right) - 1 = \varepsilon^{\gamma-1} - 1 \tag{2.14a}$$

$$\left|\frac{w_{34}}{c_{\mathrm{v}}T_1}\right| = \frac{T_3 - T_4}{T_1} = (\Theta - \Omega^{\gamma}) \tag{2.14b}$$

$$w_{41} = 0 \tag{2.14c}$$

Further, since $w_{23} = p_2 - (V_3 - V_2)/M_1$ and $M_1 = p_1 V_{\mathrm{t}}/(RT_1)$ is the mean of air inside the cylinder, we get

$$\begin{aligned}\left|\frac{w_{23}}{c_{\mathrm{v}}T_1}\right| &= (\gamma - 1)(p_2 - p_1)\frac{(V_3 - V_{\mathrm{c}})}{V_{\mathrm{t}}} = (\gamma - 1)(\Omega - 1)\varepsilon^{\gamma-1} \\ &= (\gamma - 1)\left(\Theta - \varepsilon^{\gamma-1}\right) \end{aligned} \tag{2.14d}$$

On the other hand, the *heat added* is

$$q_{\mathrm{a}} = c_{\mathrm{p}}(T_3 - T_2)$$

and the *heat rejected* is

$$q_{\mathrm{r}} = c_{\mathrm{v}}(T_4 - T_1)$$

from which, after some manipulation, we get

$$\left|\frac{q_{\mathrm{a}}}{c_{\mathrm{v}}T_1}\right| = \gamma\left(\Theta - \varepsilon^{\gamma-1}\right) \tag{2.15a}$$

and

$$\left|\frac{q_{\mathrm{r}}}{c_{\mathrm{v}}T_1}\right| = \Omega^{\gamma} - 1 \tag{2.15b}$$

From the *principle of conservation of energy*, the total heat exchanged must be equal to the sum of work of all part processes; as such, the overall specific work is

$$\frac{w}{c_{\mathrm{v}}T_1} = \gamma\left(\Theta - \varepsilon^{\gamma-1}\right) - (\Omega^{\gamma} - 1) \tag{2.16}$$

Thus, the thermodynamic cycle efficiency is given by the relation

$$\eta_{\mathrm{th}} = \frac{w}{q_{\mathrm{a}}} = 1 - \left[\frac{\Omega^{\gamma} - 1}{\gamma(\Theta - \varepsilon^{\gamma-1})}\right] = 1 - \left[\frac{\Omega^{\gamma} - 1}{\gamma\varepsilon^{\gamma-1}(\Omega - 1)}\right] \tag{2.17a}$$

Since $1 \leq \Omega \leq \varepsilon$ the two limiting cases are

$$\Omega \to 1 : \eta_{\mathrm{th}} \to \frac{\varepsilon^{\gamma-1} - 1}{\varepsilon^{\gamma-1}} \tag{2.17b}$$

which is the same as (2.8b) for the Otto cycle, and

$$\Omega \to \varepsilon : \eta_{\text{th}} \to 1 - \frac{\varepsilon^{\gamma-1} - 1}{\varepsilon^{\gamma-1}} - (\Omega - 1) \tag{2.17c}$$

Once again, as in the Otto cycle, the overall heat q depends actually on the fuel–air ratio. From (1.19), we have

$$\dot{m}_{\text{f}} \Delta H_{\text{p}} = \dot{m}_{\text{a}} c_{\text{p}} (T_3 - T_2) = \gamma \dot{m}_{\text{a}} c_{\text{v}} (T_3 - T_2)$$

and hence the *fuel–air ratio*,

$$f = \frac{\dot{m}_{\text{f}}}{\dot{m}_{\text{a}}} = \frac{c_{\text{p}} T_1}{\Delta \text{H}_{\text{p}}} \frac{T_3 - T_2}{T_1} = \frac{c_{\text{p}} T_1}{\Delta \text{H}_{\text{p}}} \left(\Theta - \varepsilon^{\gamma-1} \right) \tag{2.18}$$

Thus, although Θ, as in the Otto cycle, depends on the fuel–air ratio f, unlike the Otto cycle, it also depends on Ω and ε.

Further, as in (2.10), the *mean indicated pressure*, p_{m}, is given by the relation

$$p_{\text{m}} = w.M_1 / V_{\text{d}}$$

where M_1 is the mass of the medium (air) in the cylinder, which reduces the relation for the mean indicated pressure to

$$\frac{p_{\text{m}}}{p_1} = (\gamma - 1) \frac{\varepsilon}{\varepsilon - 1} \left[\gamma \left(\Theta - \varepsilon^{\gamma-1} \right) - (\Omega^{\gamma} - 1) \right] \tag{2.19}$$

For the power developed by the engine, (2.11) developed for the Otto cycle can be used. For the specific fuel consumption, we write SFC $= f/\Omega$, which reduces to

$$\text{SFC} = \frac{f}{(\eta_{\text{th}} q_{\text{a}})} = \frac{1}{\eta_{\text{th}} \Delta H_{\text{p}}} \tag{2.20}$$

which can be reduced further into a nondimensional form, $\text{SFC}^* = \text{SFC}.\Delta H_{\text{p}}$, which is again inversely proportional to the thermodynamic efficiency.

Now again, as for the Otto cycles, we take $f = 0$, $\Delta H_{\text{p}} = 4.2707 \times 10^4$ kJ/kg, $c_{\text{p}} = 1.005$ kJ/kg/K. and $T_1 = 298.15$ K. Thus, from (2.18), we have

$$\Theta - \varepsilon^{\gamma-1} = f \Delta H_{\text{p}} (c_{\text{p}} T_1) = 7.126358.$$

Further results are computed from (2.12), (2.16), (2.17a), and (2.19) and are given in Table 2.2. By comparing it with Table 2.1, we can see that the diesel cycle has a much better thermodynamic efficiency. In addition, because of the limitation in the value of the compression ratio for the Otto cycle, the diesel cycle has a much

Table 2.2 Some performance parameters for the diesel cycle

ε	4	5	6	8	10	15	20	35
Θ	8.867	9.030	9.174	9.420	9.638	10.080	10.440	11.020
Ω	5.093	4.743	4.480	4.102	3.837	3.412	3.150	2.828
p_m/p_1	4.033	6.672	8.443	10.750	12.240	14.060	15.770	17.800
$w/(c_v T_1)$	1.210	2.135	2.814	3.363	4.406	5.402	5.992	6.690
η_{th}	0.125	0.214	0.282	0.377	0.442	0.541	0.600	0.670
$\eta_{th,actual}$	–	–	0.200	0.275	0.330	0.420	0.470	0.500

higher compression ratio. For example, there are a reported compression ratio of 27.0 and a mean indicated pressure of 59.3 bar. These results are, of course, for an ideal cycle without any losses. In real cycles, there are losses due to incomplete entry of the combustion volume, incomplete combustion (especially for diesel cycles, where one can observe lots of smoke through the exhaust pipe, especially in cold engines), and heat transfer from the hot gas to the colder wall. Finally, the data for piston engines have been given in Appendix to this book, which may be studied in order to have an overall view of manufactured engines.

2.1.3 *Force and Moment Analysis for Piston Engines*

Because piston engines are reciprocating engines, the forces and moments acting on the piston and other components of the engine are not uniform. To develop engine power, these forces and moments have to be taken into account. For this purpose, we show the lengths and angles schematically in Fig. 2.4.

The following geometrical relations are obvious:

$$x = l(1 - \cos\varphi) + \frac{d}{2}(1 - \cos\theta) \tag{2.21a}$$

where θ is the *crank angle*, l is the *connecting rod length,* and d is the *stroke length*.

Since

$$X = \frac{d}{2} + l - x$$

it follows that

$$\frac{X}{l} = 1 + \frac{d}{2l} - \frac{x}{l} \tag{2.21b}$$

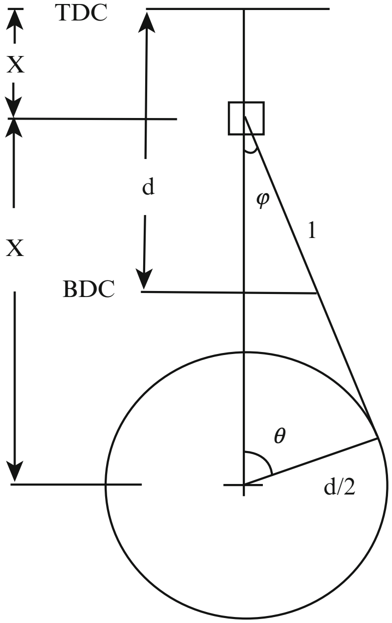

Fig. 2.4 Lengths and angles in a single-cylinder piston engine

Now,

$$l \sin \varphi = \frac{d}{2} \sin \theta \tag{2.21c}$$

and so it follows again that

$$\sin \varphi = \frac{d}{2l} \sin \theta$$

$$\cos \varphi = \sqrt{1 - \sin^2 \varphi} = \sqrt{1 - \left(\frac{d}{2l}\right) \sin^2 \theta}$$

and

$$\tan \varphi = \frac{\frac{d}{2l} \sin \theta}{\sqrt{1 - \left(\frac{d}{2l}\right)^2 \sin^2 \theta}}$$

Therefore,

$$\frac{x}{l} = \left[1 - \sqrt{1 - \left(\frac{d}{2l}\right)^2 \sin^2 \theta}\right] + \frac{d}{2l}(1 - \cos \theta) \tag{2.21d}$$

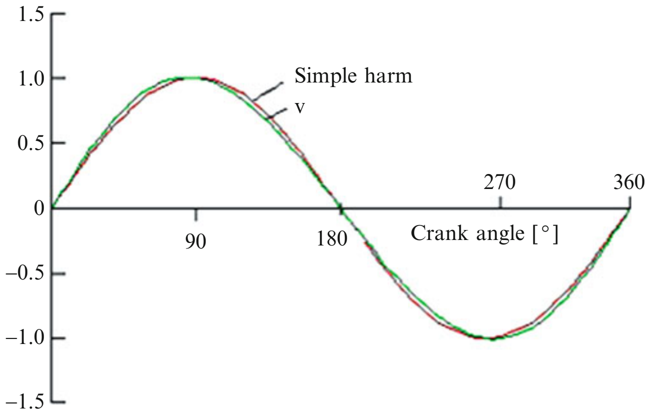

Fig. 2.5 Comparison of piston velocity for (l/d = 3) with simple harmonic motion

and

$$\frac{X}{l} = \frac{d}{2l}\cos\theta + \sqrt{1 - \left(\frac{d}{2l}\right)^2 \sin^2\theta} \tag{2.21e}$$

Noting that $\dot{\theta} = \pi n/30$ is constant, we differentiate (2.21d) to get the expression for velocity:

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = l\frac{\mathrm{d}(x/l)}{\mathrm{d}\theta}.\dot{\theta};\dot{\theta} = \frac{\pi n}{30};\frac{\mathrm{d}(x/l)}{\mathrm{d}\theta} = \frac{d}{2l}\sin\theta\left[1 - \frac{d}{2l}\frac{\cos\theta}{\sqrt{1 - \left(\frac{d}{2l}\sin\theta\right)^2}}\right]$$

and hence,

$$v = \frac{\pi n d}{60}\sin\theta\left[1 - \frac{d}{2l}\frac{\cos\theta}{\sqrt{1 - \left(\frac{d}{2l}\sin\theta\right)^2}}\right] \tag{2.22a}$$

Here the two limiting cases, described as top dead center (TDC) and bottom dead center (BDC) since these are extreme points of historically vertical machines, are

$$\begin{aligned} &\theta = 0^\circ(\mathrm{TDC}) : x = 0,\ X = 1 + d/2,\ v = 0 \\ &\theta\pi(\mathrm{BDC}) : x = d,\ X = 1 - d/2,\ v = 0 \end{aligned} \tag{2.22b}$$

and in between, the absolute piston velocity is maximum when the crank angle is 90°. The relative values of the piston's velocity, v, are plotted in Fig. 2.5, by taking the full equation (2.22a) for (l/d) =3, and also if only the first term in brackets is

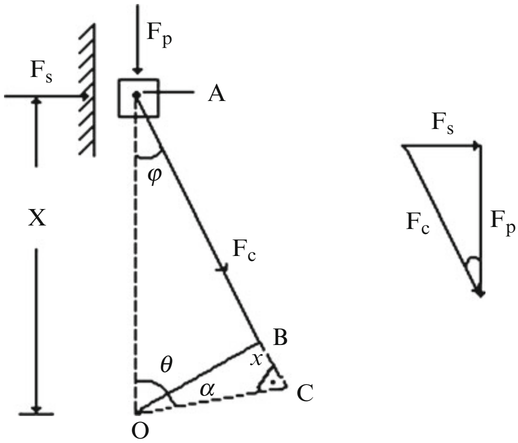

Fig. 2.6 Forces acting on cylinder wall, piston, and connecting rod

taken into consideration (simple harmonic motion). It is now possible to calculate the inertial and total forces on the piston; for the latter, it is necessary to have the data on the cylinder pressure, p, as a function of the crank angle.

Now, for the inertial and pressure force on the piston, F_{inertia} and F_{p}, respectively,

$$F_{\text{inertia}} = -M_{\text{p}}\dot{v} = -M_{\text{p}}\left(\frac{\pi^2 n^2 d}{1800}\right)\left[\cos\theta + \frac{d}{2l}\cos^2\theta\right]$$

$$F_{\text{p}} = p\frac{\pi}{4}D^2$$

and a sum of the two is the total force on the piston. In above expressions, M_{p} is the mass of the piston and D is the diameter of the piston. These forces, and two other forces—the side force on the cylinder, F_{s}, and the force on the connecting rod, F_{c}—are shown schematically in Fig. 2.6. From the condition of equilibrium of forces on the crank pin of the piston, we may write

$$F_{\text{c}} = F_{\text{s}} + F_{\text{p}};\ F_{\text{c}} = F_{\text{p}}/\cos\theta$$

Normally, the pressure force has to be evaluated separately, following which one may evaluate the torque moment. However, it can be safe to assume that at the time of combustion, the pressure force is opposite the inertial force.

The *torque moment* is evaluated with the help of the equation

$$\begin{aligned} M_{\text{t}} &= F_{\text{c}}x\sin\varphi = F_{\text{p}}X\tan\varphi \\ &= F_{\text{p}}l\left[1 - \frac{d^2}{30l^2}\sin^2\theta + \frac{d}{2l}\cos\theta\right]\frac{d}{2l}\frac{\sin\theta}{\sqrt{1-\left(\frac{d}{2l}\right)^2\sin^2\theta}} \end{aligned} \tag{2.22c}$$

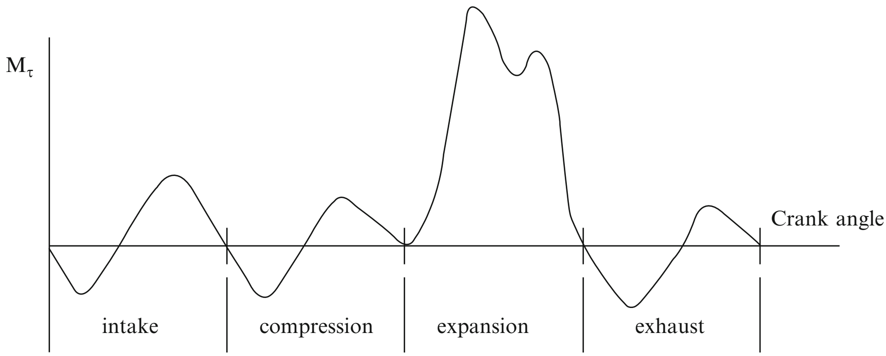

Fig. 2.7 Torque moment of the piston engine

It is obvious that for multicylinder engines the total torque moment has to be evaluated carefully by adding the moment of individual pistons, in order for the engine to run smoothly. The result of calculating (2.22c) is shown in Fig. 2.7, which shows that there are positive and negative torque moments in each part of the cycle, except in the expansion process (*power stroke*), where the positive torque moment has a large value.

2.1.4 *An Interactive Code for Performance of Otto/Diesel Engines*

The following program analyzes the performance of Otto and Diesel engines.

```
   PROGRAM OTTDIS
C  PERFORMANCE ROUTINE FOR OTTO/DIESEL CYCLE
15 FORMAT(1X,' OUTPUT UNIT NO.:',$)
16 FORMAT(1X,' DATE=',9A1)
17 FORMAT(1X,'PERFORMANCE ROUTINE FOR OTTO/DIESEL CYCLE')
   WRITE(7,15)
   READ(5,*)NOUT
   WRITE(NOUT,17)
   WRITE(NOUT,7)
10 WRITE(7,1)
1  FORMAT(1X,' READ EQUIV. RATIO,COMP. RATIO(2F6.2)=',$)
2  FORMAT(2F6.2)
   READ(5,2)EQUIV,CR
   IF(EQUIV.GE.1.0.AND.CR.GT.1.0)GOTO5
   WRITE(7,6)
6  FORMAT(1X,'PARAMETRIC ERROR : TRY AGAIN ')
7  FORMAT(1X,'EQUIV.RATIO,COMP.RATIO,T2/T1,(T3/T1,ETA,WNET,PME)OTTO'
   1,/,1X,'(T3/T1,OMEGA,ETA,WNET,PME) DIESEL')
```

```
    GOTO10
5   CONTINUE
    XA=CR**0.4
    XB=13.267*(CR-1.0)/(EQUIV*CR)
    TR=XA+XB
    TRD=XA+(XB/1.4)
    ETA=(XA-1.0)/XA
    WNET=(TR-XA)*(XA-1.0)/XA
    PME=WNET*2.5*CR/(CR-1.0)
    OM=TRD/XA
    ETAD=1.0-((OM**0.4-1.0)/((TRD-XA)*1.4))
    WNETD=((TRD-XA)*1.4)-(OM**0.4-1.0)
    PMED=WNETD*2.5*CR/(CR-1.0)
    WRITE(NOUT,20)EQUIV,CR,XA,TR,ETA,WNET,PME,TRD,OM,ETAD,
     WNETD,PMED
20  FORMAT(1X,1P12E10.3)
    GOTO10
    END
```

2.1.5 Ideal Turboprop Cycles

The engine being considered in this section is only partially a jet engine, but its main power delivery is through a rotating shaft to a propeller. As such, it can be considered in between a piston engine and the “pure” jet engines, like the ramjets, turbojets, and turbofans. In all of these jets, it is assumed, for the evaluation of an ideal cycle, that the working gas medium does not change its composition due to burning of the fuel (heat addition, for all practical purposes, is considered externally added at constant pressure), the specific heat of which is assumed to be constant. Further, all compressions and expansions are assumed to be isentropic.

