Logical Oppositions in Arabic Logic: Avicenna and Averroes

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Abstract In this paper, I examine Avicenna's and Averroes' theories of opposition and compare them with Aristotle's. I will show that although they are close to Aristotle in many aspects, their analysis of logical oppositions differs from Aristotle's by its semantic character, and their conceptions of opposition are different from each other and from Aristotle's conception. Following Al Fārābī, they distinguish between propositions by means of what they call their "matter" modalities, which are determined by the meanings of the propositions. This consideration gives rise to a precise distribution of truth-values for each kind of proposition, and leads in turn to the definitions of the logical oppositions. Avicenna admits the four traditional oppositions, while Averroes, who seems closer to Aristotle and especially to Al Fārābī, does not mention subalternation, but admits subcontrariety. Nevertheless, we can find that Averroes defends what Parsons calls SQUARE and [SQUARE], because he holds **E** and **I**-conversions and the truth conditions he admits are just those that make all the relations of the square valid, while Avicenna defends SQUARE and [SQUARE] only for the waṣfī reading of assertoric propositions. They also give a special attention to the indefinite which in Averroes' view is ambiguous, while Avicenna treats it as a particular. Some points of their analysis prefigure the medieval concepts and distinctions, but their opinion about existential import is not as clear as the medieval one and does not really escape the modern criticisms.

Keywords Logical oppositions · Matter necessity · Possibility and impossibility · Existential import · Indefinites · waṣfī vs ḍātī readings of propositions

Mathematics Subject Classification Primary 03A05 · Secondary 03B10

1 Introduction

In this paper, I will examine Avicenna's and Averroes' views on the specific topic of the so-called logical oppositions in order to compare between them both on the one hand and between them and Aristotle on the other hand. As is well known, Aristotle identifies opposition with incompatibility, since the only logical relations that he holds to be oppositions are contradiction and contrariety. But can one say the same thing about Avicenna and Averroes? Are their respective accounts of the notion of opposition distinct from Aristotle's one? Are they distinct from each other? How are logical oppositions characterized in their respective systems? And how is the notion of opposition itself viewed in these systems? In order to answer these questions, I will start by analyzing Avicenna's system then

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I will turn to Averroes' one and finally, I will compare between them both and Aristotle in order to determine the way they define this notion. This comparative analysis shows that the notion of opposition is different in the three systems, and gives rise to three quite distinct figures.

2 Avicenna's Analysis of the Logical Oppositions

The logical oppositions are analyzed by Avicenna (980–1037) in his book entitled *al-shifā'* (*The Cure*), more precisely in *al-Maqūlāt* (*Categories*) and *al-'Ibāra* (*Peri Hermeneias*). In *al-'Ibārā*, he defends his views on oppositions and presents what we would consider as a square of oppositions since he analyzes the four traditional relations which are: contradiction, contrariety, subcontrariety and subalternation. But before turning to his analysis of these relations let us first see how he defines the notion of opposition itself. This definition is presented in *al-Maqūlāt* (*Categories*) where Avicenna considers many types of oppositions which could concern the terms or the propositions. He defines opposition in general by saying that "The opposites do not conjoin in the same subject by any aspect in any time" [12, p. 241].[1] The opposites could be terms or propositions expressed by sentences, and opposition itself could be expressed by means of negation or by other means; as examples, he gives the following pairs: "horse/non-horse" and "even/odd". Avicenna follows here Aristotle's *Categories* (10) when he distinguishes between oppositions by virtue of correlation such as "double" and "half", of possession and privation, of contrariety such as "sick" and "healthy" and finally the opposition of truth-values. This last one affects propositions and is expressed by means of a negation; thus singular propositions such as "Zayd is a man" and "Zayd is not a man" are opposed because they do not have the same truth-value. The real opposition between propositions is, then, the opposition between truth-values. This opposition in truth-values is specifically made by the negation since Avicenna, like Aristotle, thinks that sentences that contain opposite predicates are not contradictories when the subject does not exist, because in that case, they are both false. Avicenna gives the following example: "Zayd who does not exist [*al ma'dūm*] is seeing" does not contradict "Zayd who does not exist is blind", but it does contradict "Zayd who does not exist is not seeing" [12, p. 259], because this last sentence is true. In the same vein, the sentence "Stones are sick" does not contradict "Stones are healthy" but it does contradict "Stones are not sick" [12, p. 258], because the first two sentences are false while the last one is true. But the notion of opposition is more general than contradiction, contrariety or correlation. It can be seen as a genus that includes several species [12, p. 245].

