

Contents

Part I Introduction and Some Relevant Preliminary Material

1	Exact Tests for Contingency Tables and Discrete Exponential Families	3
1.1	Independence Model of 2×2 Two-Way Contingency Tables.....	3
1.2	2×2 Contingency Table Models as Discrete Exponential Family	8
1.3	Independence Model of General Two-Way Contingency Tables	10
1.4	Conditional Independence Model of Three-Way Contingency Tables	14
1.4.1	Normalizing Constant of Hypergeometric Distribution for the Conditional Independence Model....	18
1.5	Notation of Hierarchical Models for m -Way Contingency Tables	19
2	Markov Chain Monte Carlo Methods over Discrete Sample Space	23
2.1	Constructing a Connected Markov Chain over a Conditional Sample Space: Markov Basis	23
2.2	Adjusting Transition Probabilities by Metropolis–Hastings Algorithm	27
3	Toric Ideals and Their Gröbner Bases	33
3.1	Polynomial Ring	33
3.2	Term Order and Gröbner Basis	35
3.3	Buchberger’s Algorithm	38
3.4	Elimination Theory.....	39
3.5	Toric Ideals	39

Part II Properties of Markov Bases

4	Definition of Markov Bases and Other Bases	47
4.1	Discrete Exponential Family	47
4.2	Definition of Markov Basis	50
4.3	Properties of Moves and the Lattice Basis	51
4.4	The Fundamental Theorem of Markov Basis	54
4.5	Gröbner Basis from the Viewpoint of Markov Basis	59
4.6	Graver Basis, Lawrence Lifting, and Logistic Regression	60
5	Structure of Minimal Markov Bases	65
5.1	Accessibility by a Set of Moves	65
5.2	Structure of Minimal Markov Basis and Indispensable Moves	66
5.3	Minimum Fiber Markov Basis	71
5.4	Examples of Minimal Markov Bases	72
5.4.1	One-Way Contingency Tables	72
5.4.2	Independence Model of Two-Way Contingency Tables	73
5.4.3	The Unique Minimal Markov Basis for the Lawrence Lifting	73
5.5	Indispensable Monomials	75
6	Method of Distance Reduction	79
6.1	Distance Reducing Markov Bases	79
6.2	Examples of Distance-Reducing Proofs	81
6.2.1	The Complete Independence Model of Three-Way Contingency Tables	81
6.2.2	Hardy–Weinberg Model	83
6.3	Graver Basis and 1-Norm Reducing Markov Bases	85
6.4	Some Results on Minimality of 1-Norm Reducing Markov Bases	86
7	Symmetry of Markov Bases	91
7.1	Motivations for Invariance of Markov Bases	91
7.2	Examples of Invariant Markov Bases	92
7.3	Action of Symmetric Group on the Set of Cells	93
7.4	Symmetry of a Toric Model and the Largest Group of Invariance	96
7.5	The Largest Group of Invariance for the Independence Model of Two-Way Tables	98
7.6	Characterizations of a Minimal Invariant Markov Basis	100

Part III Markov Bases for Specific Models

8	Decomposable Models of Contingency Tables	109
8.1	Chordal Graphs and Decomposable Models	109
8.2	Markov Bases for Decomposable Models	111