Figure 2.8a is a schematic of a turboprop engine, and Fig. 2.8b is the corresponding schematic sketch of the ideal turboprop cycle. External air is assumed to enter through the inlet (state 1); it is compressed in the compressor isentropically to state 2; heat is added at constant pressure to state 3; and it is expanded isentropically in the turbine or nozzle to the ambient pressure (state 4). Normally, maximum expansion of the gas takes place in the turbine, which drives the compressor and (through a gear box) the propeller, but a small expansion (in pressure) is required in the nozzle for a smooth flow out of the engine. In this process, the nozzle develops a small thrust, the major thrust being through the propeller. Although in principle modern-day turboprops such as propfans can run for aircrafts flying at moderately high subsonic speeds, in the present case it is assumed that the flight speed and all other speeds inside the engine are small enough so that the stagnation state and the static state are practically the same.

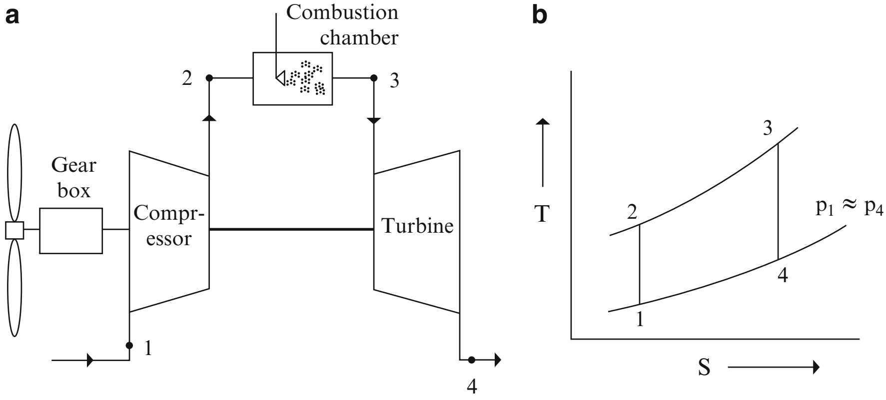

Fig. 2.8 Schematic sketch of a turboprop engine

As initial performance parameters, let's consider $\pi_c = p_2/p_1 = p_3/p_4$ as the *compression ratio* and $\Theta = T_3/T_1$ as the *temperature ratio* of the compressor given. Here we talk about the pressure compression ratio, as is usual in jet engines, instead of the volume compression ratio, as is usual for petrol engines. Obviously,

$$\pi_c = \Theta^{\gamma/(\gamma-1)}$$

Now

$$T_2/T_1 = T_3/T_4 = \pi_c^{(\gamma-1)/\gamma}$$

and thus,

$$\frac{T_4}{T_1} = \frac{\Theta}{\pi_c^{(\gamma-1)/\gamma}} \tag{2.23}$$

The technical work per unit mass gas in a flowing system for isentropic change of state is the difference of enthalpy; as such, for compression in the compressor and expansion in the turbine, these are

$$\left|\frac{w_{12}}{c_p T_1}\right| = \pi_c^{(\gamma-1)/\gamma} - 1 \tag{2.24a}$$

and

$$\left|\frac{w_{34}}{c_p T_1}\right| = \Theta \frac{\left(\pi_c^{(\gamma-1)/\gamma} - 1\right)}{\pi_c^{(\gamma-1)/\gamma}} \tag{2.24b}$$

On the other hand, heat added (process 2-3) and heat rejected (4-1) per unit mass are given by the expressions

$$\left|\frac{q_a}{c_p T_1}\right| = \left(\Theta - \pi_c^{(\gamma-1)/\gamma}\right) \tag{2.25a}$$

and

$$\left|\frac{q_r}{c_p T_1}\right| = \frac{\left(\Theta - \pi_c^{(\gamma-1)/\gamma}\right)}{\pi_c^{(\gamma-1)/\gamma}} \tag{2.25b}$$

Note that in the overall cycle analysis for an ideal case, the overall work is the same, whether it is computed from the difference of the two heats (heat added – heat rejected) or from consideration of all part work in various engine components (work developed in the turbine – work needed in the compressor). This overall work

$$\begin{aligned}\frac{|w|}{c_p T_1} = \frac{|w_{34}| - |w_{12}|}{c_p T_1} = \frac{|q|}{c_p T_1} = \frac{|q_a| - |q_r|}{c_p T_1} \\ = \left(\Theta - \pi_c^{(\gamma-1)/\gamma}\right)\left(\frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\pi_c^{(\gamma-1)/\gamma}}\right)\end{aligned} \tag{2.26a}$$

goes to drive the propeller. Now the thermodynamic efficiency is given by the ratio of overall work to heat added, and thus,

$$\eta_{th} = \frac{|w|}{|q_a|} = \left(\frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\pi_c^{(\gamma-1)/\gamma}}\right) \tag{2.26b}$$

Now two special limiting cases for the given Θ are considered. The first one is when $\pi_c \to 1$, for which $|q_a| = |q_r| = \Theta - 1, w = 0, \eta_{th} \to 0$. In the other limiting case, let $\pi_c = \Theta^{\gamma/(\gamma-1)}$. Then $|q_a| = |q_r| = 1, w = 0$, and $\eta_{th} = (\Theta - 1)/\Theta$, which is the maximum efficiency of a Carnot cycle. Between these two limiting cases, w is maximum (optimum π_c). To find this, let $x = \pi_c^{(\gamma-1)/\gamma}$, and the maximum value of w is obtained by differentiating (2.26a) with respect to x and setting it equal to 0. The result is

$$\left(\pi_c^{(\gamma-1)/\gamma}\right)_{opt} = \sqrt{\Theta} \tag{2.27a}$$

and

$$\left[\frac{w}{c_p T_1}\right]_{opt} = \left(\sqrt{\Theta} - 1\right)^2 \tag{2.27b}$$

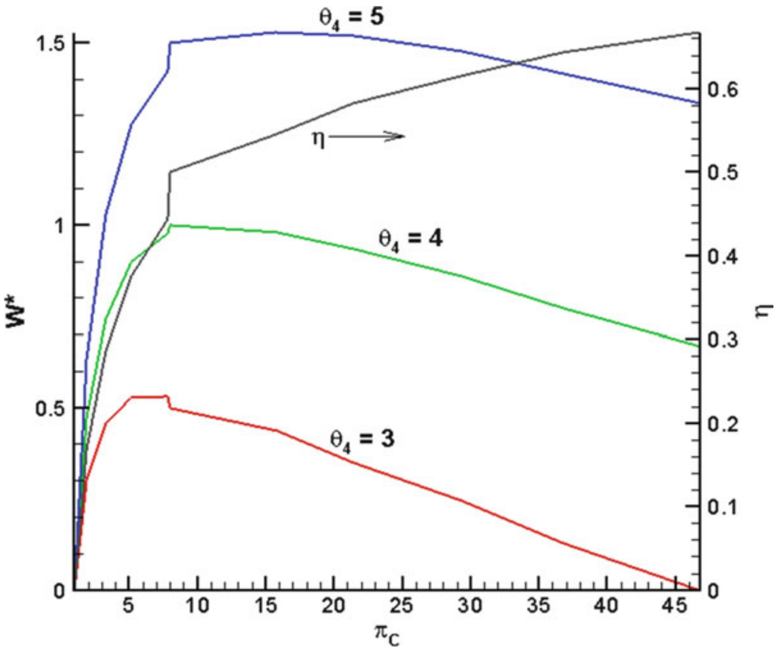

Fig. 2.9 Different values (as described in the text) vs. π_c

Table 2.3 $[w/(c_p T_1)]_{opt}$ and Corresponding $\pi_{c,opt}$ vs. Θ

Θ	3	4	5	6
π_c	46.760	128.000	279.510	529.090
$\pi_{c,opt}$	6.838	11.313	16.718	23.002
$[w/(c_p T_1)]_{opt}$	0.536	1.000	1.528	2.101

The results of calculating η_{th} and $w/(c_p T_1)$, the latter for different values of Θ, are presented vs. π_c in Fig. 2.9. It is seen that η_{th} increases monotonically with π_c, whereas $w/(c_p T_1)$ changes with π_c with a maximum (optimum) for different values of Θ. The optimum values of $[w/(c_p T_1)]_{opt}$ and the corresponding $\pi_{c,opt}$ are given in the Table 2.3.

As per the analysis of these results, it is evident that higher the compression ratio, the higher is the thermodynamic efficiency. However, increasing the compression ratio too much beyond the optimum compression ratio for a given temperature ratio results in a lower work output.

The fuel–air ratio is now given by (1.19a), and we get

$$f = \frac{c_p T_1}{\Delta H_p}\left(\frac{T_3}{T_1} - \frac{T_2}{T_1}\right) = \frac{c_p T_1}{\Delta H_p}\left(\Theta - \pi_c^{(\gamma-1)/\gamma}\right) \tag{2.28a}$$

The nondimensional *specific fuel consumption* is

$$\mathrm{SFC} = \frac{f}{(\eta_{\mathrm{th}} q_{\mathrm{a}})} = \frac{1}{(\eta_{\mathrm{th}} \Delta H_{\mathrm{p}})} \to \mathrm{SFC}^{*} = \mathrm{SFC}.\Delta H_{\mathrm{p}} = \frac{1}{\eta_{\mathrm{th}}} \tag{2.28b}$$

and is a measure of the thermodynamic efficiency.

It is also quite evident that by increasing the compression ratio as much as possible up to the limit of $\pi_{\mathrm{c}} > \pi_{\mathrm{c,opt}} > \pi_{\mathrm{c,max}} = \Theta^{\gamma/(\gamma-1)}$, the thermodynamic efficiency can increase up to the maximum of the Carnot cycle efficiency, $\eta_{\mathrm{Carnot}} = 1 - (1/\Theta) = 1 - (T_1/T_3)$, only, the specific work output is zero under the limit, and the maximum specific output is obtained under the condition of (2.27b). For this maximum, all three—that is, the heat addition, the heat rejection, and the work output—are zero, and no fuel can be introduced, but the thermodynamic efficiency is nonzero. On the other hand, for the minimum compression ratio, the heat added = heat rejected is nonzero, but both the work output and the thermodynamic efficiency are zero. For a proper understanding of turboprop engines, the data on manufactured engines given in Appendix may be studied.

2.2 Jet Engines

So far we have discussed engines that operate at comparatively low flight speeds, so that the static and stagnation states of air can be considered to be the same. Jet engines, including ramjets, straight turbojets, bypass jets, and fanjets not only operate at higher flight speeds than the propeller-driven engines, but they also have to develop high jet speeds at the exit nozzle. Later we will show that for the extraction or introduction of work in a turbine or a compressor, the change in *stagnation states* has to be considered, rather than the change of *static states*. Therefore, when analyzing jet engines, the usual concepts of the stagnation pressure and the stagnation temperature are introduced. In addition, here we again make the assumption of ideal gas (c_{p} = constant) and ideal thermodynamic cycle (compression and expansion as isentropic, and heat addition and heat rejection at constant pressure). Further, no change in the composition of gas (air) is assumed in the combustion chamber. From the equation of energy, we have

$$c_{\mathrm{p}} T^{o} = c_{\mathrm{p}} T + u^{2}/2 \tag{2.29a}$$

where T^{o} is the stagnation temperature, T is static temperature, u is (one-dimensional) gas speed, and c_{p} is the specific heat at constant pressure. With $c_{\mathrm{p}}/R = \gamma/(\gamma - 1)$, where R is the *gas constant*, and from the definition of Mach number $M = u/\sqrt{\gamma R T}$, we write

$$\frac{T^{o}}{T} = 1 + \frac{(\gamma - 1)}{2} M^{2} \tag{2.29b}$$

$$\frac{p^o}{p} = \left(\frac{T^o}{T}\right)^{\frac{\gamma}{\gamma-1}} \tag{2.29c}$$

We now introduce the definition of nondimensional stagnation temperature and stagnation pressure,

$$\Theta = \frac{T^o}{T_\infty} \tag{2.30a}$$

and

$$\delta = \frac{p^o}{p_\infty} \tag{2.30b}$$

where the subscript "∞" refers to the ambient state of air, and those of stagnation temperature and stagnation pressure ratio across components as

$$\tau = \frac{T^{o\prime\prime}}{T^\prime}, \tag{2.30c}$$

$$\pi = \frac{p^{o\prime\prime}}{p^{o\prime}} \tag{2.30d}$$

Therefore, at the inlet,

$$\Theta_\infty = \frac{T^o_\infty}{T_\infty} = 1 + \frac{(\gamma-1)}{2} M^2_\infty \tag{2.31a}$$

$$\delta_\infty = \frac{p^o_\infty}{p_\infty} = \Theta_\infty^{\gamma/(\gamma-1)} \tag{2.31b}$$

Now we'll start with the simplest of the jet engines, that is, the *ramjet*, which does not have any moving parts but operates only at high flight speeds.

2.2.1 Ideal Ramjet Cycle

Figure 2.10a shows schematically a ramjet engine and Fig. 2.10b depicts the *ideal ramjet cycle process* in a (T, s) diagram. It has an inlet, which actually has a much more complicated shape than the simple divergent diffusor shown in the figure to operate efficiently at the Mach number of operation, and it must suitably be altered at flight design and off-design Mach numbers. Fuel is introduced in the combustion

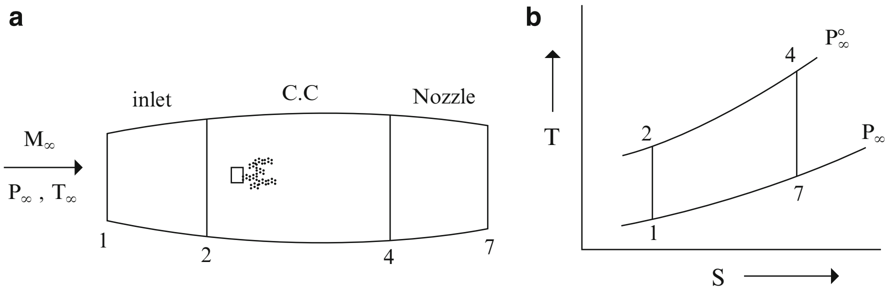

Fig. 2.10 Sketch of (**a**) a ramjet engine, and (**b**) a thermodynamic cycle process in a (T, s) chart

chamber at state 2, a complete combustion is assumed at state 4, and the gas is expelled through the nozzle (normally, it is a convergent–divergent nozzle for a supersonic exit) to reach ambient pressure at state 7.

Let's consider the role of the basic parameters for the ramjet: the approaching flow Mach number M_∞ and $\Theta_4 = T_4^o/T_\infty^o$.

Now,

$$\Theta_4 = \Theta_1 = \Theta_2 = \frac{T_\infty^o}{T_\infty} = \frac{T_1^o}{T_\infty} = \frac{T_2^o}{T_\infty} = 1 + \frac{(\gamma - 1)}{2} M_\infty^2 \tag{2.32a}$$

and

$$\delta_4 = \delta_1 = \delta_2 = \frac{p_\infty^o}{p_\infty} = \frac{p_1^o}{p_\infty} = \frac{p_2^o}{p_\infty} = \frac{p_4^o}{p_\infty} = \frac{p_4^o}{p_\infty} = \Theta_\infty^{\gamma/(\gamma-1)} \tag{2.32b}$$

Obviously, $(\Theta_4 - \Theta_\infty)$ depends on the fuel–air ratio and the efficiency with which the fuel is burned in the combustion chamber, and it will be maximum at a stoichiometric fuel–air ratio. A discussion on these has been given later in this section. One could therefore perhaps think that especially for ramjets with no moving parts, it should be possible to have a large Θ_4 for a large Θ_∞. It needs to be pointed out, however, that the static combustion chamber is limited by the temperature at which substantial dissociation may take place in carbon dioxide and oxygen molecules, which may be around 3,500 K at a gas pressure of 1 bar and may be substantially lower at lower pressures (higher altitude). Since the combustion chamber pressure is directly proportional to ambient pressure at a constant flight Mach number, but otherwise depends on about the sixth power of the flight Mach number, the dissociation temperature's dependence on the ambient pressure may not be very critical. However, a high Mach number is also associated with a very high-shock-pressure loss in the inlet region, especially if the nozzle is not operating under a fully isentropic change of state. It is therefore necessary to judiciously analyze the combination of flight Mach number, ambient temperature, and ambient pressure in an integrated way along the design of the inlet region.

Since

$$\frac{T_4^o}{T_\gamma} = \left(\frac{p_4^o}{p_\gamma}\right)^{(\gamma-1)/\gamma} = \Theta_\infty$$

we get

$$\frac{T_\gamma}{T_\infty} = \left(\frac{T_4^o}{T_\infty}\right)\left(\frac{T_\gamma}{T_4^o}\right) = \frac{\Theta_4}{\Theta_\infty} \tag{2.32c}$$

The *jet exit speed* is, therefore,

$$u_7 = \sqrt{2c_\mathrm{p}(T_4 - T_7)} = \sqrt{2c_\mathrm{p}T_\infty\left(\Theta_4 - \frac{T_7}{T_\infty}\right)} \tag{2.33a}$$

Further,

$$u_\infty = M_\infty\sqrt{\gamma R T_\infty} = M_\infty\sqrt{(\gamma-1)c_\mathrm{p}T_\infty} \tag{2.33b}$$

and the ratio of the two speeds is

$$\frac{u_7}{u_\infty} = \frac{1}{M_\infty}\sqrt{\frac{2}{(\gamma-1)}\frac{\Theta_4(\Theta_\infty - 1)}{\Theta_\infty}} \tag{2.33c}$$

Note that for $u_\infty \to 0, M_\infty \to 0, \Theta_\infty \to 1$, and $u_7 \to 0$, but the ratio $(u_7/u_\infty) \to \sqrt{\Theta_4}$ is a finite quantity.