How can we apply this to the different kinds of propositions that are expressed by singular, quantified or non quantified sentences? To answer this question let us see what Avicenna says in *al-'Ibāra*. In this book, he classifies propositions into three kinds: (1) Singulars, (2) Indefinites (i.e. not quantified) and (3) Quantified, i.e. Universal and Particular propositions. We have to notice here that Avicenna uses a specific term to indicate the presence of a quantification. The quantifier is expressed by the word '*sūr*' [14, p. 52] and

[1] Wilfrid Hodges in [19, p. 13] translates this passage in the following way: "We say: opposing pairs are those which don't combine in a single subject from a single aspect at a single time together."

the quantified propositions by '*musawwara*'.[2] This word '*sūr*' is also used by Al Fārābī in *al-Qawl fi al-'Ibārā*, for instance. Al Fārābī explicitly mentions Alexander of Aphrodisias and other commentators of Aristotle in many of his writings, which could explain the closeness of his terminology to the Greek commentators' one.[3] He defines the quantifier in the following way: "It is the word that indicates that the judgment made by the predicate is about part of the subject or about the total subject" [1, p. 118] he adds that there are four quantifiers, which are "All, None (lā' wāḥid), Some and Not all" [1, p. 118] and are used in the four kinds of propositions. Avicenna mentions the same classification in *al-'Ibāra* [14, p. 54], but Averroes seems to be clearer on that point since he talks about only two quantifiers by saying: "I mean by quantifier the words «all» and «some»" [9, p. 91]. This generic word '*sūr*' is not, however, exactly equivalent to the modern quantifier, for (1) in Avicenna's and Al Farabi's account, such words may be mixed with negation, the separation from negation occurs only in Averroes' account, (2) the particular quantifier does not stress specifically on existence, as is the case with the modern existential quantifier. But this grouping may be seen as prefiguring the medieval distinction between two separate kinds of terms: the *syncategorematic* and the *categorematic* terms. As it is expressed by Jean Buridan, this distinction occurs between terms that signify by themselves, and "may be subject or predicate *per se*" in propositions (the *categorematic* terms), and terms that do not signify in isolation but only in connection with other terms in the proposition (the *syncategorematic* terms, e.g. 'not', 'or' and the like). Terms like 'nobody', 'nothing', 'somewhere' are said to be 'mixed' [17, p. 96]. As noted by [24, section 4], the origin of the words '*syncategorematic*' and '*categorematic*' is grammatical and can be found in "Priscian's *Institutiones grammaticae* II, 15". The grammatical distinctions are also made by Arabic logicians, since they distinguish between the noun, the verb and the particle, which is defined exactly like the *syncategorematic* terms: the particle does not signify in isolation but only in connection with something else. But they do not provide a complete listing of *logical syncategorematic* terms.

Let us now turn to the logical oppositions as they are defined by Avicenna. We will focus here on his *al-'Ibāra*, since we find them stated there quite systematically, but we will mention also *al-Qiyās*. Not surprisingly, Avicenna considers the singular propositions as being contradictories, that is, they never share the same truth-value. Whenever a singular proposition is negated, it becomes false if the affirmative is true and vice versa.[4] However, with respect to other kinds of propositions, that is, the quantified and the non quantified propositions, Avicenna distinguishes between several possibilities which are treated in much detail. This treatment reveals many kinds of oppositions, which are all related in one way or another with the notion of truth-value.

Let us start by the quantified propositions. These are the particular and the universal propositions; they are explicitly quantified by adding in one case 'some' and in the other

[2] Avicenna uses also the expression 'maḍkūrat as sūr' and the word 'maḥṣūra' to designate the quantified propositions. As to Al Fārābī, he uses the expression 'ḍawāt al aswār' (i.e. those that contain quantifiers) [2, p. 121].

[3] I thank one anonymous referee who drew my attention to the fact that the word '*sūr*' is "heir to the notion of «*prosdiorismos*» in the Greek commentators of Aristotle".

[4] However in [14, p. 70], Avicenna says that the singular propositions which are in the future are not necessarily true or false and seems to agree with Aristotle in his treatment of the problem of the "future contingents", although he gives more details on the possible propositions.

case ‘all’. When we add negations, the universal negative is expressed by “No A is B” and the particular negative by “Not all A are B”. As in the Aristotelian tradition, Avicenna considers that the universal negative (**E**) and the particular affirmative (**I**) on the one side, and the universal affirmative (**A**) and the particular negative (**O**) on the other are contradictories, that is, they never share the same truth-value. But what is added is the subdivision of these quantified propositions into three kinds which are: Necessary, Impossible and Possible. Necessary, Possible and Impossible must be understood here in terms of the relation between the subject and the predicate. The necessity, possibility or impossibility is internal and is not expressed by a specific word. As an example of a necessary proposition, Avicenna gives the following: “Every man is an animal”, this proposition is necessary because of the fact that being an animal is an essential attribute of men. The example corresponding to an impossible proposition is the following: “No man is a stone” [14] which expresses the fact that “stone” cannot be a feature of the subject “man”. This sentence expresses an impossible proposition because Avicenna defines material impossibility in this way: it is “what is permanent and whose affirmation is necessarily false” [14, p. 47].[5] This means that in an impossible proposition, the predicate is never adequate for the subject, which makes the affirmative proposition always false (and consequently the negative one always true). In a possible proposition, the predicate does not express an essential attribute of the subject, but could be predicated of it; the example is a sentence where the subject is “man” and the predicate “writer”. When we add the universal quantifier, we obtain the following proposition “All men are writers” which is false as well as the corresponding universal negative which is “No man is a writer” [14, p. 46]. These modalities are called “matter”[6] modalities because they are related to the essences of the objects concerned and express material necessity or impossibility or possibility. When the inherence of the predicate into the subject is permanent, the proposition is necessary, when it is not, the proposition is possible, when the predicate is never convenient for the subject, the proposition is impossible. Necessity and impossibility are related to the notion of permanence, while in the notion of possibility, there is no permanence. These modalities must, however, be distinguished from the explicit (verbal) modalities which are expressed by specific words such as “necessary” (= *wājib*), “possible” (= *mumkin*) and “impossible” (= *mumtana‘*). Avicenna makes a clear distinction between the two kinds of modalities by saying that a sentence which contains an explicit modality could be false as is the case with the following example “All men are necessarily writers” [14, p. 112], while a sentence with a matter modality is never false when it is affirmative and necessary, for instance. The falsity of this sentence is explained by the fact that it has “a modality which disagrees with its matter” [14, p. 112]. “Matter” modalities are considered in the general theory of (categorical) syllogisms while explicit modalities are studied in the theory of