8.3	Structure of Degree 2 Fibers	113
8.4	Minimal Markov Bases for Decomposable Models	115
8.5	Minimal Invariant Markov Bases	119
8.6	The Relation Between Minimal and Minimal Invariant Markov Bases	127
9	Markov Basis for No-Three-Factor Interaction Models and Some Other Hierarchical Models	129
9.1	No-Three-Factor Interaction Models for $3 \times 3 \times K$ Contingency Tables	129
9.2	Unique Minimal Markov Basis for $3 \times 3 \times 3$ Tables	130
9.3	Unique Minimal Markov Basis for $3 \times 3 \times 4$ Tables	139
9.4	Unique Minimal Markov Basis for $3 \times 3 \times 5$ and $3 \times 3 \times K$ Tables for $K > 5$	142
9.5	Indispensable Moves for Larger Tables	145
9.6	Reducible Models	149
9.7	Markov Basis for Reducible Models	150
9.8	Markov Complexity and Graver Complexity	153
9.9	Markov Width for Some Hierarchical Models	156
10	Two-Way Tables with Structural Zeros and Fixed Subtable Sums ...	159
10.1	Markov Bases for Two-Way Tables with Structural Zeros	159
10.1.1	Quasi-Independence Model in Two-Way Incomplete Contingency Tables	159
10.1.2	Unique Minimal Markov Basis for Two-Way Quasi-Independence Model	161
10.1.3	Enumerating Elements of the Minimal Markov Basis	164
10.1.4	Numerical Example of a Quasi-Independence Model	167
10.2	Markov Bases for Subtable Sum Problem	168
10.2.1	Introduction of Subtable Sum Problem	168
10.2.2	Markov Bases Consisting of Basic Moves	169
10.2.3	Markov Bases for Common Diagonal Effect Models	172
10.2.4	Numerical Examples of Common Diagonal Effect Models	176
11	Regular Factorial Designs with Discrete Response Variables	181
11.1	Conditional Tests for Designed Experiments with Discrete Observations	181
11.1.1	Conditional Tests for Log-Linear Models of Poisson Observations	181
11.1.2	Models and Aliasing Relations	184
11.1.3	Conditional Tests for Logistic Models of Binomial Observations	191
11.1.4	Example: Wave-Soldering Data	193

11.2	Markov Bases and Corresponding Models for Contingency Tables	194
11.2.1	Rewriting Observations as Frequencies of a Contingency Table	194
11.2.2	Models for the Two-Level Regular Fractional Factorial Designs with 16 Runs	200
11.2.3	Three-Level Regular Fractional Factorial Designs and 3^{s-k} Contingency Tables	203
12	Groupwise Selection Models	209
12.1	Examples of Groupwise Selections.....	209
12.1.1	The Case of National Center Test in Japan	209
12.1.2	The Case of Hardy–Weinberg Models for Allele Frequency Data.....	212
12.2	Conditional Tests for Groupwise Selection Models	213
12.2.1	Models for NCT Data	214
12.2.2	Models for Allele Frequency Data	215
12.3	Gröbner Basis for Segre–Veronese Configuration	217
12.4	Sampling from the Gröbner Basis for the Segre–Veronese Configuration	219
12.5	Numerical Examples	219
12.5.1	The Analysis of NCT Data	219
12.5.2	The Analysis of Allele Frequency Data	221
13	The Set of Moves Connecting Specific Fibers	229
13.1	Discrete Logistic Regression Model with One Covariate	229
13.2	Discrete Logistic Regression Model with More than One Covariate	231
13.3	Numerical Examples	238
13.3.1	Exact Tests of Logistic Regression Model	238
13.4	Connecting Zero-One Tables with Graver Basis	240
13.5	Rasch Model	241
13.6	Many-Facet Rasch Model	242
13.7	Latin Squares and Zero-One Tables for No-Three-Factor Interaction Models	245
 Part IV Some Other Topics of Algebraic Statistics		
14	Disclosure Limitation Problem and Markov Basis	251
14.1	Swapping with Some Marginals Fixed	251
14.2	<i>E</i> -Swapping	252
14.3	Equivalence of Degree-Two Square-Free Move of Markov Bases and Swapping of Two Records	253
14.4	Swappability Between Two Records	254
14.5	Searching for Another Record for Swapping	257

15 Gröbner Basis Techniques for Design of Experiments	261
15.1 Design Ideals	261
15.2 Identifiability of Polynomial Models and the Quotient with Respect to the Design Ideal	262
15.3 Regular Two-Level Designs	267
15.4 Indicator Functions	269
16 Running Markov Chain Without Markov Bases	275
16.1 Performing Conditional Tests When a Markov Basis Is Not Available	275
16.2 Sampling Contingency Tables with a Lattice Basis	275
16.3 A Lattice Basis for Higher Lawrence Configuration	277
16.4 Numerical Experiments	278
16.4.1 No-Three-Factor Interaction Model	278
16.4.2 Discrete Logistic Regression Model	282
References	287
Index	295



<http://www.springer.com/978-1-4614-3718-5>

Markov Bases in Algebraic Statistics

Aoki, S.; Hara, H.; Takemura, A.

2012, XII, 300 p., Hardcover

ISBN: 978-1-4614-3718-5