The heat added in the combustion chamber and the heat rejected are

$$|q_\mathrm{a}| = c_\mathrm{p}\left(T_4^o - T_2^o\right) \quad \text{and} \quad |q_\mathrm{r}| = c_\mathrm{p}(T_7 - T_\infty)$$

from which we write

$$\frac{|q_\mathrm{a}|}{c_\mathrm{p}T_\infty} = \Theta_4 - \Theta_\infty \tag{2.34a}$$

and

$$\frac{|q_\mathrm{r}|}{c_\mathrm{p}T_\infty} = \frac{T_7}{T_\infty} - 1 = \frac{\Theta_4 - \Theta_\infty}{\Theta_\infty} \tag{2.34b}$$

Thus, the *overall specific work* in the system is

$$\frac{|w|}{c_\mathrm{p}T_\infty} = \frac{|q_\mathrm{a}| - |q_1|}{c_\mathrm{p}T_\infty} = \frac{(\Theta_4 - \Theta_\infty)(\Theta_\infty - 1)}{\Theta_\infty} \tag{2.34c}$$

and the *thermodynamic efficiency* is

$$\eta_\mathrm{th} = \frac{|w|}{|q_\mathrm{a}|} = \frac{(\Theta_\infty - 1)}{\Theta_\infty} \tag{2.35a}$$

Note further from (2.33a) and (2.33b) that

$$\frac{1}{2}\left(u_7^2 - u_\infty^2\right) = c_\mathrm{p}T_\infty = \frac{(\Theta_4 - \Theta_\infty)(\Theta_\infty - 1)}{\Theta_\infty} = |w| \tag{2.34d}$$

and hence the effective work gained out of the ramjet is equal to the increase in the kinetic energy. This result is important, since for the first time in a jet machine it is shown that work output is not with the help of a mechanical engine, but through the change in kinetic energy.

For $M_\mathrm{inf} \to 0, \Theta_\infty \to 1$, both w and η_th tend to zero. On the other hand, for $\Theta_4 = \Theta_\infty$ (no fuel introduced), $w = 0$, but η_th is the maximum value of the Carnot cycle efficiency. Between these two extreme cases, we can examine the condition for maximum specific work by differentiating (2.33b) with respect to Θ_∞ and set it equal to zero to get the result that $\lfloor w/(c_\mathrm{p}T_\infty)\rfloor$will be maximum, and $\Theta_\infty = \sqrt{\Theta_4}$. Substituting back into (2.34c), we therefore get the maximum work

$$[w/(c_\mathrm{p}T_\infty)]_\mathrm{max} = \left(\sqrt{\Theta_4} - 1\right)^2 \tag{2.36}$$

Now, the *propulsive efficiency* is given by the relation

$$\eta_\mathrm{p} = \frac{2}{\left[1 + \left(\frac{u_7}{u_\infty}\right)\right]} = \frac{2}{\left[1 + \sqrt{\frac{\Theta_4}{\Theta_\infty}}\right]} \tag{2.37a}$$

Since $\Theta_4 > \Theta_\infty$, $\eta_\mathrm{p} < 1$. On the other hand, for $\Theta_\infty > 1$, $\eta_\mathrm{p} = 2/\lfloor 1 + \sqrt{\Theta_4}\rfloor$ and $\Theta_4 = \Theta_\infty$, $\eta_\mathrm{p} = 1$. It is evident that for $M_\infty \to 0, \Theta_\infty \to 1$, and the jet speed goes to zero, but η_p is finite.

Since the thrust is given by the expression

$$F = \dot{m}_\mathrm{a}(u_7 - u_\infty) = \dot{m}_\mathrm{a}u_\infty\left(\frac{u_7}{u_\infty} - 1\right)$$

we can write for the *specific thrust,* after some manipulation,

$$\frac{F}{\dot{m}_\mathrm{a}u_\infty} = \sqrt{\frac{2}{\gamma - 1}(\Theta_\infty - 1)}\left[\sqrt{\frac{\Theta_4}{\Theta_\infty}} - 1\right] = M_\infty\left[\sqrt{\frac{2\Theta_4}{2 + (\gamma - 1)M_\infty^2}} - 1\right]$$

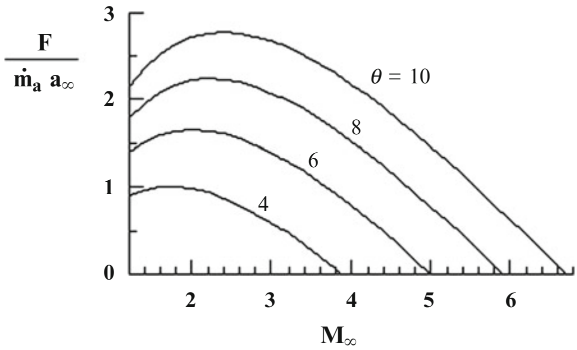

Fig. 2.11 Specific thrust vs. flight Mach number

From the above equation, it can be seen that the specific thrust is equal to zero if $M_\infty \to 0, \Theta_\infty \to 1$, or $\Theta_4 = \Theta_\infty$. While the first case refers to zero flow, where the thrust has to be zero anyway, the second condition refers to the case that by just gas dynamic compression the combustion chamber temperature is reached and no additional fuel can be added. In between is the condition for maximum specific thrust. By differentiating the above equation with respect to M_∞ and setting it equal to zero, we find the specific thrust optimum (maximum) if $\Theta_\infty = \Theta_4^{1/3}$ and the maximum specific thrust becomes

$$\left(\frac{F}{\dot{m}_a u_\infty}\right)_{\max} = \sqrt{\frac{2}{(\gamma-1)}\left(\Theta_4^{1/3}-1\right)^3} \tag{2.37b}$$

We should mention, however, that since the mass flow rate is directly proportional to the flight speed (or flight Mach number), determining the flight Mach number at which the thrust is maximum requires a slightly different modification of the specific thrust equation. In this case, the dependent variable is $F/(p_{\mathrm{int}}A_{\mathrm{inlet}})$, where A_{inlet} is the inlet diffuser entry cross section. Both of these are plotted in Figs. 2.11 and 2.12 as a function of the flight Mach number, respectively. Figure 2.13 shows the results of calculating the specific work as a function of the flight Mach number.

We therefore have two values of Θ_∞ for optimization: One is for the specific thrust if $\Theta_\infty = \Theta_4^{1/3}$, and the other is for maximum specific work if $\Theta_\infty = \sqrt{\Theta_4}$. The two corresponding optimum Mach numbers are designated as $M_{1\infty,\mathrm{opt}}$ and $M_{2\infty,opt}$, respectively. These are given in Table 2.4.

Now, we calculate the fuel–air ratio from (1.19) to get

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{c_p \Delta T}{\Delta H_p} = \frac{c_p T_\infty}{\Delta H_p}(\Theta_4 - \Theta_\infty) \tag{2.38a}$$

which decreases continuously from $\Theta_\infty = 1$ (that is, $M_\infty \to 0$) to $\Theta_\infty = \Theta_4$. On the other hand, the specific thrust first increases and then decreases. Usually, the

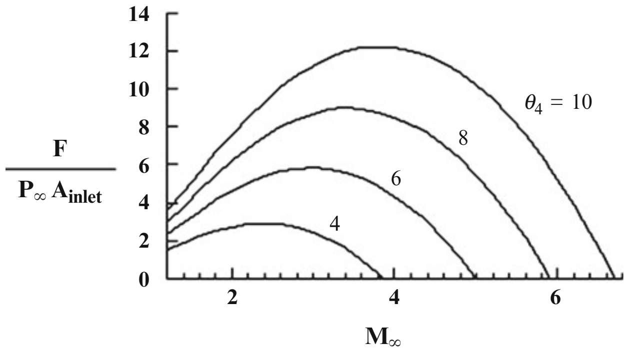

Fig. 2.12 Thrust per unit inlet area vs. flight Mach number

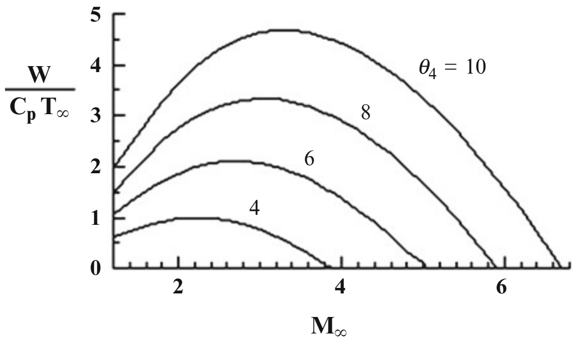

Fig. 2.13 Specific work vs. flight Mach number

Table 2.4 Two corresponding optimum Mach numbers for different Θ_4

Θ_4	3	4	5	6	10
$M_{1\infty,\mathrm{opt}}$	1.913	2.236	2.486	2.692	3.288
$\lfloor w_{\mathrm{opt}}/(c_{\mathrm{p}}T_{\infty})\rfloor$	0.536	1.000	1.527	2.101	4.675
$M_{2\infty,\mathrm{opt}}$	1.487	1.713	1.884	2.021	2.402
$\left(\frac{F}{\dot{m}_{\mathrm{a}}a_{\infty}}\right)_{\mathrm{opt}}$	0.658	1.007	1.338	1.652	2.773

specific fuel consumption [kg/J] is computed by dividing the fuel flow rate $\dot{m}_{\mathrm{f}}$ [kg/s] by the work produced per kg of air [J/kg]. However, for aircraft engine applications, it is computed by dividing the fuel mass flow rate by the thrust [N], and hence the specific fuel consumption $[\mathrm{kg}/(\mathrm{N.s}) \equiv \mathrm{s/m}]$ is defined as

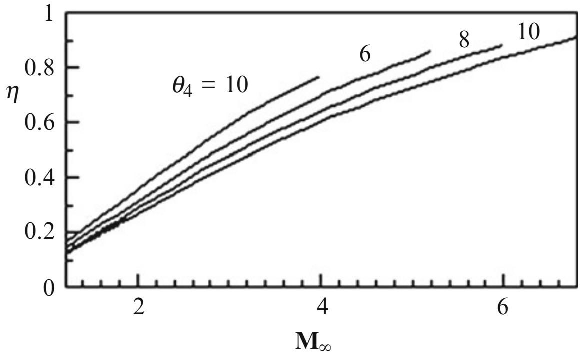

Fig. 2.14 Overall efficiency of ramjet vs. flight Mach number and Θ_4 as parameter

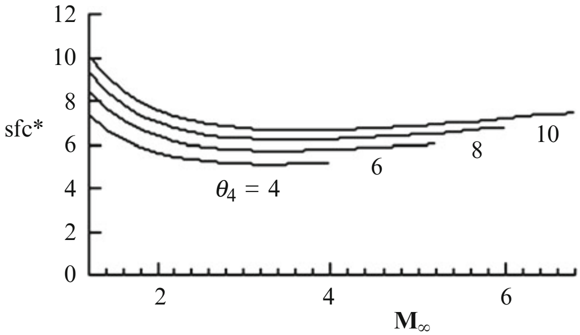

Fig. 2.15 Nondimensional SFC of ramjet vs. flight Mach number and Θ_4 as parameter

Thus, the, which is the fuel mass flow rate per unit thrust, is a somewhat complicated relation as follows:

$$\mathrm{SFC} = \frac{\dot{m}_\mathrm{f}}{F} = \frac{f}{M_\infty a_\infty \left(\frac{u_7}{u_\infty}\right)} = \frac{f}{a_\infty \left[\frac{2\Theta_4}{(\gamma-1)} \frac{(\Theta_\infty - 1)}{\Theta_\infty} - M_\infty\right]} \tag{2.38b}$$

For $M_\infty = 0$, both f ($\Theta_4 = \Theta_\infty$: if there is no airflow, then there is also no fuel flow either) and $F = 0$, and SFC is indeterminate. From (2.38b) we can now define a *nondimensional specific fuel consumption*:

$$\mathrm{SFC}^* = \mathrm{SFC}.a_\infty = \frac{f}{a_\infty \left[\sqrt{\frac{2\Theta_4}{(\gamma-1)} \frac{(\Theta_\infty - 1)}{\Theta_\infty}} - M_\infty\right]} \tag{2.38c}$$

The optimum values are given in Table 2.4. Further various parameters have been computed as a function of M_∞ and Θ_4 (all calculations have been done for $\gamma = 1.4$), as shown in Figs. 2.14 and 2.15. While the thermodynamic efficiency for a

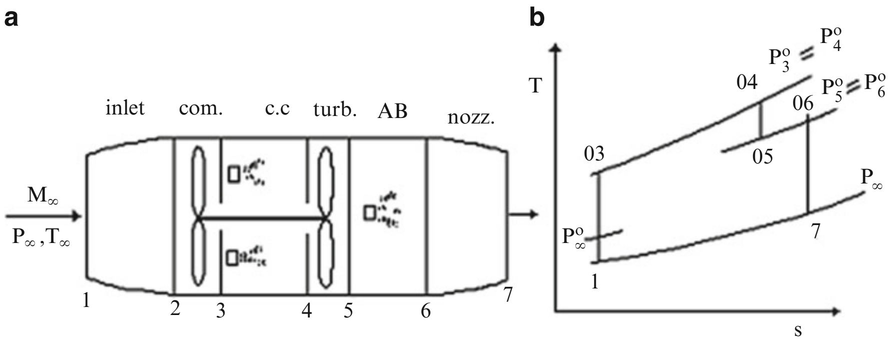

Fig. 2.16 Schematic sketch of an ideal straight-jet engine with (T, s) chart

ramjet is a function of the flight Mach number alone (the higher the flight Mach number, the better the thermodynamic efficiency), the overall efficiency depends on both M_∞ and Θ_4. Similarly, SFC* depends on both.

2.2.2 *Ideal Straight-Jet Cycle with Afterburner*

A schematic sketch of the straight-jet engine is shown again in Fig. 2.16a, and the ideal process is shown in Fig. 2.16b. Once again, it has an inlet, which actually may look quite different because of a supersonic inlet flow, compression in a compressor, expansion in a turbine, a combustion chamber, an afterburner, and a nozzle. For example, a supersonic inlet may have a built-in multishock system designed to have a minimum loss at the inlet, and a supersonic exit nozzle of convergent–divergent type. Similarly, both the compressor and turbine will have multiple stages with many blades in each stage, the design of which will depend on the axial or radial flow of the gas. Again, the combustion chamber and the afterburner are designed to have the best possible combustion characteristics with minimal pressure loss. In this system, air at state (p_∞, T_∞) is compressed in the inlet diffuser to state 02 (state: $0\infty = 01 = 02$); it is compressed further with the help of the compressor to state 03; fuel is introduced and burned at constant (stagnation) pressure to state 04; it is expanded through a turbine to state 05 (for without the afterburner operation, $05 = 06 = 07$, and for the case of the afterburner operation to reach state $06 = 07$); and, finally, the exhaust gas is expanded in the nozzle to reach state 07 at the ambient pressure p_∞. Thus, the state 04 is the turbine inlet state.

Let's assume the parameters given are $\pi_c = p_3^o/p_2^o$ as the compression ratio, $\Theta_4 = T_4^o/T_\infty$ as the temperature ratio, M_∞ as the approaching flow Mach number, and

$$\Delta\Theta_{AB} = (\Theta_6 - \Theta_5) = \frac{\left(T_6^o - T_5^o\right)}{T_\infty}$$

as the stagnation temperature increase in the afterburner (which is, of course, zero if the afterburner is not in operation). Thus, even without the afterburner, we have an

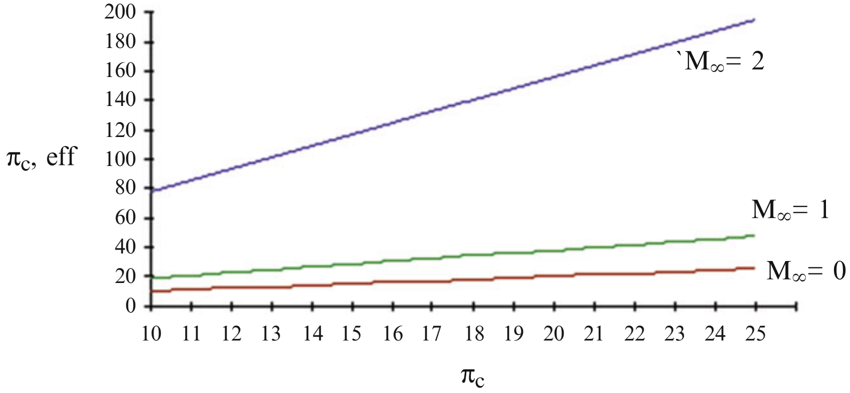

Fig. 2.17 Effective compression ratio vs. compression ratio with Mach number as parameter

increase in operational parameters by 1 over these for ramjets, although, as we will show, by defining an effective compression ratio $\pi_{c,eff}$, we would reduce the number of operational parameters to 2 again (without afterburner), excluding M_∞.

Now,

$$\Theta_\infty = \Theta_1 = \Theta_2 = \frac{T^o_\infty}{T_\infty} = \frac{T^o_1}{T_\infty} = \frac{T^o_2}{T_\infty} = 1 + \frac{\gamma - 1}{2} M^2_\infty \tag{2.39a}$$

and

$$\delta_\infty = \delta_1 = \delta_2 = \frac{p^o_\infty}{p_\infty} = \frac{p^o_1}{p_\infty} = \frac{p^o_2}{p_\infty} = \Theta_\infty^{\gamma/(\gamma-1)} \tag{2.39b}$$

Further,

$$\delta_3 = \delta_4 = \frac{p^o_3}{p_\infty} = \frac{p^o_4}{p_\infty} = \pi_c \delta_\infty = \pi_c \Theta_\infty^{\gamma/(\gamma-1)} \tag{2.39c}$$

which we call the *effective compression pressure ratio*, $\pi_{c,eff}$, and

$$\Theta_3 = \frac{T^o_3}{T_\infty} = \pi_c \delta_\infty = \delta^{(\gamma-1)/\gamma} = \Theta_\infty \pi_c^{(\gamma-1)/\gamma} \tag{2.39d}$$

which we call the *effective compression temperature ratio*, $\tau_{c,eff}$ (for the ideal case only, $p^o_3 = p^o_4$, since here there would not be any pressure loss being considered due to friction and heat addition in the combustion chamber, but in the actual case, $p^o_3 \geq p^o_4$). It is strongly dependent on the approaching flow Mach number and compression pressure ratio and may indicate the limitation in both for a given temperature ratio. Hence, the relationship is plotted in Fig. 2.17.

Further, the temperature ratios across the compressor and combustion chamber are

$$\tau_c = \frac{T_3^o}{T_2^o} = \frac{\Theta_3}{\Theta_2} = \frac{\Theta_3}{\Theta_\infty} = \pi_c^{(\gamma-1)/\gamma} \geq 1 \tag{2.40a}$$

and

$$\tau_b = \frac{T_4^o}{T_3^o} = \frac{\Theta_4}{\Theta_3} = \frac{\Theta_4}{\tau_c \Theta_\infty} \tag{2.40b}$$

and the *effective compression pressure ratio* and *effective temperature ratio* across the compressor are, respectively,

$$\pi_{c,\text{eff}} = \pi_c \delta_\infty = \delta_3; \tag{2.40c}$$

$$\tau_{c,\text{eff}} = \tau_c \Theta_\infty = \Theta_3 \tag{2.40d}$$

Since the turbine and the compressor sit on the same shaft, work required by the compressor is supplied by the turbine, and thus,

$$T_3^o - T_2^o = T_4^o - T_5^o$$

from which it follows that

$$\Theta_3 - \Theta_2 = \Theta_\infty(\tau_c - 1) = \Theta_4 - \Theta_5 \tag{2.40e}$$

Therefore,

$$\Theta_3 = \frac{T_5^o}{T_\infty} = \Theta_4 - \Theta_\infty(\tau_c - 1) = \Theta_4 + \Theta_\infty - \tau_{c,\text{eff}} \tag{2.40f}$$

and the (stagnation) temperature ratio across the turbine is

$$\tau_t = \frac{\Theta_5}{\Theta_4} = 1 - \Theta_\infty \frac{(\tau_c - 1)}{\Theta_4} \leq 1 \tag{2.40g}$$

since $\tau_c \geq 1$.