[5]The Arabic sentence is the following: “... yadūmu wa-yajibu kadhibu ījābihi ... yusammā māddat al- imtinā‘.”

[6]We find the expression “*matter modalities*” in Al Fārābī’s text too and the word *matter* seems to have a long history starting from Aristotle and his Greek commentators. The *matter* (*hūlē*) in Aristotle is opposed to the *tropos*, which has a rather vague sense and could mean the *form* of the proposition (see [18, p. 298]). In Ammonius’ text, it is related to modalities since he says in his *De Interpretatione*: “These relations <between subject and predicate> they call the matter of propositions and they say they are necessary, impossible or possible” (cited in [8, p. 233]). The necessity, possibility or impossibility are, according to this author, “due to the very nature of the objects” [8, p. 233]. In Avicenna’s text, the modal sense is clear as we have seen.

modal syllogisms. This notion of "matter" modalities could be related to the medieval notions "*materia necessaria*", "*materia contingenti*", and "*materia impossibili*", which were discussed by many authors "in early medieval logic and <were> dealt with in mid-thirteenth-century books", for instance, in Thomas Aquinas' writings "who wrote that universal propositions are false and particular propositions are true in contingent matter (*In Perihem.*I. 13, 168)" [21, Sect. 3]. Similar distinctions are made by William of Sherwood, according to the same author [21]. This distribution of truth-values is exactly the same in Avicenna's (and Al Fārābī's) analysis, as will appear in the following list:

A necessary: True	E necessary: False
A impossible: False	E impossible: True
A possible: False	E possible: False
I necessary: True	O necessary: False
I impossible: False	O impossible: True
I possible: True	O possible: True

These truth conditions follow directly from what Avicenna says about the truth-values of the considered propositions [14, p. 47]. It shows that the contradictory propositions which never share the same truth-value in any matter are, when they are quantified, **A** and **O** on the one hand and **E** and **I** on the other. We have thus six contradictory propositions which are: (1) Necessary **A** and Necessary **O**, (2) Impossible **A** and Impossible **O**, (3) Possible **A** and Possible **O**, (4) Necessary **E** and Necessary **I**, (5) Impossible **E** and Impossible **I**, and (6) Possible **E** and Possible **I**.[7]

All these oppositions are contradictions since in all these pairs, only one proposition is true, the other being false. Avicenna explains this very precisely by saying exactly which one is true and which one is false in all the cases. They are then totally opposed *whatever matter they may have* as he notes in the chapter devoted to the analysis of the contradictory propositions [14, pp. 66–75]. This means that contradiction is the strongest kind of opposition.

The second kind of opposition is contrariety. The contrary propositions are as in the Aristotelian tradition the two universal propositions, that is, **A** and **E**. As we can see, they do not share the same truth-value when they are Necessary and when they are Impossible since Necessary **A**: "Every man is an animal" is true and opposed to Necessary **E**: "No man is an animal" which is false. Impossible **A**: "Every man is a stone" is false and opposed to Impossible **E**: "No man is a stone" which is true. But these propositions do share the same truth-value when they are Possible, since Possible **A**: "Every man is a writer" and Possible **E**: "No man is a writer" are both false. Contrariety is, then, defined in the traditional way: it is the relation between propositions that are never true together but might be false together. But the cases of truth and falsity are determined more precisely by taking into consideration the matter of the propositions. It is less strong than contradiction because it concerns only two propositions (**A** and **E**) opposed in two modes.

Then, he considers subcontrary propositions which are the two particular propositions. As the table shows, these do not share the same truth-value when they are Necessary and when they are Impossible; but they are both true when they are Possible. Thus, **I** necessary: "Some men are animals" is true and opposed to **O** Necessary: "Not all men are

[7]Unlike Aristotle, Avicenna does not use the word "contingency" in [14, p. 122].

animals" which is false, and **I** impossible: "Some men are stones" is false and opposed to **O** Impossible: "Not all men are stones" which is true. Possible particulars are exemplified by: "Some men are writers" and "Not all men are writers" which are both true. Subcontrariety is then an opposition which makes the propositions never false together but sometimes true together. This is even less strong than contrariety since we have two propositions opposed in two modes, but the propositions are true in the third mode, thus not opposed at all in that mode: this makes subcontrariety less strong than contrariety.

Finally, we have the subaltern propositions which are **A** and **I** on the one hand and **E** and **O** on the other, the word used for subalternation being "*Tadākhul*". These are opposed when they are possible as we can see in the following examples: **A** possible: "Every man is a writer" is false, while **I** possible: "Some men are writers" is true. The same holds with **E** possible: "No man is a writer", which is false, while **O** Possible: "Not all men are writers", is true. But they do share the same truth-value when they are Necessary (they are both true) and when they are Impossible (they are both false).

What is interesting here is the way Avicenna expresses this relation, since he says: "As to those that differ in quantity but not in quality, ***let us call them*** subalterns, we ***find*** that those which are affirmative are true in the Necessary, and that the negative subalterns are true in the Impossible, and both do not share the same truth-value in the Possible, but the particulars are true in that case, and examine that by yourself" [14, p. 48, my emphasis]. This shows that his characterization of this kind of opposition and the other ones follows from the observation of the distribution of the propositions' truth-values which in turn depends upon the senses of the propositions involved: this makes it even *more semantic* than in Aristotle's account. The semantic character is related both to the consideration of the meanings of the propositions and to the distribution of truth-values which is quite systematic in Avicenna's account, while it is not in Aristotle's and the traditional logicians' one, even if they define also the oppositions by considering the truth-values of the propositions. Avicenna, unlike Aristotle and the traditional logicians, does not say: since the contrary propositions are never true together *therefore*, when one of them is true, the other must be false; rather, he *finds that*, in consideration of the distribution of the truth-values of all kinds of propositions that *they are never true together*. He presents then a kind of truth table similar to the well known contemporary semantic method, though less achieved since only the cases of truth are considered, but not all the cases. We see then that his method is the ancestor of the semantic contemporary method of truth tables.

Regarding subalternation, it is not defined exactly as in Aristotle's text since Aristotle says: "For in demolishing or establishing a thing universally we also prove it in particular; for if it belongs to all, it belongs also to some, and if to none, not to some" [5, III, 6, 119a, 34–36]. Aristotle does not mention any differences in truth-values between the propositions, but rather relations of implication. Moreover, he does not consider subalternation as an opposition at all. But we can notice that, according to Avicenna, subalternation, even if characterized in a different way, is the less strong opposition since only two pairs of propositions in the Possible matter are opposed by their truth-values. We can notice also that the Arabic word used by Avicenna to express this relation, which is "*Tadākhul*" is *not* synonymous with the traditional word "Subalternation" since it does not have the same linguistic meaning. While "Subalternation" derives from the Latin words "*alte*r" which means "other" and "*sub*" which means "under" and evokes the notion of dependence (upon the other) and thus implication, the Arabic word comes from the root "*dakhala*" which is a verb meaning "to enter", the other verb, which is closer to the word used, is "*tadākhala*" and means "to enter into each other". The ideas involved then are the ideas of inclusion and of the relation between the whole and the part: the part is included into the whole, therefore what is true of the whole is true of the part. It seems that Avicenna has chosen this word by himself without relying on a specific tradition, which appears clearly in the preceding quotation where he says: "*let us* call them [...]".[8]

The oppositions involve more or less differences in truth-values but the differences are either total or partial i.e. concern only some cases. The strongest opposition is contradiction since it involves all the pairs of propositions concerned and all the modes or matters but the other ones are different in degree so that we can say that subalternation is the less strong one, while contrariety, which involves one pair of propositions and two matters (the propositions being false in the third matter), and subcontrariety, which involves also one pair of propositions and two matters, are intermediates. As we can see, the oppositions between quantified propositions lead to a Square of oppositions in Avicenna's view since he admits the four oppositions.

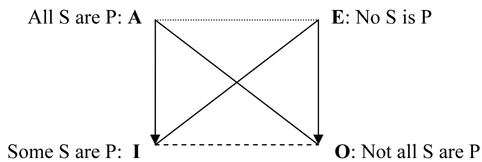
Regarding the indefinites, Avicenna tends to defend an Aristotelian position according to which these propositions should be considered as particulars even though they do not contain explicitly any quantification. This opinion is expressed explicitly at page 51 of *al-'Ibāra* where he says "the indefinite has the force of the particular". But he spends much time and place to explain why this should be so. Being perfectly aware that this kind of propositions might be considered, in ordinary usage, as universal propositions, he tries to explain that one should avoid this kind of interpretation because it might be confusing and even misleading. For when we add a universal quantifier to an indefinite proposition, it might become false while it was true without quantification. For instance, when we say "White is necessarily white" this is true, but when a quantifier is added, we obtain the following sentence: "All what is described as white is necessarily white" [14, p. 52] which is false according to him, since what is white now might not be white later.