At the lower limit, $\tau_c = \pi_c = 1$, there is no compression in the compressor and no expansion in the turbine ($\pi_t = 1, \tau_t = 1$). On the other hand, it is possible that the (given) turbine inlet temperature is reached by compression alone and that there is no injection of the fuel ($\Theta_4 = \Theta_3 = \Theta_\infty \tau_c$).

We will now first analyze the rest of process *without the afterburner*, for which $\Theta_5 = \Theta_6 = \Theta_7$.

Now, since

$$\begin{aligned}\frac{T_5^o}{T_7} &= \left(\frac{p_5^o}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} = \delta_5^{\frac{\gamma-1}{\gamma}} = \tau_t\delta_4^{\frac{\gamma-1}{\gamma}} = \tau_t\delta_5^{\frac{\gamma-1}{\gamma}} = \tau_t\Theta_3 \\ &= \tau_t\tau_c\Theta_\infty = \left[1 - \Theta_\infty\frac{(\tau_c - 1)}{\Theta_4}\right]\tau_c\Theta_\infty = \tau_c\Theta_\infty\frac{\Theta_5}{\Theta_4}\end{aligned}$$

we get

$$\frac{T_7}{T_\infty} = \frac{T_5^o}{T_\infty}\cdot\frac{T_7}{T_5^o} = \frac{\Theta_5\Theta_4}{\tau_c\Theta_\infty\Theta_5} = \frac{\Theta_4}{\tau_c\Theta_\infty} = \frac{\Theta_4}{\tau_{c,\text{eff}}} \tag{2.40h}$$

Heat added to and *rejected* by the system are

$$\begin{aligned}|q_a| &= c_p\left(T_4^o - T_4^o\right) = c_pT_\infty(\Theta_4 - \Theta_3) = c_pT_\infty(\Theta_4 - \tau_{c,\text{eff}}) \\ &= c_pT_\infty(\Theta_5 - \Theta_\infty\tau_c - \Theta_\infty - \tau_c\Theta_\infty) = c_pT_\infty\end{aligned}$$

and

$$|q_r| = c_p(T_7 - T_\infty) = c_pT_\infty\left(\frac{T_7}{T_\infty} - 1\right) = c_pT_\infty\frac{(\Theta_4 - \tau_{c,\text{eff}})}{\tau_{c,\text{eff}}} \tag{2.41b}$$

Therefore, the nondimensional work and thermodynamic efficiency are

$$\frac{w}{c_pT_\infty} = \frac{(|q_a| - |q_r|)}{c_pT_\infty} = \left(\Theta_4 - \tau_{c,\text{eff}}\right)\frac{\left(\tau_{c,\text{eff}} - 1\right)}{\tau_{c,\text{eff}}} \tag{2.41c}$$

and

$$\eta_{th} = \frac{w}{q_q} = \frac{\left(\tau_{c,\text{eff}} - 1\right)}{\tau_{c,\text{eff}}} = \frac{\pi_{c,\text{eff}}^{\frac{\gamma-1}{\gamma}} - 1}{\pi_{c,\text{eff}}^{\frac{\gamma-1}{\gamma}}} \tag{2.41d}$$

which for $\tau_{c,\text{eff}} = 1$ (no compression in inlet or compressor), both w and η_{th} are zero, for $\left(\Theta_4 - \tau_{c,\text{eff}}\right) = 0$ (no fuel injection), $w = 0$, but η_{th} attains the maximum possible efficiency of a Carnot cycle. It is interesting to note from (2.41d) that the two limiting values of the thermal efficiency are $\pi_{c,\text{eff}} = 1 : \eta_{th} = 0$ and $\pi_{c,\text{eff}} \to \infty : \eta_{th} \to 1$, and thus it would appear that the compression ratio must be as large as possible. However, that is not possible, since the maximum effective compression ratio is restricted by the maximum given temperature ratio with no fuel injection, where again the specific work goes to zero. Hence, the two limits of $\tau_{c,\text{eff}}$ are $1 \le \tau_{c,\text{eff}} \le \Theta_4$. At the second limit, $\tau_{c,\text{eff}} = 1 : \eta_{th} = (\Theta_4 - 1)/\Theta_4$thermodynamic efficiency of the *Carnot cycle* (*Carnot cycle efficiency*).

Between these two extreme cases, we can now find the condition for maximum specific work by differentiating (2.41c) with respect to $\tau_{c,eff}$ and setting it equal to zero to get

$$\left(\tau_{c,eff}\right)_{opt} = \sqrt{\Theta_4} \tag{2.42a}$$

and the optimum (maximum) nondimensional work is

$$\frac{w_{opt}}{c_p T_\infty} = \left(\sqrt{\Theta_4} - 1\right)^2 \tag{2.42b}$$

Note that (2.41c) and (2.41d) about work output and thermodynamic efficiency here are exactly the same as (2.26a) and (2.26b) for turboprops, except that the variables $\pi_{c,eff}$ and $\tau_{c,eff}$ here are replaced in the latter by π_c and τ_c, respectively.

Now we'll calculate the jet exhaust speed. From the principle of energy consumption,

$$c_p T_7^o = c_p T_7 + \frac{1}{2} u_7^2$$

we write

$$\begin{aligned}
\frac{u_7}{u_\infty} &= \frac{1}{M_\infty} \frac{\sqrt{c_p (T_5^o - T_7)}}{\gamma R T_\infty} = \frac{1}{M_\infty} \sqrt{\frac{2}{\gamma - 1}\left(\Theta_5 - \frac{T_7}{T_\infty}\right)} \\
&= \frac{1}{M_\infty} \sqrt{\frac{2}{\gamma - 1}\left[\Theta_4 + \Theta_\infty - \tau_{c,eff} - \frac{\Theta_4}{\tau_{c,eff}}\right]} \\
&= \frac{1}{M_\infty} \sqrt{\frac{2}{\gamma - 1}\left[\Theta_4 \frac{(\tau_{c,eff} - 1)}{\tau_{c,eff}} - \Theta_\infty (\tau_c - 1)\right]} \\
&= \frac{1}{M_\infty} \sqrt{\frac{2}{\gamma - 1}\left[\Theta_4 \frac{(\Theta_\infty \tau_c - 1)}{\Theta_\infty \tau_c} - \Theta_\infty (\tau_c - 1)\right]}
\end{aligned} \tag{2.43a}$$

and we get an expression for the *propulsive efficiency*:

$$\eta_p = \frac{2}{\left[1 + \frac{u_7}{u_\infty}\right]} \tag{2.43b}$$

On the other hand, work is gained through the difference in the kinetic energy of the jet and the inlet flow, which are

$$\frac{u_7^2}{2 c_p T_\infty} = \Theta_5 - \frac{T_7}{T_\infty} = \Theta_4 + \Theta_\infty - \tau_{c,eff} - \frac{\Theta_4}{\tau_{c,eff}} \tag{2.43c}$$

and

$$\frac{u_\infty^2}{2c_pT_\infty} = \frac{\gamma-1}{2}M_\infty^2 = \Theta_\infty - 1 \tag{2.43d}$$

Subtracting one from the other, we get

$$\begin{aligned}\frac{u_7^2 - u_\infty^2}{2c_pT_\infty} &= \Theta_4 + 1 - \tau_{c,\mathrm{eff}} - \frac{\Theta_4}{\tau_{c,\mathrm{eff}}} = \Theta_4\frac{\tau_{c,\mathrm{eff}} - 1}{\tau_{c,\mathrm{eff}}} - (\tau_{c,\mathrm{eff}} - 1)\\ &= (\Theta_4 - \tau_{c,\mathrm{eff}})\frac{(\tau_{c,\mathrm{eff}} - 1)}{\tau_{c,\mathrm{eff}}} = \frac{w}{c_pT_\infty}\end{aligned}$$

and we show again, just like for ramjets, that the overall work out of the thermodynamic cycle is due to the increase in the kinetic energy.

Now, the thrust to be developed is

$$F = \dot{m}_a(u_7 - u_\infty)$$

In the above expression, and contrary to that for the ramjet engine, the mass flow rate and u_∞ are not completely on the flight speed of the engine, since even at zero flight speed on ground, air is sucked into the engine due to the rotating gas turbine engine.

Now, the nondimensional specific thrust, with the help of (2.43a), is

$$\begin{aligned}\frac{F}{\dot{m}_a a_\infty} &= M_\infty\left(\frac{u_7}{u_\infty} - 1\right) = M_\infty\left[\frac{1}{M_\infty}\sqrt{\frac{2}{\gamma-1}\Theta_4\frac{\Theta_\infty\tau_c - 1}{\Theta_\infty\tau_c} - \Theta_\infty(\tau_c - 1)} - 1\right]\\ &= \sqrt{\frac{2}{\gamma-1}\Theta_4\left(\frac{\Theta_\infty\tau_c - 1}{\Theta_\infty\tau_c} - \Theta_\infty(\tau_c - 1)\right)} - M_\infty\end{aligned} \tag{2.44a}$$

Equation 2.44a is a somewhat complicated function of $(M_\infty, \pi_c, \Theta_4)$. However from (2.44a), it can be seen that the specific thrust is maximum at zero Mach number $(\Theta_\infty = 1)$ and decreases linearly with increasing flight Mach number. This is because although the mass flow rate increases with the flight Mach number, the difference in the two velocities continuously decreases with the increasing flight Mach number. Thus, thrust is maximal at zero Mach number, as follows:

$$\begin{aligned}\left(\frac{F}{\dot{m}_a a_\infty}\right)_{M_\infty=0} &= \sqrt{\frac{2}{\gamma-1}\Theta_4\left(\frac{\tau_c - 1}{\tau_c} - (\tau_c - 1)\right)}\\ &= \sqrt{\frac{2}{\gamma-1}\left(\frac{w}{c_pT_\infty}\right)_{M_\infty=0}}\end{aligned} \tag{2.44b}$$

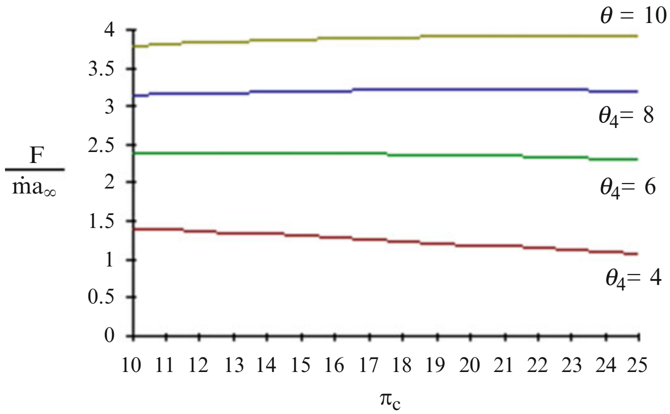

Fig. 2.18 Results of calculating (2.44a) for different Θ_4

Results of calculating (2.44a) at zero Mach number are given in Fig. 2.18.

Thus, the condition for maximum thrust for $M_\infty = 0$ is the same as that for maximum work, as we can verify by comparing with (2.41c).

Now, for the energy input in the combustion chamber,

$$\dot{m}_a c_p (T_4^o - T_3^o) = \dot{m}_f \Delta H_p = \dot{m}_a q_a = \frac{\dot{m}_a w}{\eta_{th}} \tag{2.44c}$$

which gives the relation for the *fuel–air ratio* as

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{c_p T_\infty}{\Delta H_p}(\Theta_4 - \Theta_3) = \left(\frac{c_p T_\infty}{\Delta H_p}\right)(\Theta_4 - \tau_{c,eff}) \tag{2.45a}$$

Let's discuss the specific fuel combustion further. Usually, the specific fuel consumption [kg/J] is computed by dividing the fuel flow rate $\dot{m}_f$ [kg/s] by the work produced per kg of air [J/kg]. However, for aircraft engine applications, it is computed by dividing the fuel flow rate by the thrust [N].

Hence, the specific fuel consumption is defined as

$$\mathrm{SFC} = \frac{\dot{m}_f}{F} = \frac{f}{a_\infty\left[\sqrt{\frac{2}{\gamma-1}\left\{\Theta_4 \frac{(\Theta_\infty \tau_c - 1)}{\Theta_\infty \tau_c}\right\}} - M_\infty\right]} \tag{2.45b}$$

and one can define a *nondimensional specific fuel consumption*,

$$\mathrm{SFC}^* = \mathrm{SFC}.a_\infty = \frac{f}{\left[\sqrt{\frac{2}{\gamma-1}\left\{\Theta_4 \frac{(\Theta_\infty \tau_c - 1)}{\Theta_\infty \tau_c}\right\}} - M_\infty\right]} \tag{2.45c}$$

We have therefore shown that although the fuel–air ratio (without afterburner operation) is reduced linearly with τ_c, it is also proportional to Θ_4. On the other hand, the nondimensional specific fuel consumption is a complicated function of $(\Theta_4|\Theta_\infty)$, that is, $M_\infty,\ \tau_c$, and η_{th}, the last variable again being dependent on τ_c and Θ_∞.

We will now consider the situation with *afterburner*. Instead of (2.41a), the *heat added* is

$$|q_a| = c_p\left(T_4^o - T_3^o + T_6^o - T_5^o\right) = c_p T_\infty(\Theta_4 - \Theta_3 + \Theta_6 - \Theta_5)$$

Since from (2.40d) and (2.40f),

$$\Theta_3 = \tau_c\Theta_\infty \ \text{ and } \ \Theta_5 = \Theta_4 - \Theta_\infty(\tau_c - 1)$$

we get

$$\begin{aligned}|q_a| &= c_p T_\infty(\Theta_4 - \Theta_\infty\tau_c + \Theta_6 - \Theta_4 + \Theta_\infty\tau_c - \Theta_\infty) = c_p T_\infty(\Theta_6 - \Theta_\infty)\\ &= c_p T_\infty[(\Theta_6 - \Theta_5) + (\Theta_4 - \Theta_\infty - \tau_c)]\\ &= \frac{a_\infty^2}{(\gamma-1)}[(\Theta_6 - \Theta_5) + (\Theta_4 - \Theta_\infty - \tau_c)]\end{aligned} \tag{2.46a}$$

Since

$$\frac{T_7^o}{T_7} = \left(\frac{p_7^o}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = \delta_5^{\frac{\gamma-1}{\gamma}} = \tau_t\delta_3^{\frac{\gamma-1}{\gamma}} = \tau_t\Theta_3 = \tau_c\Theta_5\frac{\Theta_\infty}{\Theta_4} = \Theta_7 = \Theta_6 \text{ and } \tau_t = \frac{\Theta_5}{\Theta_4}$$

we get

$$\begin{aligned}\frac{T_7}{T_\infty} &= \frac{\Theta_6}{(\Theta_\infty\tau_t\tau_c)} = \frac{\Theta_6 - \Theta_5 + \Theta_5}{(\Theta_\infty\tau_t\tau_c)} = \Theta_4\frac{\Theta_6 - \Theta_5 + \Theta_5}{(\Theta_\infty\tau_c\Theta_5)}\\ &= \frac{\Theta_4}{\Theta_\infty\tau_c} + \Theta_4\frac{\Theta_6 - \Theta_5}{(\Theta_\infty\tau_c\Theta_5)}\end{aligned} \tag{2.47}$$

and hence, the *reject heat* is

$$\begin{aligned}|q_r| &= c_p T_\infty\left(\frac{T_7}{T_\infty} - 1\right) = c_p T_\infty\left[\frac{\Theta_4}{\Theta_\infty\tau_c} - 1 + \Theta_4\frac{\Theta_6 - \Theta_5}{\Theta_\infty\tau_c\Theta_5}\right]\\ &= c_p T_\infty\left[\frac{\Theta_4 - \Theta_\infty\tau_c}{\Theta_\infty\tau_c} + \Theta_4\frac{\Theta_6 - \Theta_5}{\Theta_\infty\tau_c\Theta_5}\right]\end{aligned} \tag{2.46b}$$

Thus, the specific work gained is

$$\begin{aligned}\frac{w}{c_\mathrm{p}T_\infty} &= \frac{|q_\mathrm{a}|-|q_\mathrm{r}|}{c_\mathrm{p}T_\infty} = (\Theta_6-\Theta_5)+(\Theta_4-\Theta_\infty\tau_\mathrm{c}) - \frac{(\Theta_4-\Theta_\infty\tau_\mathrm{c})}{\Theta_\infty\tau_\mathrm{c}} - \frac{(\Theta_6-\Theta_5)}{\Theta_\infty\tau_\mathrm{c}\Theta_5}\\ &= (\Theta_4-\Theta_\infty\tau_\mathrm{c})\frac{(\Theta_\infty\tau_\mathrm{c}-1)}{\Theta_\infty\tau_\mathrm{c}} + (\Theta_6-\Theta_5)\frac{(\Theta_\infty\tau_\mathrm{c}\Theta_5-\Theta_4)}{\Theta_\infty\tau_\mathrm{c}\Theta_5}\\ &= (\Theta_4-\Theta_\infty\tau_\mathrm{c})\frac{(\Theta_\infty\tau_\mathrm{c}-1)}{\Theta_\infty\tau_\mathrm{c}} + (\Theta_6-\Theta_5)\left(1-\frac{\Theta_4}{\Theta_\infty\tau_\mathrm{c}\Theta_5}\right)\\ &= \left(\Theta_4-\tau_\mathrm{c,eff}\right)\frac{\left(\tau_\mathrm{c,eff}-1\right)}{\tau_\mathrm{c,eff}} + (\Theta_6-\Theta_5)\left(1-\frac{\Theta_4}{\tau_\mathrm{c,eff}\Theta_5}\right)\end{aligned} \tag{2.46c}$$

Now,

$$\begin{aligned} w = \frac{1}{2}\left(u_7^2-u_\infty^2\right)\frac{w}{c_\mathrm{p}T_\infty} &= \frac{\gamma-1}{2}M_\infty^2\left[\left(\frac{u_7}{u_\infty}\right)^2-1\right]\\ &= \left(\Theta_4-\tau_\mathrm{c,eff}\right)\frac{\left(\Theta_4-\tau_\mathrm{c,eff}-1\right)}{\tau_\mathrm{c,eff}} + (\Theta_6-\Theta_5)\left(1-\frac{\Theta_4}{\tau_\mathrm{c,eff}\Theta_5}\right)\end{aligned}$$

Hence,

$$\frac{u_7}{u_\infty} = \sqrt{1+\frac{2}{(\gamma-1)}M_\infty^2\left[\left(\Theta_4-\tau_\mathrm{c,eff}\right)\frac{\left(\tau_\mathrm{c,eff}-1\right)}{\tau_\mathrm{c,eff}} + (\Theta_6-\Theta_5)\left(1-\frac{\Theta_4}{\tau_\mathrm{c,eff}\Theta_5}\right)\right]}$$

and the *nondimensional specific thrust* is

$$\begin{aligned}\frac{F}{\dot m_\mathrm{a}a_\infty} &= M_\infty\left(\frac{u_7}{u_\infty}-1\right)\\ &= M_\infty\left[\sqrt{1+\frac{2}{(\gamma-1)}M_\infty^2\left[\left(\Theta_4-\tau_\mathrm{c,eff}\right)\frac{\left(\tau_\mathrm{c,eff}-1\right)}{\tau_\mathrm{c,eff}} + (\Theta_6-\Theta_5)\left(1-\frac{\Theta_4}{\tau_\mathrm{c,eff}\Theta_5}\right)\right]}-1\right]\end{aligned} \tag{2.46d}$$

Now, the maximum of (2.46d) is again similar to that given by (2.44a), and its maximum in comparison to (2.44b), which is without an afterburner, is shifted toward a high effective compression ratio.