But what happens if we negate an indefinite proposition? Does it become false if its corresponding affirmative is true? According to Avicenna, the indefinite when negated, is *not* the contradictory of its corresponding affirmative. He says that at page 67 where he claims: "the indefinite has no contradictory" and also "the indefinites [...] are like the particulars, they should be said to be subcontraries" [14, p. 66, my translation] since they might be true together as witnessed by the two following sentences: "Men are beautiful", "Men are not beautiful" [14, p. 67]. But Avicenna gives examples of indefinite sentences which do not share the same truth-value, such as "Stones are sick" and "Stones are not sick" [12, p. 258], and he says explicitly that such sentences are contradictory, as appears in the following quotation: "Two contradictories are not false together [such as] when we say: 'Stones (or Zayd who does not exist) are seeing', 'Stones (or Zayd who does not exist) are not seeing' " [12, p. 259]. In other passages of *al-'Ibāra*, he defends the opinion that, although the indefinites seem contradictory in impossible and necessary matters, one should treat them in general without focusing on the different matters, since he says: "One must treat the indefinites as propositions, which is more general than the three matters, and not consider them matter by matter. So the indefinite in the necessary matter, in so far as it is an indefinite, is a particular judgment" [14, p. 69]. From these quotations, one can conclude the following: the indefinites are true in the possible, but they do not

[8] As far as I know, there is no mention of this word in Al Fārābī's texts; Al Fārābī does not mention nor include subalternation in his treatment of oppositions.

share the same truth-value in the impossible and the necessary. Their truth conditions are then the same as those of the two particulars. But this is not quite satisfying since if we consider the indefinite as a particular and only as a particular, first it should behave as such in all circumstances, which is not obvious nor warranted, secondly, the particular has a contradictory which is the universal negative and this does not fit with what Avicenna says about the fact that the indefinite has no contradictory and makes his opinion somewhat confused as we will show in the last part. But we could say the same thing about Aristotle himself who tends to consider the indefinite as a particular without treating it exactly as a particular.

The shape corresponding to this analysis of the opposition is then a square since the four oppositions are admitted and the indefinite is not characterized with enough preciseness. This square is the following:



It seems then that Avicenna defends what Terence Parsons calls SQUARE, that is, all the relations of the square for the propositions he examined in that first treatise. But the question remains to determine whether or not he defends SQUARE, as well as [SQUARE] i.e. the relations of the square plus **E** and **I** conversions, for all the readings of the propositions that he talks about in *al-Qiyās*. We have also to examine his treatment of the question of existential import. We will return to both topics in the last section.

3 Averroes' Views on the Oppositions

Regarding Averroes (1126–1198), things are different because his aim in writing his treatises was to comment on Aristotle's logical writings. These treatises are the following: (1) *Talkhīṣ Kitāb al-Maqūlāt* [Paraphrase of the *Categories*], (2) *Talkhīṣ Kitāb al-'Ibāra* [Paraphrase of the *Peri Hermeneias*], (3) *Talkhīṣ Kitāb al Analitīqa al Awwil* (or *al-Qiyās*) [Paraphrase of the *Prior Analytics*], (4) *Talkhīṣ Kitāb al Analitīqa at thānī* (or *al-Burhān*) [Paraphrase of the *Posterior Analytics*), (5) *Talkhīṣ Kitāb al-Jadal* [Paraphrase of *The Topics*], (6) *Talkhīṣ Kitāb al-Moughālaṭa* [Paraphrase of the *Sophistical Refutations*]. They have been grouped and edited recently by Gérard Jehamy under the title *Talkhīṣ Mantiq Aristū* (Lebanon, 1982). We will also mention the Cairo editions of these different treatises.

As we can see from the very beginning, Averroes follows Aristotle's text faithfully. But this does not mean that his opinions are exactly similar to Aristotle's as we will see in the following. For it happens to him to diverge from Aristotle's text even if his aim is to explain it. For instance, he writes sometimes "He (i.e. Aristotle) says... and we say..." [9, p. 92, p. 137, etc.] which shows that when commenting the text, he gives his opinion on it.