Now, similar to for the case without an afterburner, (2.44c), we have

$$\dot m_\mathrm{f}\Delta H_\mathrm{p} = \dot m_\mathrm{a}q_\mathrm{a} = \dot m_\mathrm{a}c_\mathrm{p}T_\infty\left\lfloor(\Theta_6-\Theta_5)+\left(\Theta_4-\tau_\mathrm{c,eff}\right)\right\rfloor$$

Thus, the *fuel–air ratio* is

$$f = \frac{\dot{m}_\mathrm{f}}{\dot{m}_\mathrm{a}} = \frac{c_\mathrm{p} T_\infty}{\Delta H_\mathrm{p}}[(\Theta_6 - \Theta_5) + (\Theta_4 - \tau_{\mathrm{c,eff}})]$$

and the thermodynamic efficiency is

$$\eta_\mathrm{th} = \frac{\left\{\frac{(\Theta_4 - \Theta_\infty\tau_\mathrm{c})(\Theta_\infty\tau_\mathrm{c}-1}{\Theta_\infty\tau_\mathrm{c}} + (\Theta_6 - \Theta_5)\left[1 - \frac{\Theta_4}{\Theta_\infty\tau_\mathrm{c}\Theta_5}\right]\right\}}{\{(\Theta_4 - \Theta_\infty\tau_\mathrm{c}) + (\Theta_6 - \Theta_5)\}}$$
$$= \frac{\left\{\frac{(\Theta_4 - \tau_{\mathrm{c,eff}})(\tau_{\mathrm{c,eff}}-1)}{\tau_{\mathrm{c,eff}}} + (\Theta_6 - \Theta_5)\left[1 - \frac{\Theta_4}{\tau_{\mathrm{c,eff}}\Theta_5}\right]\right\}}{\{(\Theta_4 - \tau_{\mathrm{c,eff}}) + (\Theta_6 - \Theta_5)}$$

Now, the *thrust* is given by

$$F = M_\infty \dot{m}_\mathrm{a} a_\infty \left[\sqrt{1 + \frac{2}{(\gamma-1}M_\infty^2\left[(\Theta_4 - \tau_{\mathrm{c,eff}})\frac{(\tau_{\mathrm{c,eff}}-1)}{\tau_{\mathrm{c,eff}}} + (\Theta_6 - \Theta_5)\left(1 - \frac{\Theta_4}{\tau_{\mathrm{c,eff}}\Theta_5}\right)\right]} - 1\right], N$$

Hence, the nondimensional specific fuel consumption is

$$\mathrm{SFC} = \frac{\dot{m}_\mathrm{f}}{F}\frac{f}{a_\infty\left[\sqrt{1 + \frac{2}{(\gamma-1}M_\infty^2\left[(\Theta_4 - \tau_{\mathrm{c,eff}})\frac{(\tau_{\mathrm{c,eff}}-1)}{\tau_{\mathrm{c,eff}}} + (\Theta_6 - \Theta_5)\left(1 - \frac{\Theta_4}{\tau_{\mathrm{c,eff}}\Theta_5}\right)\right]} - 1\right]}$$

and the nondimensional specific fuel combustion is

$$\mathrm{SFC}^* = \mathrm{SFC}.a_\infty\frac{f}{\left[\sqrt{1 + \frac{2}{(\gamma-1}M_\infty^2\left[(\Theta_4 - \tau_{\mathrm{c,eff}})\frac{(\tau_{\mathrm{c,eff}}-1)}{\tau_{\mathrm{c,eff}}} + (\Theta_6 - \Theta_5)\left(1 - \frac{\Theta_4}{\tau_{\mathrm{c,eff}}\Theta_5}\right)\right]} - 1\right]}$$

For the afterburner operation, f increases, but with increasing flight Mach number (increasing Θ_∞) it decreases, since for a given Θ_4 the scope for fuel flow rate decreases (before the combustion chamber, the air is heated), and the specific fuel consumption changes in a similar fashion.

It is seen that the thermodyamic efficiency is considerably reduced against that without an afterburner. As before, the optimum specific work is obtained by differentiating (2.46c) with respect to τ_c and equating it to zero to get

$$(\tau_\mathrm{c})_\mathrm{opt} = \sqrt{\Theta_4|\Theta_6/\Theta_5} \tag{2.48a}$$

and the *optimum specific work* is

$$\frac{w_{\mathrm{opt}}}{c_{\mathrm{p}}T_{\infty}} = (\Theta_6 - \Theta_5)\left[1 - \sqrt{\frac{\Theta_4\Theta_6}{\Theta_5}} - \Theta_6\right] + \sqrt{\frac{\Theta_5}{\Theta_6}\left(\Theta_4\sqrt{\frac{\Theta_6}{\Theta_5}}\right)}\left(\sqrt{\Theta_4\frac{\Theta_6}{\Theta_5}} - 1\right) \tag{2.48b}$$

From these equations, we can see that for a given combustion chamber temperature ratio, π_{c} can be considerably increased at higher flight Mach numbers if the afterburner is operational; however, it has the penalty of a higher specific fuel consumption. An aircraft, like a supersonic transporter to operate at Mach >2, can therefore take off the ground and increase its speed with or without an afterburner. With increasing flight speed, the thrust will come to zero sooner if the afterburner does not work. Therefore, with the afterburner working, it can maintain its thrust level sufficiently high to reach the required level. At the time the afterburner is operating, the tailpipe's cross-sectional area has to be increased to accommodate a higher-volume flow rate; otherwise, pressure can be fed back upstream through the turbine, combustion chamber, and compressor to reduce the mass flow rate.

In order to show the effect from the afterburner working, results of $w/(c_{\mathrm{p}}T_{\infty})$, $F/(\dot{m}_{\mathrm{a}}a_{\infty})$, and SFC* are computed for $(\Theta_4 = 4)$ for two values each of M_{∞} and $\Delta\Theta_{\mathrm{AB}} = (\Theta_6 - \Theta_5)$ and are shown in Figs. 2.19, 2.20, and 2.21.

2.2.3 *Ideal Bypass Jet Cycle with Afterburner*

We have shown earlier that in order to get better propulsive efficiency, it is necessary to have the average jet speed lower and as close to the flight speed as possible. For this purpose, two different methods have been considered. In the first method, the incoming air after the low-pressure compressor is split into two parts: one going through the high-pressure compressor, the combustion chamber, and the turbine (internal bypass and mixing), and the other bypasses both the combustion chamber and the turbine. For the first, there are problems because in order to match the air stream pressure after the turbine, the bypass ratio is limited. But this has a comparatively small front area and can be useful for military engines. On the other hand, if a very high volume of bypassed air is exited right after the low-pressure compressor, then one can considerably increase the bypass ratio (fanjet), which is the prevalent engine for transonic flying commercial jets. For the first type, a schematic sketch of a bypass (internal mixing) jet with an afterburner is shown in Fig. 2.22a, and the corresponding ideal thermodynamic cyclic process appears in Fig. 2.22b. Here b is the bypass ratio, which is the ratio of bypassed (cold) air to the core (hot) air, $b = \dot{m}_{\mathrm{C}}/\dot{m}_{\mathrm{H}}$. Since $\dot{m}_{\mathrm{C}} + \dot{m}_{\mathrm{H}} = \dot{m}_{\mathrm{a}}$, which is the total air mass flow ratio, it is evident that

$$\dot{m}_{\mathrm{H}} = \frac{\dot{m}_{\mathrm{a}}}{1+b} \tag{2.50a}$$

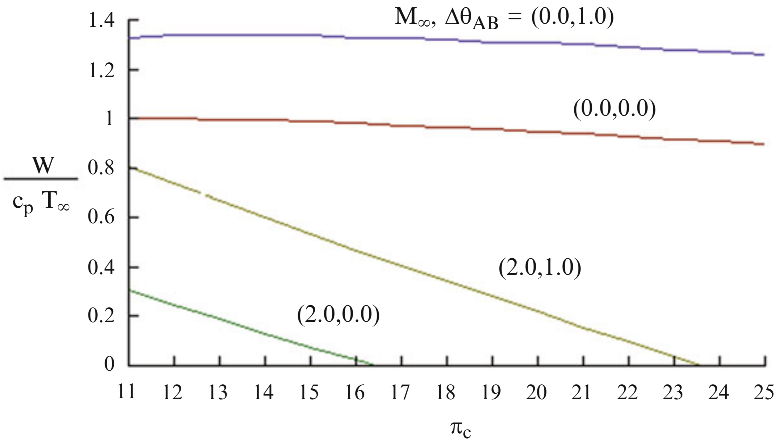

Fig. 2.19 Specific work vs. pressure ratio for afterburner operation

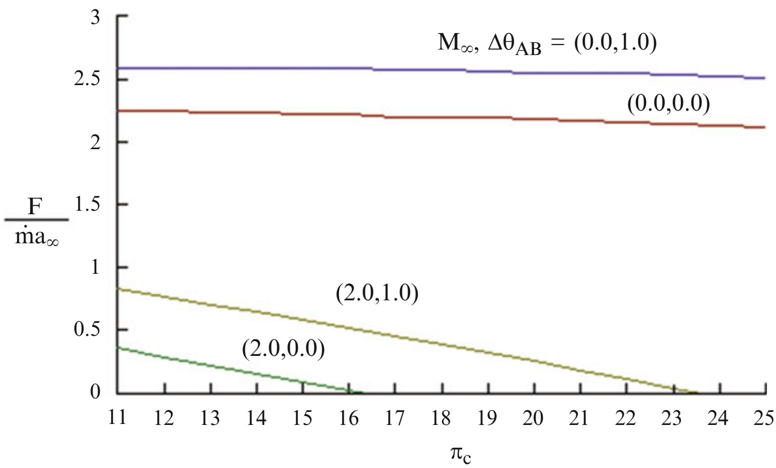

Fig. 2.20 Specific thrust vs. pressure ratio for afterburner operation

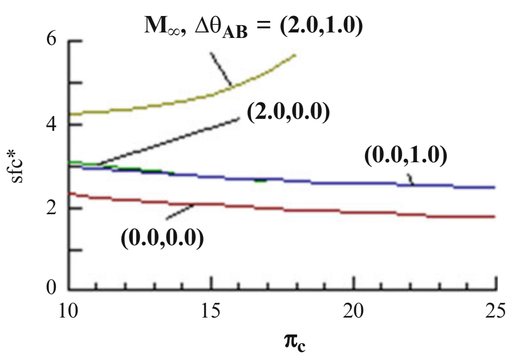

Fig. 2.21 Nondimensional SFC vs. pressure ratio for afterburner operation

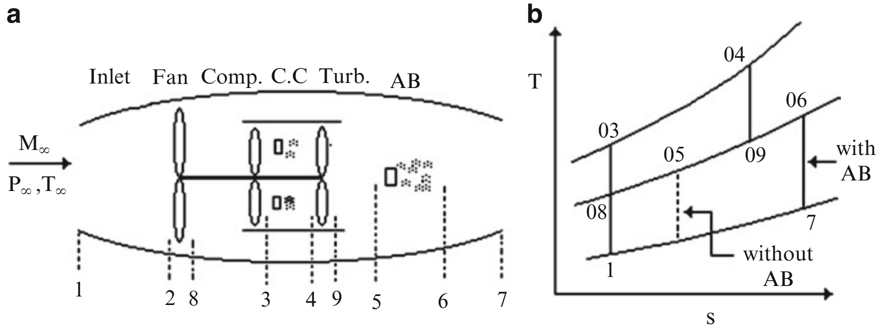

Fig. 2.22 (**a**) Schematic sketch of a bypass jet (with internal mixing); (**b**) corresponding sketch of the thermodynamic process in a (T, s) chart

and

$$\dot{m}_{\mathrm{C}} = \frac{\dot{m}_{\mathrm{a}} b}{1 + b} \tag{2.50b}$$

For this case, the bypass ratio b is being introduced as a parameter in addition to the parameters already introduced in the previous case, that is, the approaching flow Mach number, the combustion chamber temperature ratio, and the afterburner stagnation temperature increases.

Thus, let's assume that the parameters given are $\pi_{\mathrm{c}} = p_3^o/p_2^o$ as the *overall compressor ratio*, $\Theta_4 = T_4^o/T_\infty$ as the main *combustion chamber temperature ratio* = *turbine inlet temperature ratio*, M_∞ as the *approaching flow Mach number*, b as the *bypass ratio*, and $\Delta\Theta_{\mathrm{AB}} = \left(T_6^o - T_5^o\right)/T_\infty$ as the *stagnation temperature increase in the afterburner*. Thus, we have an increase in the basic parameter by 1 over those for the straight-jet engine.

Now,

$$\Theta_\infty = \Theta_1 = \Theta_2 = \frac{T_\infty^o}{T_\infty} = \frac{T_1^o}{T_\infty} = \frac{T_2^o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \tag{2.51a}$$

and

$$\delta_\infty = \delta_1 = \delta_2 = \frac{p_\infty^o}{p_\infty} = \frac{p_1^o}{p_\infty} = \frac{p_2^o}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2.51b}$$

Let $\pi_{\mathrm{f}} = p_8^o/p_\infty^2$ be the *fan compression ratio*, which is not an independent parameter, but depends on b to match the fan exit pressure with the turbine exit pressure. Thus, the following relations are obvious:

$$\begin{aligned} \delta_8 = \delta_9 = \delta_5 = \delta_6 &= p_8^o/p_\infty = p_9^o/p_\infty = p_5^o/p_\infty = p_6^o/p_\infty = \pi_{\mathrm{f}}\delta_\infty \\ &= \pi_{\mathrm{f}}\Theta^{\gamma(\gamma-1)} = \pi_{\mathrm{f,eff}} \end{aligned} \tag{2.51c}$$

$$\delta_3 = \delta_4 = p_3^o/p_\infty = p_4^o/p_\infty = \pi_c \delta_\infty = \pi_c \Theta_\infty^{\gamma/(\gamma-1)} = \pi_{c,eff} \tag{2.51d}$$

$$\Theta_8 = T_8^o/T_\infty = \pi_{c,eff}^{(\gamma-1)/\gamma} = \Theta_\infty \pi_f^{(\gamma-1)/y} \tag{2.51e}$$

and

$$\Theta_3 = T_3^o/T_\infty = \pi_{c,eff}^{(\gamma-1)/y} = \Theta_\infty \pi_c^{(y-1)/y} \tag{2.51f}$$

In above equation, $\pi_{c,eff}$ and $\pi_{f,eff}$ are the *effective overall compression ratio* and the *effective fan compression ratio*, respectively.

Noting that the turbine is driving the compressor and assuming a *single-spool engine*, we have

$$T_4^o - T_9^o = \left(T_3^o - T_2^o\right) + b\left(T_8^o - T_2^o\right)$$

from which it follows that

$$\begin{aligned}\Theta_9 &= \Theta_4 - [(\Theta_3 - \Theta_2) + b(\Theta_8 - \Theta_2)] \\ &= \Theta_4 - \Theta_\infty\left\{\left(\pi_c^{(y-1)/y} - 1\right) + b\left(\pi_f^{(y-1)/y} - 1\right)\right\} \\ &= \Theta_4 - \left\{\left(\pi_{c,eff}^{(y-1)/y} - 1\right) + \left(\pi_{f,eff}^{(y-1)/y} - 1\right) + (1+b)\Theta_\infty\right\}\end{aligned} \tag{2.52a}$$

Furthermore,

$$\frac{T_4^o}{T_9^o} = \frac{\Theta_4}{\Theta_9} = \left(\frac{p_2^o}{p_9^o}\right)^{\frac{\gamma-1}{\gamma}} = \left[\frac{\pi_c \Theta_\infty}{\pi_f \Theta_\infty}\right]^{\frac{y-1}{y}} = \left(\frac{\pi_{c,eff}}{\pi_{f,eff}}\right)^{\frac{y-1}{y}} \tag{2.52b}$$

From (2.52a) and (2.52b), we can therefore write

$$\Theta_9 = \Theta_4\left(\frac{\pi_f}{\pi_c}\right)^{\frac{y-1}{y}} = \{\Theta_4 - \Theta_\infty\}\left[\left(\pi_c^{(y-1)/y} - 1\right) + b\left(\pi_f^{(y-1)/y}\right)\right]$$

from which it follows that

$$\left(\frac{\pi_f}{\pi_c}\right)^{\frac{y-1}{y}} = \frac{\{\Theta_4 - \Theta_\infty\}}{\Theta_4}\left[\left(\pi_c^{(\gamma-1)/y} - 1\right) + b(\pi_f^{(y-1)/y} - 1\right] \tag{2.53a}$$

Thus,

$$\frac{\pi_f}{\pi_c} = f(\Theta_4, \Theta_\infty, \pi_c, b, \gamma) \tag{2.53b}$$

Table 2.5 Effect of Bypass Ratio on the Fan Pressure Ratio

b	0.4	0.8	1.2	1.6
π_f/π_c	0.1720	0.0522	0.0291	0.0042

For $\gamma = 1.4$ and $M_\infty \to 0$, that is, $\Theta_\infty = 1$, we have still three independent parameters on which the ratio (π_f/π_c) depends. Obviously, the limit of $b \to 0$ for the ratio (π_f/π_c) has no meaning, since then the pressure behind the turbine is not related to the pressure behind the fan, although we get a finite ratio. However, (π_f/π_c) is computed for nominal values of $\gamma = 1.4, \Theta_4 = 4.0$ and $\pi_c = 16.0$, and the results are presented for $b = 0.4$, 0.8, 1.2, and 1.6 in Table 2.5.