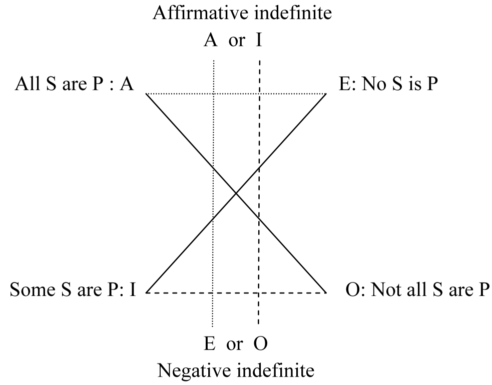
Averroes says that there are exactly six oppositions between the different kinds of propositions. These kinds are the following: (1) The singular propositions, (2) The indefinite propositions, (3) The quantified propositions which are the particular and the universal propositions. The first opposition is the one between the two singular propositions which is, as in Avicenna and Aristotle's views, a contradiction without any doubt. The second is the opposition between the two indefinites or non quantified propositions: these are in Averroes' view either contraries when they are meant to be universal propositions, or subcontraries if they are meant to be particular propositions. The indefinites could be true in possible "matters", in which case they are particular and would be subcontraries, but if in that same matter they are interpreted as universal, then they are contraries [9, pp. 92–93]. The example he gives to illustrate this fact is "Men are white" and "Men are not white" both propositions are true if they are interpreted as particular, but if they are interpreted as universal, both are false. Their relation depends then upon how one interprets the possible indefinites, since when the matters of the indefinite propositions are impossible or necessary, they never share the same truth-value, whether they are universal or particular. For instance "Men are animals" and "Men are not animals" differ in their truth-values: the first is true, the second is false, whether they are particular or universal, and we can say the same thing about "Men are stones" which is false and "Men are not stones" which is true. This means that, according to Averroes, the indefinites are ambiguous because in the vernacular language both interpretations are admissible. He does not share Aristotle's opinion, nor Avicenna's and Al Fārābī's [1, p. 122] one, because all of them treat the indefinite as a particular, which makes the two indefinites be subcontraries. However, this position is not quite convincing for several reasons that we will examine in the last section.

The third is the opposition between quantified propositions. These are the following: the contradictories, the contraries and the subcontraries. If we consider that there are two pairs of contradictories, we have really six oppositions, which are: (**1**) *Singulars*, (**2**) *Indefinites*, (**3**) *Contradictories*$_1$, (**4**) *Contradictories*$_2$, (**5**) *Contraries*, and (**6**) *Subcontraries*.

The contradictories are those which never share the same truth-value such as the singular propositions. Other contradictories are the quantified propositions which never share the same truth value "in all matters" [9, p. 92]. These are **A** and **O** on the one side and **E** and **I** on the other. To illustrate this, he gives the following example: "All men are white" and "Not all men are white". Then we have the contraries which are the two universal propositions and do not share the same truth value "in the Necessary and the Impossible" [9, p. 92] but are both false when they are Possible. The examples given are the following: "Every man is white", "No man is white" which are both false. The subcontraries are the two particular propositions, which are never true or false together in the Impossible and the Necessary but are true together in the Possible.

However, like Aristotle (and Al Fārābī), Averroes does *not* mention the subalterns which he seems to ignore completely. This might be explained by the fact that he is commenting on the *Peri Hermeneias* in which we don't find any mention of the subalterns. Regarding the treatise corresponding to the *Topics*, which is *al-Jadal* [10, p. 558], Averroes does not say anything different from Aristotle for he just summarizes his ideas by saying that what is true of the whole (or is false of it) is also true (or false) of the part, without entering into more details. He does not even use the word "*Tadākhul*" which was used by Avicenna and so known by the Arabic logicians. This shows that he probably does not consider subalternation as an opposition or else that he did not want to adopt Avicenna's views but rather to return to Aristotle's ones (even if it is with some modifications) and probably to Al Fārābī's opinions, since Al Fārābī admits exactly the same kinds of oppositions: contradiction, contrariety and subcontrariety and differs from Averroes only in his treatment of the indefinites which, according to him, are subcontrary.

It seems then that he follows Al Fārābī's and Avicenna's views by adopting the same classification of matters, but he is closer to Aristotle and especially to Al Fārābī in his classification of the kinds of oppositions. He differs from Aristotle by considering subcontrariety as a *real* opposition while it is only a *verbal* opposition in Aristotle's view and from Aristotle, Al Fārābī and Avicenna by considering explicitly the indefinites as ambiguous while in these authors' view they ought to be considered as particulars. The shape corresponding to this classification is, then, the following if we do not include the singulars:



This shape is different from Aristotle's and Avicenna's ones. It shows that the indefinite might be included inside the square since it has the specificity of being ambiguous and could not be assimilated either to the particular or to the universal. If we add the singular propositions, we have an even more extended shape which will contain one more horizontal line similar to the diagonals. However, if 'or' is taken as an inclusive disjunction, the proposition '**A** or **I**' is equivalent to '**I**' and the proposition '**E** or **O**' is equivalent to **O**,[9] when **A** and **E** have existential import, given that both **I** and **O** have existential import. This makes these two vertices superfluous, hence there is no hexagon since the ambiguous character of the indefinites disappears. However when **A** and **E** do not have existential import, while **I** and **O** do have it, there is no equivalence and the propositions '**A** or **I**' and '**E** or **O**' remain ambiguous and different respectively from **I** and **O**. The problem is then to determine whether **A**, **E, I** and **O** have existential import or not in Averroes' theory. It makes no doubt that existential import is attributed to **A** and to **I** as is the case with Aristotle and Al Fārābī, but what about **E** and **O**? According to many authors, Aristotle himself gives to both universals an existential import, but others such as Terence Parsons, say on the contrary that only affirmatives have existential import in Aristotle's theory, the two negatives being free of it, because **O** is expressed in the following way: "Not all S are P", which allows it not to have an import (see [23]). As to Averroes, he says that **O** might be expressed either by "Not all S are P" or by "Some S are not P" (the example given is: "Not all men are white", and "Some men are not white" [9, p. 92]) which suggests that, according to him, both formulations are equivalent since he adds "as to the particular negative, it is expressed in both ways" [9, p. 92]. But the second formulation means that **O** does have an existential import; if it