The results therefore show that with the increasing bypass ratio, the fan pressure ratio decreases. This can be explained due to the fact that with the increasing bypass ratio, the turbine has to produce more work to allow compression of more and more air flow, and hence the turbine pressure must decrease, which, to balance the pressure at that point, must reduce the fan pressure.

From energy balance, we now have

$$bc_p(T_5^o - T_8^o) = c_p(T_9^o - T_5^o)$$

which results in the relation

$$b(\Theta_5 - \Theta_8) = \Theta_9 - \Theta_5$$

Hence,

$$\Theta_5(b+1) = \Theta_9 + \Theta_8 = \Theta_4\left(\frac{\pi_f}{\pi_c}\right)^{\frac{\gamma-1}{\gamma}} + b\Theta_\infty \pi_f^{\frac{\gamma-1}{\gamma}} = \pi_f^{\frac{\gamma-1}{\gamma}}\left[b\Theta_\infty + \frac{\Theta_4}{\pi_c^{\frac{\gamma-1}{y}}}\right]$$

Since

$$\pi_f^{\frac{\gamma-1}{\gamma}}\left[b\Theta_\infty + \frac{\Theta_4}{\pi_c^{\frac{\gamma-1}{\gamma}}}\right] = \Theta_4 - \Theta_\infty\left(\pi_c^{(\gamma-1)/y} - 1 - b\right)$$

we get

$$\Theta_5 = \frac{\Theta_4 - \pi_{c,\text{eff}}^{(\gamma-1)/\gamma} + \Theta_\infty(1-b)}{1+b} \tag{2.54a}$$

It follows further that

$$\begin{aligned}\delta_5 &= \frac{p_5^o}{p_\infty} = \left(\frac{p_4^o}{p_\infty}\right)\left(\frac{p_5^o}{p_4^o}\right) = \left(\frac{p_4^o}{p_\infty}\right)\left(\frac{p_9^o}{p_4^o}\right) = \left(\frac{p_3^o}{p_\infty}\right)\left(\frac{p_8^o}{p_3^o}\right)\\ &= \delta_8\frac{p_8^\infty}{p_\infty} = \pi_{c,\text{eff}}\left(\frac{p_9^o}{p_4^o}\right)^{\frac{\gamma}{\gamma-1}} = \pi_{c,\text{eff}}\left(\frac{\Theta_9}{\Theta_4}\right)^{\frac{\gamma}{\gamma-1}}\frac{\pi_f}{\pi_c} = \pi_{f,\text{eff}}\end{aligned} \tag{2.54b}$$

We will now first study the case *without* an afterburner, that is, $T_5^o = T_6^o = T_7^o$. Thus,

$$u_7 = \sqrt{2c_\mathrm{p}\left(T_5^o - T_7\right)}$$

from which it follows that

$$\frac{u_7}{u_\infty} = \frac{1}{M_\infty}\sqrt{\frac{2}{\gamma-1}\left(\Theta_5 - \frac{T_7}{T_\infty}\right)}$$

Now since

$$\frac{T_7}{T_\infty} = \left(\frac{T_5^o}{T_\infty}\right)\left(\frac{T_7}{T_5^o}\right) = \Theta_5\left(\frac{p_\infty}{p_5^o}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\Theta_5}{\left(\Theta_\infty \pi_\mathrm{f}^{(\gamma-1)/\gamma}\right)} = \frac{\Theta_5}{\left(\pi_\mathrm{f,eff}^{(\gamma-1)/\gamma}\right)} = \frac{\Theta_5}{\tau_\mathrm{f,eff}} \tag{2.55a}$$

it follows that

$$\left(\frac{u_7}{u_\infty}\right) = \frac{1}{M_\infty}\sqrt{\frac{2}{\gamma-1}\left(1-\pi_\mathrm{f,eff}^{(\gamma-1)/\gamma}\right)} = \frac{1}{M_\infty}\sqrt{\frac{2}{\gamma-1}\left(1-\tau_\mathrm{f,eff}^{(\gamma-1)/\gamma}\right)} \tag{2.55b}$$

Now

$$\frac{u_7^2}{2c_\mathrm{p}T_\infty} = \frac{\left(T_5^o - T_7\right)}{T_\infty} = \Theta_5 - \frac{T_7}{T_\infty} = \Theta_5\frac{\left(\tau_\mathrm{f,eff}-1\right)}{\tau_\mathrm{f,eff}} \tag{2.55c}$$

and

$$\frac{u_\infty^2}{2c_\mathrm{p}T_\infty} = M_\infty^2\frac{(\gamma-1)}{2} = \Theta_\infty - 1 \tag{2.55d}$$

From the difference in (2.55c) and (2.55d), we get the overall work (per kg of mixture) as

$$\frac{|w|}{c_\mathrm{p}T_\infty} = \frac{u_7^2 - u_\infty^2}{2c_\mathrm{p}T_\infty} = \Theta_\infty\frac{\left(\tau_\mathrm{f,eff}-1\right)}{\tau_\mathrm{f,eff}} - (\Theta_\infty - 1) \tag{2.56a}$$

We get the same result from the overall balance of the heat input and output. First, the heat added (per unit mass of hot gas) is q_a, which, when written in nondimensional form, is

$$\frac{|q_\mathrm{a}|}{c_\mathrm{p}T_\infty} = \Theta_4 - \Theta_3 = \Theta_4 - \tau_\mathrm{c,eff}$$

which, by taking (2.54a) into consideration, becomes

$$\frac{|q_{\mathrm{a}}|}{c_{\mathrm{p}}T_{\infty}} = (\Theta_5 - \Theta_{\infty}).(1+b) \tag{2.56b}$$

On the other hand, the heat added (per unit of total gas mixture) is

$$|\bar{q}_{\mathrm{a}}| = \frac{|q_{\mathrm{a}}|}{1+b}$$

and thus,

$$\frac{|\bar{q}_{\mathrm{a}}|}{c_{\mathrm{p}}T_{\infty}} = (\Theta_5 - \Theta_{\infty}). \tag{2.56c}$$

However, the rejected heat (per unit mass of total gas mixture) is

$$|q_{\mathrm{r}}| = c_{\mathrm{p}}(T_7 - T_{\infty})$$

and, written in nondimensional form, it is

$$\frac{|\bar{q}_{\mathrm{a}}|}{c_{\mathrm{p}}T_{\infty}} = \left(\frac{T_7}{T_{\infty}} - 1\right) = \left(\frac{\Theta_5 - \tau_{\mathrm{f,eff}}}{\tau_{\mathrm{f,eff}}}\right) \tag{2.56d}$$

From (2.56c) and (2.55d), we get the overall work (per kg of mixture),

$$\begin{aligned}\frac{|w|}{c_{\mathrm{p}}T_{\infty}} &= \frac{|\bar{q}_{\mathrm{a}}|}{c_{\mathrm{p}}T_{\infty}} - \frac{|\bar{q}_{\mathrm{r}}|}{c_{\mathrm{p}}T_{\infty}} = (\Theta_5 - \Theta_{\infty}) = \left(\frac{\Theta_5 - \tau_{\mathrm{f,eff}}}{\tau_{\mathrm{f,eff}}}\right) \\ &= \Theta_5\left(\frac{\tau_{\mathrm{f,eff}} - 1}{\tau_{\mathrm{f,eff}}}\right) - (\Theta_{\infty} - 1)\end{aligned}$$

which is the same as (2.56a). We therefore prove, once again, that the overall work in a low-bypass jet, as in the case of the ramjet and straight turbojet (with or without afterburner), is equal to the increase in the kinetic energy.

Now the *thermodynamic efficiency* is

$$\eta_{\mathrm{th}} = \frac{|w|}{|\bar{q}_{\mathrm{a}}|} = \left\{\Theta_5\left(\frac{\tau_{\mathrm{f,eff}} - 1}{\tau_{\mathrm{f,eff}}}\right) - \frac{(\Theta_{\infty} - 1}{(\Theta_5 - \Theta_{\infty})}\right\} = 1 - \frac{(\Theta - \tau_{\mathrm{f,eff}})}{(\Theta_5 - \Theta_{\infty})\tau_{\mathrm{f,eff}}} \tag{2.57a}$$

and the *propulsive efficiency* is

$$\eta_{\mathrm{p}} = \frac{2}{1 + \frac{u_7}{u_{\infty}}} = \frac{2}{1 + \frac{1}{M_{\infty}}\sqrt{\frac{2\Theta_5}{\gamma-1}\left(1 - \frac{1}{\tau_{\mathrm{f,eff}}}\right)}} \tag{2.57b}$$

which can, of course, give the *overall efficiency* as the product of the two. Now since the thrust is

$$F = \dot{m}_a(u_7 - u_\infty)$$

it can easily be shown that

$$\frac{F}{\dot{m}_a a_\infty} = M_\infty\left(\frac{u_7}{u_\infty} - 1\right) = \sqrt{\frac{2\Theta_5}{\gamma - 1}\left(1 - \frac{1}{\tau_{f,eff}}\right)} - M_\infty \tag{2.57c}$$

which gives the surprising result that the specific thrust depends mainly on how much the temperature ratio at point 5 is across the far, which, of course, depends on various parameters, as shown in (2.55b). It is therefore evident, that a high specific thrust in a low-bypass jet requires a smaller $\tau_{f,eff}$ and a high Θ_5. This last one, according to (2.54a), depends on $\Theta_4, \pi_{c,eff}$, and b.

We will now consider the situation with the afterburner operating. Let's specify the temperature increase in the afterburner, $\Delta\Theta_{AB} = \Theta_6 - \Theta_5$. Thus, the least heat added (per unit mass of mixed air) is

$$\frac{\bar{q}_a}{c_p T_\infty} = \Theta_5 - \Theta_\infty + \Theta_6 - \Theta_5 = \Theta_6 - \Theta_\infty \tag{2.58a}$$

Now,

$$\frac{T_7}{T_\infty} = \left(\frac{T_6^o}{T_\infty}\right)\left(\frac{T_7}{T_6^o}\right) = \Theta_6\left(\frac{p_\infty}{p_6^o}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\Theta_5}{\pi_{f,eff}^{(\gamma-1)/\gamma}} = \frac{\Theta_6}{\tau_{f,eff}} \tag{2.58b}$$

and thus the reject heat in nondimensional form (per unit mass of air) is

$$\frac{|\bar{q}_r|}{c_p T_\infty} = \frac{T_7}{T_\infty} - 1 = \frac{(\Theta_6 - \tau_{f,eff})}{\tau_{f,eff}} \tag{2.58c}$$

The *exhaust kinetic energy* in nondimensional form is now

$$\frac{u_7^2}{2c_p T_\infty} = \Theta_6 - \frac{T_{67}}{T_\infty} = \Theta_6 \frac{(\tau_{f,eff} - 1)}{\tau_{f,eff}} \tag{2.58d}$$

and the approaching flow kinetic energy in nondimensional form, before in (2.55d), is

$$\frac{u_\infty^2}{2c_p T_\infty} = \Theta_\infty - 1 \tag{2.58e}$$

Thus, we can show that the overall work (per unit mass of mixed air) in nondimensional form is

$$\frac{w}{c_\mathrm{p}T_\infty} = \frac{|\bar{q}_\mathrm{a}| - |\bar{q}_\mathrm{r}|}{c_\mathrm{p}T_\infty} = (\Theta_6 - \Theta_\infty) - \frac{(\Theta_6 - \tau_\mathrm{f,eff})}{\tau_\mathrm{f,eff}} = \frac{(u_7^2 - u_\infty^2)}{2c_\mathrm{p}T_\infty} \tag{2.59a}$$

Now, the *thermodynamic efficiency* is

$$\eta_\mathrm{th} = \frac{|w|}{|\bar{q}_\mathrm{a}|} = 1 - \frac{(\Theta_6 - \tau_\mathrm{f,eff})}{\tau_\mathrm{f,eff}(\Theta_6 - \Theta_\infty)} \tag{2.59b}$$

which replaces (2.57a) *without an afterburner*. From (2.58d) and (2.58e), the relation for the *speed ratio* is

$$\frac{u_7}{u_\infty} = \sqrt{\frac{\Theta_6}{(\Theta_\infty - 1)} \cdot \frac{(\tau_\mathrm{f,eff} - 1)}{\tau_\mathrm{f,eff}}} = \frac{1}{M_\infty}\sqrt{\frac{2}{\gamma - 1}\Theta_6\left(1 - \frac{1}{\tau_\mathrm{f,eff}}\right)} \tag{2.58f}$$

and, thus, the *propulsion efficiency* is

$$\eta_\mathrm{p} = \frac{2}{1 + \sqrt{\frac{\Theta_6}{(\Theta_\infty - 1)}\left(1 - \frac{1}{\tau_\mathrm{f,eff}}\right)}} \tag{2.59c}$$

which replaces (2.57b). The *specific thrust* relation with afterburner operating is now

$$\frac{F}{\dot{m}_\mathrm{a}a_\infty} = M_\infty\left(\frac{u_7}{u_\infty} - 1\right) = \sqrt{\frac{2}{\gamma - 1}\Theta_6\left(1 - \frac{1}{\tau_\mathrm{f,eff}}\right)} - M_\infty \tag{2.59d}$$

which replaces (2.57c).

Now, the fuel mass flow rate in the combustion chamber (without afterburner operating) is obtained (in term of unit mass of hot gas) by writing

$$\dot{m}_\mathrm{fcc} = \dot{m}_\mathrm{H} c_\mathrm{p} T_\infty \left(\frac{\Theta_4 - \tau_\mathrm{c,eff}}{\Delta H_\mathrm{p}}\right)$$

which gives the *fuel–air ratio* in the combustion chamber as

$$f_\mathrm{cc} = \frac{\dot{m}_\mathrm{fcc}}{\dot{m}_\mathrm{H}} = \frac{c_\mathrm{p}T_\infty}{\Delta H_\mathrm{p}}(\Theta_4 - \tau_\mathrm{c,eff}) \tag{2.60a}$$

The above fuel–air ratio in the combustion chamber must not be more than the maximum fuel–air ratio allowed for a particular fuel. Since, however, $\dot{m}_{\mathrm{H}} = \dot{m}_{\mathrm{a}}/(b+1)$, we get the fuel–air ratio after mixing the two cold and hot air streams as

$$\bar{f}_{\mathrm{cc}} = \frac{\dot{m}_{\mathrm{fcc}}}{\dot{m}_{\mathrm{a}}} = \frac{f_{\mathrm{cc}}}{(b+1)} = \frac{c_{\mathrm{p}} T_{\infty}}{\Delta H_{\mathrm{p}}} \frac{\left(\Theta_4 - \tau_{\mathrm{c,eff}}\right)}{(b+1)} \tag{2.60b}$$

While the afterburner is in operation,

$$\dot{m}_{\mathrm{fAB}} = \dot{m}_{\mathrm{a}} c_{\mathrm{p}} \frac{\left(T_6^o - T_5^o\right)}{\Delta H_{\mathrm{p}}}$$

and thus, the fuel–air ratio added in the afterburner is

$$\overline{\Delta f}_{\mathrm{AB}} = \frac{\dot{m}_{\mathrm{fAB}}}{\dot{m}_{\mathrm{a}}} \left[\frac{c_{\mathrm{p}} T_{\infty}}{\Delta H_{\mathrm{p}}} (\Theta_6 - \Theta_5)\right] \tag{2.60c}$$

The total fuel flow rate is now

$$\dot{m}_{\mathrm{f}} = \dot{m}_{\mathrm{fcc}} + \dot{m}_{\mathrm{fAB}} = \dot{m}_{\mathrm{a}} \frac{c_{\mathrm{p}} T_{\infty}}{\Delta H_{\mathrm{p}}} \left[\frac{\left(\Theta_4 - \tau_{\mathrm{c,eff}}\right)}{(b+1)} + (\Theta_6 - \Theta_5)\right] \tag{2.60d}$$

Thus, the specific fuel consumption (SFC) is given by the relation

$$\begin{aligned}
\mathrm{SFC} &= \frac{\dot{m}_{\mathrm{f}}}{F} = \frac{\dot{m}_{\mathrm{a}}}{\dot{m}_{\mathrm{a}}(u_7 - u_{\infty})} \cdot \frac{c_{\mathrm{p}} T_{\infty}}{\Delta H_{\mathrm{p}}} \left[\frac{\left(\Theta_4 - \tau_{\mathrm{c,eff}}\right)}{1+b} + (\Theta_6 - \Theta_5)\right] \\
&= \frac{1}{M_{\infty}} \cdot \frac{a_{\infty}}{(\gamma - 1)\Delta H_{\mathrm{p}}} \left[\frac{\left(\Theta_4 - \tau_{\mathrm{c,eff}}\right)}{1+b} + (\Theta_6 - \Theta_5)\right] \cdot \frac{1}{\left(\frac{u_7}{u_{\infty}} - 1\right)} \\
&= \frac{a_{\infty}}{M_{\infty}(\gamma - 1)\Delta H_{\mathrm{p}}} \frac{\left[\frac{\left(\Theta_4 - \tau_{\mathrm{c,eff}}\right)}{1+b} + (\Theta_6 - \Theta_5)\right]}{\sqrt{\frac{2\Theta_6}{\gamma - 1}\left\{1 - \frac{1}{\tau_{\mathrm{f,eff}}}\right\}} - 1} \\
&= \frac{a_{\infty}}{(\gamma - 1)\Delta H_{\mathrm{p}}(1+b)} \frac{\left[(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5)(1+b)\right]}{\sqrt{\frac{2\Theta_6}{\gamma - 1}\left\{1 - \frac{1}{\tau_{\mathrm{f,eff}}}\right\}} - M_{\infty}}
\end{aligned} \tag{2.60e}$$

and the nondimensional specific fuel consumption is given by the relation

$$\mathrm{SFC}^* = \frac{\mathrm{SFC}.\Delta H_{\mathrm{p}}}{a_{\infty}} = \frac{1}{(\gamma - 1)(1+b)} \frac{\left[(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5)(1+b)\right]}{\sqrt{\frac{2\Theta_6}{\gamma - 1}\left\{1 - \frac{1}{\tau_{\mathrm{f,eff}}}\right\}} - M_{\infty}} \tag{2.60f}$$

Now, from (2.53b), we have

$$\frac{1}{\tau_{\mathrm{f,eff}}} = \frac{\Theta_4 + b\tau_{\mathrm{c,eff}}}{\tau_{\mathrm{c,eff}}}\left[\Theta_4 + \Theta_\infty(1+b) - \tau_{\mathrm{c,eff}}\right]$$

which can be substituted in (2.60f) to get an explicit expression for (SFC^*) as a function of $\left(\Theta_4, b, \tau_{\mathrm{c,eff}}, M_\infty\right)$ and an increase in the (nondimensional) stagnation temperature in the afterburner, $(\Theta_6 - \Theta_5)$.