[9]This observation is due to Fabien Schang. I thank him for having pointed it out to me in an informal discussion.

is equivalent to the first one, then **O** has an import according to Averroes. As to Aristotle's text, if we follow Tricot, the French translator of *De Interpretatione*, then **O** has an import since he says in [6, note 1, p. 90]: "The following Aristotle's example: $o\acute{\upsilon}$ $\pi\alpha\varsigma$ $\alpha\nu\theta\rho\omega\pi o\varsigma$ $\lambda\varepsilon\upsilon\varkappa o\varsigma$ is translated in Latin by *non omnis homo est halbus*, which is equivalent to *quidam homo non est albus*, which we have expressed in French by *quelque homme n'est pas blanc*". In the English translation, however, the same example is translated simply as: "Not every man is white" [3, p. 5]. And the ancient Arabic translation is also the same as the English one since Aristotle's example is expressed as: "Not all men are white" [7, p. 106]. This means that regarding Aristotle's text, the Latin translation is ambiguous, but the other ones corroborate Parsons' theory. As to Averroes, however, the text is clear: **O** could be expressed both ways, and this means that it can have an existential import. Regarding the singular proposition, Averroes defends Aristotle's position, saying that whenever the subject does not exist, the sentences "Socrates is sick" and "Socrates is healthy" are both false, while the negative sentence "Socrates is not sick" is true [9, p. 66]. So if we generalize this to the universal negative, we would say that it does not have an existential import in Averroes' account. But things are not so clear for Averroes seems to treat both universals in the same way in *al-Qiyās*, for instance, since he states that "the universal negative is the one where the predicate is negated from the whole of the subject as when we say 'no single man is a stone' " ([9, p. 138], [11, p. 62]), while "the universal affirmative is the one where the predicate is affirmed of the whole subject" ([11, p. 138], [9, p. 62]). This way of expressing things suggests that in the negative universal, the negation puts on the predicate. If this is so, then **E** has an import as well as **A.** Therefore if the disjunction is inclusive, '**E** or **O**' will be equivalent to **O**.

But if we consider 'or' as an exclusive disjunction i.e. '**A** or **I** but not both' and '**E** or **O** but not both', the indefinites remain also ambiguous and different from the quantified propositions. However, this creates other problems which we will consider in our last part. Anyway the figure is more complex then Aristotle's and Avicenna's ones since we could also add the singular propositions which are explicitly included by Averroes into the class of opposed propositions and construct a new kind of hexagon where the new line is horizontal. The problem is that Averroes did not specify precisely the logical relations between the singulars and the other types of propositions, so we could not really credit him with the discovery of this kind of hexagon. Furthermore, he did not consider subalternation as an opposition which makes his figure different from a closed hexagon.

But Averroes' position about the notion of opposition itself is different from both Avicenna's and Aristotle's ones. For in Aristotle's view, the notion of opposition is meant to be incompatibility since he admits only contradiction and contrariety while in Avicenna's view, the opposition is defined by the difference in truth-values but there are different oppositions which are more or less strong depending on the number of propositions involved. In Averroes' view subcontrariety is considered as an opposition but subalternation is not, which means that he considers opposition as a plural notion but it is limited to three main patterns: it is thus less extended than Avicenna's notion but more extended than Aristotle's. In the last part, we will try to characterize the differences and the affinities between these three theories.

4 Differences and Affinities Between the Three Views

According to Aristotle, only contradiction and contrariety are considered as oppositions and the shape he admits is not really a square but just a fragment of it as has been shown by Terence Parsons [23] and other people. This means that the notion of opposition according to him must be understood in the following way:

Two propositions are opposed to each other if and only if:

(1) They have the same subject and the same predicate but one of them is affirmative and the other is negative
(2) Either they never share the same truth-value or they are never true together

By admitting only contradictory propositions which never share the same truth-value, and contrary propositions which are never true together, Aristotle considers opposition as being incompatibility. This shows as Jean-Yves Béziau, for instance, has noted that Aristotle "defends an asymmetrical view, privileging the principle of contradiction over the principle of excluded middle" [15, p. 224] since contrariety respects the first principle but not the second (contrariety is true when the two propositions are false and false when they are true, which is not in accordance with the principle of excluded middle), while subcontrariety respects the second principle. This creates according to J.-Y. Béziau, some kind of "asymmetry" which is not legitimate if we consider that both principles have the same importance and are "dual". Moreover, we can demonstrate that the two principles are equivalent to each other by using De Morgan's laws and the law of double negation. If we consider this equivalence, J.-Y. Béziau is right in saying that "it makes no sense" [15, p. 224] to admit contrariety and reject subcontrariety at least in a bivalent system, which makes Aristotle's view incomplete or even somewhat incoherent.