2.2.4 *Ideal Fanjet Cycle with Afterburner*

While low-bypass jets, with or without an afterburner, are used for supersonic flights, giving it an adequate thrust with good performance, an increase in the bypass ratio is, of course, limited because of the minimum enthalpy difference required in the nozzle. In addition, in low-bypass jets, since the fan outlet pressure is limited to the nozzle inlet pressure, a high-bypass ratio in such engines means inevitably that the fan pressure ratio must go to 1 or very near 1. This second problem can be solved if the fan pressure ratio is made independent of the overall compression ratio by keeping the two streams separated after the fan stage. In addition, for the engine operating at moderate flight speeds, a high-bypass fan can ensure a high propulsive efficiency with adequate thrust to fly large-sized commercial aircrafts. A sketch of a fanjet engine that can operate at moderate transonic speeds and is capable of both high efficiency (low specific fuel consumption) and high thrust is shown in Fig. 2.23a, and the corresponding ideal thermodynamic process appears in Fig. 2.23b. Since in the fanjet, part of the air, which goes through the main combustion chamber and the turbine, has already been used to supply oxygen for the combustion of fuel, not much oxygen is left to burn in the afterburner. Therefore, this type of engine, used mainly for commercial aircrafts, has little scope for an afterburner. However, consistent with the previous analysis for straight jets and low-bypass jets, an afterburner has been included in Fig. 2.23a, b schematically.

We now have, contrary to the low-bypass jets, a change in all the basic parameters from Points 5 to 6. These parameters are now $\pi_{\mathrm{c}} = p_3^o/p_2^o$ as the *overall compression ratio*, $\Theta_4 = T_4^o/T_\infty$ as the main *combustion chamber temperature* ratio, M_∞ as the *approaching flow Mach number*, b as the *bypass ratio*, $\Delta\Theta_{\mathrm{AB}}$ as the *stagnation temperature increase in the afterburner,* and $\pi_{\mathrm{f}} = p_8^o/p_7^o$ as the *fan compression ratio*.

Now,

$$\Theta_\infty = \Theta_1 = \Theta_2 = \frac{T_\infty^o}{T_\infty} = \frac{T_1^o}{T_\infty} = \frac{T_2^o}{T_\infty} = 1 + \frac{\gamma-1}{2}M_\infty^2 \tag{2.61a}$$

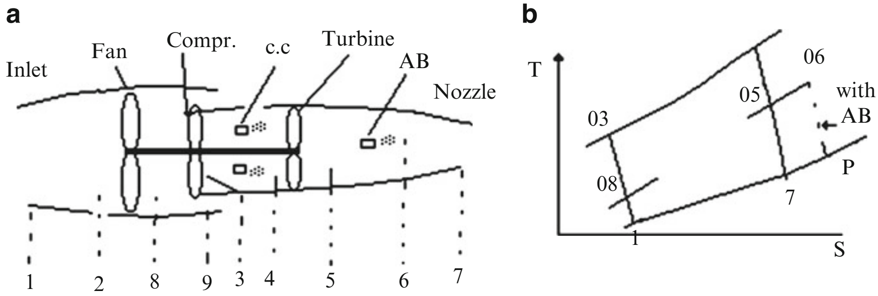

Fig. 2.23 (**a**) Schematic sketch of a fanjet engine with an afterburner; (**b**) the corresponding sketch of the thermodynamic process in a (T, s) chart

and

$$\delta_\infty = \delta_1 = \delta_2 = p^o_\infty = p^o_1 = p^o_2 = \left[1 + \frac{\gamma - 1}{2} M^2_\infty\right]^{\frac{\gamma}{\gamma-1}} \tag{2.61b}$$

Further,

$$\Theta_8 = \frac{T^o_8}{T_\infty} = \left[\left(\frac{p^o_8}{p^o_2}\right)\left(\frac{p^o_2}{p_\infty}\right)\right]^{\frac{\gamma-1}{\gamma}} = \Theta_\infty.\pi_{\mathrm{f}}^{(\gamma-1)/\gamma} = \pi_{\mathrm{f,eff}}^{(\gamma-1)/\gamma} = \tau_{\mathrm{f,eff}} = \Theta_\infty \tau_{\mathrm{f}} \tag{2.61c}$$

and

$$\delta_8 = \frac{p^o_8}{p_\infty} = \delta_\infty \pi_{\mathrm{f}} = \pi_{\mathrm{f,eff}} \tag{2.61d}$$

and similarly,

$$\Theta_3 = \frac{T^o_3}{T_\infty} = \Theta_\infty.\pi_{\mathrm{c}}^{(\gamma-1)/\gamma} = \pi_{\mathrm{c,eff}}^{(\gamma-1)/\gamma} = \tau_{\mathrm{c,eff}} = \Theta_\infty \tau_{\mathrm{c}} \tag{2.61e}$$

and

$$\delta_3 = \frac{p^o_3}{p_\infty} = \frac{p^o_4}{p_\infty} = \delta_\infty \pi_{\mathrm{c}} = \pi_\infty \pi_{\mathrm{c,eff}} \tag{2.61f}$$

Now, since the turbine drives the fan and the main compressor, we write from the energy balance that

$$c_{\mathrm{p}}\left[\left\{b\left(\left\{T^o_8 - T^o_2\right\}\right) + \left(\left\{T^o_3 - T^o_2\right\}\right)\right\}\right] = c_{\mathrm{p}}\left(T^o_4 - T^o_5\right)$$

and further,

$$b\left(\tau_{\mathrm{f,eff}} - \Theta_\infty\right) + \left(\tau_{\mathrm{c,eff}} - \Theta_\infty\right) = \Theta_4 - \Theta_5$$

Thus,

$$\Theta_5 = \frac{T_5^o}{T_\infty} = \Theta_4 - \Theta_\infty[(\tau_{\mathrm{f}} - 1)b + (\tau_{\mathrm{c}} - 1)] \tag{2.61g}$$

Defining a turbine expansion pressure ratio, $\pi_{\mathrm{t}} = p_5^o/p_4^o$, and a turbine expansion temperature ratio,

$$\tau_{\mathrm{t}} = \frac{T_5^o}{T_4^o} = \pi_{\mathrm{t}}^{(\gamma-1)/\gamma}$$

we write

$$\tau_{\mathrm{t}} = \frac{\Theta_5}{\Theta_4} = 1 - \frac{\Theta_\infty}{\Theta_4}[(\tau_{\mathrm{f}} - 1)b + (\tau_{\mathrm{c}} - 1)] \tag{2.61h}$$

and

$$\delta_5 = \delta_6 = \frac{p_5^o}{p_\infty} = \frac{p_6^o}{p_\infty} = \delta_4\left(\frac{\Theta_5}{\Theta_4}\right)^{\frac{\lambda}{\gamma-1}} = \delta_4\tau_{\mathrm{t}}^{\gamma/(\gamma-1)} \tag{2.61i}$$

On the other hand, $\Theta_6 = T_6^o/T_\infty = \Theta_5 + \Delta\Theta_{\mathrm{AB}}$.
Now,

$$\begin{aligned}\frac{T_6^o}{T_7} = \frac{T_7^o}{T_7} = \left(\frac{p_5^o}{p_\infty}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} &= \delta_5^{(\gamma-1)/\gamma} = \left(\frac{\Theta_5}{\Theta_4}\right)\delta_4^{(\gamma-1)/\gamma} \\ &= \pi_{\mathrm{c,eff}}^{(\gamma-1)/\gamma}\left[1 - \frac{\Theta_\infty}{\Theta_4}\{(\tau_{\mathrm{f}} - 1)b + (\tau_{\mathrm{c}} - 1)\}\right] \\ &= \tau_{\mathrm{c}}\Theta_\infty\left[1 - \frac{\Theta_\infty}{\Theta_4}\{(\tau_{\mathrm{f}} - 1)b + (\tau_{\mathrm{c}} - 1)\}\right]\end{aligned} \tag{2.61j}$$

and

$$\frac{T_7}{T_\infty} = \left(\frac{T_6^o}{T_\infty}\right)\left(\frac{T_7}{T_6^o}\right) = \Theta_6\frac{\Theta_4}{\left(\Theta_5\delta_4^{(\gamma-1)/\gamma}\right)} = \Theta_6\frac{\Theta_4}{\Theta_5\tau_{\mathrm{c,eff}}} \tag{2.61k}$$

Now, the kinetic energy of the exhaust gas in the main nozzle and the fan nozzle and that of the inlet gas are

$$\frac{u_7^2}{2c_{\mathrm{p}}T_\infty} = \frac{T_6^o}{T_\infty} - \frac{T_7}{T_\infty} = \Theta_6\left[1 - \frac{\Theta_4}{\Theta_5\tau_{\mathrm{c,eff}}}\right] \tag{2.62a}$$

$$\frac{u_9^2}{2c_\mathrm{p}T_\infty} = \Theta_\mathrm{s} - 1 = \tau_\mathrm{f,eff} - 1 \tag{2.62b}$$

$$\frac{u_\infty^2}{2c_\mathrm{p}T_\infty} = \Theta_\infty - 1 \tag{2.62c}$$

Therefore, the kinetic energy (nondimensional) of the gas per unit mass of total air is

$$\begin{aligned}\frac{\Delta E_\mathrm{kin}}{c_\mathrm{p}T_\infty} &= \frac{1}{(1+b)}\left[\frac{u_7^2 - u_\infty^2}{2c_\mathrm{p}T_\infty} + b\frac{\left(u_9^2 - u_\infty^2\right)}{2c_\mathrm{p}T_\infty}\right] \\ &= \frac{1}{1+b}\left[\Theta_6\left\{1 - \frac{\Theta_4}{\Theta_5\tau_\mathrm{c,eff}}\right\} + b\left(\tau_\mathrm{f,eff} - 1\right) - (1+b)(\Theta_\infty - 1)\right]\end{aligned} \tag{2.63}$$

Further, the heat added and rejected per unit mass of hot air are

$$\frac{|\overline{q}_\mathrm{a}|}{c_\mathrm{p}T_\infty} = (\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5) \tag{2.64a}$$

and

$$\frac{|q_\mathrm{r}|}{c_\mathrm{p}T_\infty} = \frac{T_7}{T_\infty} - 1 = \Theta_6 - \frac{\Theta_4}{\Theta_5\tau_\mathrm{c,eff}} - 1 \tag{2.64b}$$

The fuel–air ratio for hot gas is

$$f = \frac{\dot{m}_\mathrm{f}}{\dot{m}_\mathrm{H}} = \frac{c_\mathrm{p}T_\infty}{\Delta H_\mathrm{p}}[(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5)] \tag{2.65}$$

in which the first part is for the combustion chamber and the second part is for the afterburner, and the sum of the two must be equal to or less than the maximum fuel–air ratio allowed for the particular fuel.

Now, the work done *per unit mass of total air* is obtained from (2.64a) and (2.64b) as

$$\begin{aligned}\frac{w}{c_\mathrm{p}T_\infty} &= \frac{1}{(1+b)}\left[\frac{|\overline{q}_\mathrm{a}| - |q_\mathrm{r}|}{c_\mathrm{p}T_\infty}\right] \\ &= \frac{1}{(1+b)}\left[(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5) - \Theta_6\frac{\Theta_4}{\Theta_5\tau_\mathrm{c,eff}} + 1\right] \\ &\quad \frac{1}{(1+b)}\left[\left(1 + b\tau_\mathrm{f,eff} - (1+b)\Theta_\infty\right) + \Theta_6\left(1 - \frac{\Theta_4}{\Theta_5\tau_\mathrm{c,eff}}\right)\right]\end{aligned} \tag{2.66}$$

Adding and subtracting b within the brackets and performing some algebraic manipulation, it is easy for us to show that (2.66) is exactly the same as (2.63). We get also from (2.62a) to (2.62c) that

$$\frac{u_7}{u_\infty} = \frac{1}{M_\infty}\sqrt{\frac{2\Theta_6}{\gamma - 1}\left(1 - \frac{\Theta_4}{\Theta_5 \tau_{\mathrm{c,eff}}}\right)} \tag{2.67a}$$

and

$$\frac{u_9}{u_\infty} = \frac{1}{M_\infty}\sqrt{\frac{2}{\gamma - 1}\left(\tau_{\mathrm{f,eff}} - 1\right)} \tag{2.67b}$$

Further,

$$\begin{aligned}\frac{F}{\dot{m}_{\mathrm{a}} a_\infty} &= \frac{M_\infty}{1+b}\left[b\left(\frac{u_9}{u_\infty} - 1\right) + \left(\frac{u_7}{u_\infty} - 1\right)\right]\\ &= \frac{1}{1+b}\left[b\sqrt{\frac{2}{\gamma-1}\left(\tau_{\mathrm{f,eff}} - 1\right)} + \sqrt{\frac{2\Theta_6}{\gamma - 1}\left(1 - \frac{\Theta_4}{\Theta_5 \tau_{\mathrm{c,eff}}}\right)} - (1+b)M_\infty\right]\end{aligned} \tag{2.68a}$$

The *thermodynamic efficiency* is $\eta_{\mathrm{th}} = \overline{w}(1+b)/q_{\mathrm{a}}$, and with the help of (2.64a) and (2.66), we get

$$\begin{aligned}\eta_{\mathrm{th}} = &\frac{1}{(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5)}\\ &\times \left[1 + b\tau_{\mathrm{f,eff}} - (1+b)\Theta_\infty + \Theta_6\left\{1 - \frac{\Theta_4}{\Theta_5 \tau_{\mathrm{c,eff}}}\right\}\right]\end{aligned} \tag{2.68b}$$

Similarly, for the *propulsive efficiency,* we can write

$$\eta_{\mathrm{p}} = \frac{2\left[\left(\frac{u_7}{u_\infty} - 1\right)\right] + b\left(\frac{u_9}{u_\infty} - 1\right)}{\left[\left(\frac{u_7}{u_\infty}\right)^2 - 1\right] + b\left[\left(\frac{u_9}{u_\infty}\right)^2 - 1\right]} \tag{2.68c}$$

where for the speed ratio we can substitute an expression from (2.67a) and (2.67b). For the specific fuel consumption,

$$\mathrm{SFC} = \frac{\dot{m}_{\mathrm{f}}}{F} = \frac{c_{\mathrm{p}} T_\infty}{\Delta H_{\mathrm{p}} u_\infty}\,\frac{\left[(\Theta_4 - \Theta_3) + (\Theta_6 - \Theta_5)\right]}{\left[b\left(\frac{u_9}{u_\infty} - 1\right) + \left(\frac{u_7}{u_\infty} - 1\right)\right]} \tag{2.68d}$$

Table 2.6 Results of SFC* (Θ_4, π_c, π_f, b)

Θ_4	π_c	π_f	b	Θ_3	Θ_f	SFC*
4	12	1.2	3	2.034	3.615	0.0543
4	12	1.2	4	2.034	3.561	0.0347
4	12	1.2	5	2.034	3.508	0.0241
4	12	1.4	3	2.034	3.472	0.0543
4	12	1.4	4	2.034	3.371	0.0347
4	12	1.4	5	2.034	3.270	0.0241
4	16	1.2	3	2.208	3.585	0.0466
4	16	1.2	4	2.208	3.532	0.0297
4	16	1.2	5	2.208	3.479	0.0206
4	16	1.4	3	2.208	3.443	0.0466
4	16	1.4	4	2.208	3.342	0.0297
4	16	1.4	5	2.208	3.241	0.0206
4	20	1.2	3	2.353	3.456	0.0410
4	20	1.2	4	2.353	3.509	0.0260
4	20	1.2	5	2.353	3.455	0.0180
4	20	1.4	3	2.353	3.420	0.0410
4	20	1.4	4	2.353	3.319	0.0260
4	20	1.4	5	2.353	3.218	0.0180
5	12	1.2	3	2.034	4.615	0.0819
5	12	1.2	4	2.034	4.561	0.0523
5	12	1.2	5	2.034	4.508	0.0363
5	12	1.4	3	2.034	4.472	0.0819
5	12	1.4	4	2.034	4.371	0.0523
5	12	1.4	5	2.034	4.270	0.0363
5	16	1.2	3	2.208	4.585	0.0726
5	16	1.2	4	2.208	4.532	0.0463
5	16	1.2	5	2.208	4.479	0.0320
5	16	1.4	3	2.208	4.443	0.0726
5	16	1.4	4	2.208	4.342	0.0463
5	16	1.4	5	2.208	4.241	0.0320
5	20	1.2	3	2.353	4.562	0.0659
5	20	1.2	4	2.353	4.509	0.0419
5	20	1.2	5	2.353	4.455	0.0289
5	20	1.4	3	2.353	4.402	0.0659
5	20	1.4	4	2.353	4.319	0.0419
5	20	1.4	5	2.353	4.218	0.0289

from which we can write for the nondimensional specific fuel consumption

$$\begin{aligned}\mathrm{SFC}^* &= \mathrm{SFC}.\frac{\Delta H_\mathrm{p}}{a_\infty} = \frac{1}{(\gamma-1)M_\infty}\frac{[(\Theta_4-\Theta_3)+(\Theta_6-\Theta_5)]}{\left[b\left(\frac{u_9}{u_\infty}-1\right)+\left(\frac{u_7}{u_\infty}-1\right)\right]}\\ &= \frac{1}{1+b}\frac{[(\Theta_4-\Theta_3)+(\Theta_6-\Theta_5)]}{\left[b\sqrt{\frac{2}{\gamma-1}\left(\tau_{\mathrm{f,eff}}-1\right)}+\sqrt{\frac{2}{\gamma-1}\left(1-\frac{\Theta_4}{\Theta_5\tau_{\mathrm{c,eff}}}\right)}-(1+b)M_\infty\right]}\end{aligned} \tag{2.68e}$$

To anayze the results, we set $\Theta_\infty = 1$ or $M_\infty = 1$. Further, we let $\Delta\Theta_{AB} = 0$. We now evaluate the results for SFC* with four variables, namely, $(\Theta_4, \pi_c, \pi_f, b)$, and present them in Table 2.6. For these, Θ_4= 4 and 5, π_c= 12, 16, and 20, π_f= 1.2 and 1.4, and b = 3, 4, and 5.

By analyzing the results, we see that SFC* does not depend on π_f, but on the other three. It increases substantially with Θ_4 and decreases with an increase in π_c and the bypass ratio b. However, for every Θ_4, there is an optimum π_c, which can be investigated by the reader to get better specific fuel consumption. It is of historical interest to note that when in Boeing 707s, the old straight JT3 engine was replaced by the equivalent PW3D fanjet engine, there was an immediate 15% fuel savings. Needless to say, the trend in design of modern fanjets is to have a high turbine inlet temperature, high compressor pressure ratio, and high bypass ratio, which, again, the reader can verify by examining the most recent engines.

2.2.5 An Interactive Computer Program for Ideal Jet Engine Analysis (PAGIC)

An interactive computer program has been created to run on personal computers to study ideal jet engines' thermodynamic cycle. It is called PAGIC, and a listing is iven below. The program was tested with Professional Fortran compiler, but it should work with other FORTRAN compilers also. It works in an interactive manner by asking for relevant data and giving self-explanatory results. Before running the code, an unformatted binary data file containing six single-precision real data called AGRDT2 has to be created, which will be called by this code under Unit 7.

```
C     Interactive gasturbine performance program for ideal case (PAGIC)
      Program PAGIC
      CHARACTER IANS
      CHARACTER*10 AGTRD2
      DIMENSION VI(6),VF(9,2)
      G=1.4
      Write(6,*)' Interactive Gasturbine Program for Ideal Gas'
10    WRITE(6,*)' Type 1/2/3/4 for RJ/SJ/BJ/FJ: '
      READ(5,*)NSCOPE
      IF(NSCOPE.LT.1.OR.NSCOPE.GT.4)GOTO10
      IF(NSCOPE.EQ.1)WRITE(6,*)' Gasturbine type: Ramjet '
      IF(NSCOPE.EQ.2)WRITE(6,*)' Gasturbine type:
      Straightjet '
      IF(NSCOPE.EQ.3)WRITE(6,*)' Gasturbine type: Bypassjet '
      IF(NSCOPE.EQ.4)WRITE(6,*)' Gasturbine type: Fanjet '
      NNEW=1
      WRITE(6,*)' Is it o.k.? '
      READ(5,1001)IANS
      IF(IANS.NE.'y'.and.IANS.NE.'Y')GOTO10
14    OPEN(7,FILE='AGTRD2',STATUS='OLD',ACCESS='SEQUENTIAL',
        FORM=1'UNFORMATTED')
      READ(7)(VI(I),I=1,6)
```

```
11    CONTINUE
      IF(NSCOPE.EQ.1)WRITE(6,2001)(VI(I),I=1,2)
      IF(NSCOPE.EQ.2)WRITE(6,2002)(VI(I),I=1,4)
      IF(NSCOPE.EQ.3)WRITE(6,2003)(VI(I),I=1,5)
      IF(NSCOPE.EQ.4)WRITE(6,2004)(VI(I),I=1,6)
      IF(NNEW.EQ.1)WRITE(6,*)'any change? '
      IF(NNEW.EQ.1)READ(5,1001)IANS
      IF(NNEW.EQ.0)IANS='Y'
      IF(IANS.NE.'Y'.and.IANS.NE.'y')GOTO12
      NNEW=1
      WRITE(6,*)' Parameter no.,value: '
      READ(5,*)I,VAL
       VI(I)=VAL
      GOTO11
12    NTEST=0
      IF(NSCOPE.NE.1.AND.(VI(2).LE.1.OR.VI(3).LE.1.))NTEST=1
      IF(NSCOPE.EQ.4.AND.VI(6).LE.1.)NTEST=1
      IF(NTEST.EQ.1)WRITE(6,*)' input parameter error. Reenter '
      IF(NTEST.EQ.1)GOTO11
      REWIND(7)
      WRITE(7)(VI(I),I1,6)
      CLOSE(7)
      IF(NSCOPE.NE.1)GOTO13
      VI(3)=1.
      VI(4)=0.
      VI(5)=0.
      VI(6)=0.
13    CONTINUE
      EA=(G-1.)/G
      EB=1./EA
      TTRI=((G-1.)*.5*VI(1)**2)+1.
      TPRI=TTRI**EB
      DO16I=1,2
      VF(I,1)=TTRI
16    VF(I,2)=TPRI
      VF(3,1)=TTRI*VI(3)**EA
      IF(VF(3,1).GT.VI(2))GOTO50
      VF(3,2)=VI(3)*VF(1,2)
      VF(4,1)=VI(2)
      VF(4,2)=VF(3,2)
      PIC=VI(3)
      XMU=0.
      IF(NSCOPE.GT.2)XMU=VI(5)/(VI(5)+1.)
      GOTO(25,25,26,27),NSCOPE
```

```
25    VF(5,1)=VF(4,1)-(TTRI*(PIC**EA-1.))
      XA=VF(5,1)
      GOTO30
26    PIF=((((VF(4,1)-VF(3,1))*(1.-XMU))+TTRI)/(VF(4,1)-(XMU*(VF(4,1)
      1 -VF(3,1)))))**EB*PIC
      VI(6)=PIF
      VF(8,1)=TTRI*PIF**EA
      VF(9,1)=VF(4,1)-(TTRI*((PIC**EA-1.)+(XMU/(1.-XMU)*(PIF**EA
        1 -1.))))
      VF(8,2)=PIF*TTRI**EB
      VF(9,2)=VF(3,2)*(VF(9,1)/VF(4,1))**EB
      VF(5,1)=(XMU*VF(8,1))+(1.-XMU)*(VF(4,1)-(TTRI*((PIC**EA-1.)+
      1 (XMU/(1.-XMU)*(PIF**EA-1.)))))
      XA=VF(9,1)
      GOTO30
27    PIF=VI(6)
      VF(8,1)=TTRI*PIF**EA
      VF(8,2)=PIF*TTRI**EB
      VF(9,1)=VF(8,1)
      VF(9,2)=VF(8,2)
      VF(5,1)=VF(4,1)-(TTRI*((PIC**EA-1.)+XMU/(1.-XMU)*  (PIF**EA-1.)))
      XA=VF(5,1)
30    VF(5,2)=PIC*(TTRI*XA/VF(4,1))**EB
      VF(6,2)=VF(5,2)
      VF(7,2)=VF(6,2)
      AFBURN=0.
      IF(NSCOPE.GT.1)AFBURN=VI(4)
      VF(6,1)=VF(5,1)+AFBURN
      VF(7,1)=VF(6,1)
c     U7R=uinf/u7; T7R=T7/Tinf;T7RI=Tinf/T7
      T7R=VF(7,1)/VF(7,2)**EA
      T7RI=1./T7R
      XM7=SQRT(2./(G-1.)*((VF(7,1)*T7RI)-1.))
      U7R=VI(1)/XM7*SQRT(T7RI)
      QIN=VF(4,1)-VF(3,1)
      IF(NSCOPE.LT.3)QIN=QIN+AFBURN
      IF(NSCOPE.EQ.3)QIN=(QIN*(1.-XMU))+AFBURN
      IF(NSCOPE.EQ.4)QIN=(QIN+AFBURN)*(1.-XMU)
      QREJ=T7R-1.
      IF(NSCOPE.EQ.4)QREJ=QREJ*(1.-XMU)
      WJET=QIN-QREJ
      ETATH=WJET/QIN
       IF(NSCOPE.EQ.4)GOTO32
       ETAP=2.*U7R/(U7R+1.)
```

```
c     FPA=F/(pinf*A7)
      FPA=G*XM7**2*(1.-U7R)
      GOTO35
c     U9R=uinf/u9,UI9R=u9/u7=U7R/U9R
32    XM9=SQRT(2./(G-1.)*(VF(9,1)-1.))
      U9R=VI(1)/XM9
      UI9R=XM9/XM7*SQRT(T7RI)
      ETAP=2.*U7R*((1.-XMU)+(XMU*U9R)-U7R)/(1.-XMU+(XMU*U9R**2)
       -U7R 1 **2)
      FPA=G*XM7**2*(1.-U7R+(VI(5)*UI9R*(1.-U9R)))
35    A7AFR=SQRT(1.+(AFBURN/VF(5,1)))
c     FS=F/(mair*ainf); XMF=mfuel*fuelheat/(mair*ainf**2)
      FS=SQRT(2.*VF(6,1)/(G-1.)*(1.-(1./VF(6,2)**EA)))-VI(1)
      IF(NSCOPE.EQ.4)FS=(FS+(VI(5)*SQRT(2.*VF(8,1)/(G-1.)*(1.-(1./VF
      1 (8,2)**EA)))-VI(1)))/(VI(5)+1.)
      IF(NSCOPE.NE.3)XMF=(VI(2)-VF(3,1)+AFBURN)/(G-1.)
      IF(NSCOPE.EQ.3)XMF=(((VI(2)-VF(3,1))/(VI(5)+1.))+AFBURN)/(G-1.)
      IF(NSCOPE.EQ.4)XMF=XMF/(VI(5)+1.)
      SFCS=XMF/FS
      ETAT=ETAP*ETATH
      WRITE(6,*)' Performance of an Ideal Aircraft Gasturbine'
      IF(NSCOPE.EQ.1)WRITE(6,*)' Gasturbine type: Ramjet '
      IF(NSCOPE.EQ.2)WRITE(6,*)' Gasturbine type:
      Straightjet '
      IF(NSCOPE.EQ.3)WRITE(6,*)' Gasturbine type:
      Bypassjet '
      IF(NSCOPE.EQ.4)WRITE(6,*)' Gasturbine type: Fanjet '
      WRITE(6,1026)
      IF(NSCOPE.EQ.1)WRITE(6,2001)(VI(I),I=1,2)
      IF(NSCOPE.EQ.2)WRITE(6,2002)(VI(I),I=1,4)
      IF(NSCOPE.EQ.3)WRITE(6,2003)(VI(I),I=1,5)
      IF(NSCOPE.EQ.4)WRITE(6,2004)(VI(I),I=1,6)
      WRITE(6,1022)T7R,QIN,QREJ,WJET,FS,SFCS
      WRITE(6,1023)ETATH,ETAP,ETAT,XM7
      IF(NSCOPE.NE.4)WRITE(6,1030)U7R,FPA
      IF(NSCOPE.EQ.4)WRITE(6,1031)U7R,FPA,U9R,XM9,UI9R
      IF(AFBURN.NE.0.)WRITE(6,1024)A7AFR
      WRITE(6,1026)
      WRITE(6,1027)
      N=7
      IF(NSCOPE.GT.2)N=9
      DO37I=1,N
      WRITE(6,1028)I,VF(I,1),VF(I,2)
      CONTINUE
      WRITE(6,1026)
```

```
      WRITE(6,*)' New parameters? '
      NNEW=0
      READ(5,1001)IANS
      IF(IANS.EQ.'Y'.OR.IANS.EQ.'y')GOTO14
      WRITE(6,*)' New Gasturbine type? '
      READ(5,1001)IANS
      IF(IANS.EQ.'y'.OR.IANS.EQ.'Y')GOTO10
      STOP
50    WRITE(6,1032)VF(3,1),VI(2)
      GOTO 14
1001  FORMAT(A)
1022  FORMAT(1x,'T7/Tinf=',1pe10.3,' Heatinp=',1pe10.3,' Heatrej='
      1,1pe10.3,' Workjet=',1pe10.3,' F/(mair*ainf)=',1pe10.3,' sfc*='
      2,1pe10.3)

1023  FORMAT(1x,'etather=',1pe10.3,' etaprop=',1pe10.3,' etatot='
      1,1pe10.3,' M7=',1pe10.3)
1024  FORMAT(1X,'A7AB/A7=',1pe10.3)
1026  FORMAT(1H)
1027  FORMAT(3X,'I',3X,'T0*',7X,'P0*')
1028  FORMAT(2X,I2,1P2E10.3)
1030  FORMAT(1x,'Uinf/U7=',1pe10.3,' F/(pinf*A7AB)=',1pe10.3)
1031  FORMAT(1x,'Uinf/U7=',1pe10.3,' F/(pinf*A7AB)=',1pe10.3,' Uinf/',
      1 'U7=',1pe10.3,' M9=',1pe10.3,' U9/U7=',1pe10.3)
1032  FORMAT(1x,'error since compr. stagn. temp=',1pe10.3,' is larger'
      1,' than comb. exit temp.=',1pe10.3)
2001  FORMAT(1x,'1. Appr. Mach=',F6.3,/,1x,'2. Temp.ratio=',F6.3)
2002  FORMAT(1x,'1. Appr. Mach=',F6.3,/,1x,'2. Temp.ratio=',F6.3,/,1x,
      1 '3. Overall compr. ratio=',F6.3,/,1x,'4. Afterburner delta theta'
      2,'=',F6.3)
2003  FORMAT(1x,'1. Appr. Mach=',F6.3,/,1x,'2. Temp.ratio=',F6.3,/,1x,
      1 '3. Overall compr. ratio=',F6.3,/,1x,'4.
      Afterburner delta theta'
      2,'=',F6.3,/,1x,'5. Bypass ratio=',F6.3)
2004  FORMAT(1x,'1. Appr. Mach=',F6.3,/,1x,'2.
      Temp.ratio=',F6.3,/,1x,
      1 ' 3. Overall compr. ratio=',F6.3,/,1x,'4.
      Afterburner delta theta'
      2,'=',F6.3,/,1x,'5. Bypass ratio=',F6.3,/,1x,'6.
      Fan compr.ratio'
      3,'=',F6.3)
      END
```

2.3 Exercises

1. Explain graphically the difference in the cyclic process in a two-stroke and a four-stroke engine. What are the advantages and disadvantages of each of them? Given no. of cylinders = 6, displacement volume per cylinder = 0.995 l, weight = 148.0 kgf, specific fuel consumption = 72.7 μg/J, rpm = 2,600 min^{-1}. Assume a combustion chamber temperature. Compute the gas state (p, T, V) at the four corners of the ideal thermodynamic cycle and determine per cylinder the work done, heat added, heat rejected, air and fuel mass flow rates, and specific fuel consumption, and compare the results given in the piston engine databank.
2. For the Kawasaki KT5311A Japanese turboprop/turboshaft engine for helicopter applications, the following data are given in the databank for aircraft turboprop engines: mass flow rate = 5.0 kg/s, takeoff thrust = 600 N, shaft power = 819 kW, overall compression ratio in two spools = 6.1, weight = 225 kgf, specific fuel consumption = 116.3 μg/J, no. of axial compressor stages = 5 + 1, rpm = 25,200 and 21,200 min^{-1}, and no. of turbine stages = 1 + 1. Compute the work done, heat added, heat rejected, air and fuel mass flow rate, and the specific fuel consumption, and compare these with the values in turboprop databank.
3. Running the interactive code PAGIC given in Section 2.2.5, investigate the performance parameters of the various types of jet engines.
4. A ramjet is to propel an aircraft at Mach 3 (entry is designed to be shock-free under design condition), where the ambient pressure is 0.085 bar and the ambient temperature is 220 K. The combustion chamber exit temperature is 1,800 K and the usual aviation gasoline (heating value: ΔH_p = 42,700 kJ/kg) is used. Compute the following: (a) the fuel–air ratio f (make sure that $f < f_{\text{stoich.}}$), (b) exhaust velocity u_e, (c) specific impulse, (d) thermodynamic efficiency, (e) propulsive efficiency, and (f) overall efficiency.
5. The performance of an ideal ramjet burning aviation gasoline at stoichiometric fuel–air ratio is to be calculated for different supersonic Mach numbers. The engine is to fly at an altitude of 15 km, where the ambient pressure is 0.116 bar and the ambient temperature is 205 K.
6. Let's consider the following specification of the Canadian straight-jet engine from United Aircrafts Canadian Ltd. (UACL), model no. JT115D-4, mass flow rate = 34.1 kg/s, takeoff thrust = 10,600 N, overall compression ratio = 10.0, turbine inlet temperature = 960°C. Consider running the engine near the ground with ambient pressure = 1.0 bar, ambient temperature = 298 K, approaching flow velocity = 250 m/s. Compute the gas state at the characteristic points of the engine for an ideal cycle analysis, the thrust, specific thrust, fuel-to-air mass ratio, work output, heat added and heat released, and thermodynamic, propulsive, and overall efficiency. Examine if the compression ratio is optimum for the given temperature ratio. Draw the cycle in a (T, s) chart.
7. The idling engines of a landing turbojet produce forward thrust when operating in a normal manner, but they can produce reverse thrust if the jet is properly deflected.

Suppose that while the aircraft rolls down the runway at 150 km/h, the idling engine consumes air at 50 kg/s and produces an exhaust velocity of 150 m/s.

(a) What is the forward thrust of this engine?
(b) What are the magnitude and direction (that is, forward or reverse) if the exhaust is deflected 90°?
(c) What are the magnitude and direction of the thrust (forward or reverse) after the plane has come to a stop, with 90° exhaust deflection and an airflow of 40 kg/s? (Problem taken from Hill and Peterson (1992), p. 209, with thanks.)

8. Consider the following specifications of the French SNECMA low-bypass turbojet engine LARZAC04: mass flow rate = 27.6 kg/s, takeoff thrust = 13.2 kN, overall compression ratio = 10.7, turbine inlet temperature = 1,130°C, SFC = 20.10 mg/N·s, bypass ratio = 1.1. For the given bypass ratio, obtain the fan pressure ratio. Compute the gas state at the characteristic points of the engine for an ideal cycle analysis, the thrust, specific thrust, SFC, fuel-to-mass ratio, work output, heat added and heat released, and thermodynamic, propulsive, and overall efficiency. Draw the cycle in a (T, s) chart.
9. Consider the specifications of the Pratt & Whitney P&W JT9D-3A high-bypass turbofan engine: mass flow rate = 684.0 kg/s, takeoff thrust = 169.9 kN, overall compression ratio = 21.5, fan pressure ratio = 1.6, turbine inlet temperature = 1,243°C, SFC = 17.84 mg/N·s, and bypass ratio = 5.2. Compute the gas state at the characteristic points of the engine for an ideal cycle analysis, the thrust, specific thrust, SFC, fuel-to-mass ratio, work output, heat added and heat released, and thermodynamic, propulsive, and overall efficiency. How much air is sent through the core engine and how much through the outer fan? Draw the cycle in a (T, s) chart.

Springer

http://www.springer.com/978-1-4614-3531-0

Airbreathing Propulsion
An Introduction
Bose, T.
2012, X, 314 p., Hardcover
ISBN: 978-1-4614-3531-0