Regarding Avicenna, as we have seen, the notion of opposition could be seen as the difference of truth-values. Whenever there is such a difference, there is some kind of opposition; the propositions might then be opposed to each other totally or partially and there is some kind of graduation in the oppositions as we have seen in the first section. The notion of opposition itself seems then to be plural unlike Aristotle's notion and it goes from the strongest kind to the less strong one, which is subalternation. We could say that in this theory the opposition is either total (complete) or partial. This shows that both principles are respected: there is accordingly no asymmetry in the theory. The oppositions are characterized semantically by the distribution of the truth-values in the table lines corresponding to each of them. In his distribution of truth-values, Avicenna makes contradiction correspond to classical negation, contrariety to incompatibility ($|$) (or to the negation of conjunction $\sim(\wedge)$) since the values he retains are the three lines where this relation is true, subcontrariety to inclusive disjunction ($\vee$), and subalternation to implication ($\supset$), for the values he retains are just those which make these operators true. The distribution that Avicenna presents corresponds to the truth cases of these different operators, which shows that his characterization of the oppositions does not differ from the classical one but is more precise in that it determines exactly the cases of truth and falsity of the propositions.

We could then define opposition by distinguishing between a complete opposition and partial oppositions in the following way:

(I) Two propositions are opposed completely if and only if:
  (1) They have the same subject and the same predicate but one of them is affirmative and the other is negative
  (2) They never share the same truth-value whatever matter they have
(II) Two propositions are partially opposed if and only if:
  (1) They have the same subject and the same predicate but one of them may deny the other
  (2) They do not share the same truth-value in one or two matters

This distinction between the two kinds of oppositions is justified by the fact that Avicenna considers contradiction as the most important opposition and that he says that the opposition is "a genus which could be divided into species" [12, p. 245]. The attention he pays to the matter of the propositions could be justified by his definition of logic which, in his view, is a very general study which analyzes not only the forms of the arguments but also the matter of the propositions involved in them. He compares the logician to an architect who must take care not only of the shapes of his buildings but also of the materials he is using in order to arrive to a good result. In the same way the logician must take care of the form and the matter in order for the argument to be conclusive, for if the form is good but not convenient for the matter the argument will not be conclusive [13, pp. 6–7].

Avicenna agrees with Aristotle in giving a great importance to the principle of contradiction since he devotes a whole chapter [14, chapter 10] to the notion of contradiction and defines opposition by means of it. In his view, contradiction is the most complete opposition because it respects the principle of contradiction. He agrees with him also by considering that the opposition between propositions is introduced (most of the times) by negation.

However, his position regarding the indefinites, which are considered as particulars and are said to have no contradictory, is not very convincing. First, there is no reason why one kind of propositions could not have a contradictory, secondly if the affirmative indefinite is particular, its negation would be universal, which contradicts the general opinion that the indefinites (affirmative or negative) must be seen as particulars.

Averroes seems to be closer to Aristotle and to Al Fārābī than Avicenna is, for as we have seen, he follows Aristotle and his text faithfully and agrees with him in many points, for instance, in not considering subalternation as an opposition. Furthermore, the figure that corresponds to what he says about oppositions is very close to the one that could represent Al Fārābī's opinion about oppositions, the only difference being related to the indefinite, which would not be included in Al Fārābī's figure if it were drawn. But the theory he defends about opposition is, in the final analysis, slightly different from Aristotle's theory. For he admits subcontrariety as an opposition, which distinguishes him from Aristotle and he uses the same way as Avicenna in classifying the propositions into Necessary, Possible and Impossible, which we do not find in Aristotle's texts. Moreover, he distributes the truth-values in the same way as Avicenna. Regarding the indefinites, his opinion is different from the other ones since he considers it explicitly as ambiguous and not only as a particular. His opinion seems to be plural but restricted to three main kinds of oppositions, which makes it less limited than Aristotle's notion but more limited than Avicenna's one. The reason for that may be that he is not convinced by Aristotle's claim that subcontrariety is a verbal opposition, although he does not comment explicitly on this claim. But he thinks like Aristotle, that opposition involves a difference in the quality of the propositions concerned: this is clear from the conditions he states himself in order for an opposition to hold. These conditions are the following: (1) The subject and the predicate must be the same in all aspects in both propositions, (2) There must be only one affirmation and one negation, (3) There is only one negation opposed to a single affirmation [9, p. 94]. We could, then, define his notion of opposition in the following way:

Two propositions are opposed to each other if and only if:

(1) They have the same subject and the same predicate but one of them is affirmative and the other is negative
(2) Either they never share the same truth-value or they are never true together or they are never false together

But his treatment of the indefinites is not very convincing even if it does not contain incoherencies. For in his view, the indefinite has either a subcontrary or a contrary proposition depending on what it says; he does not say what its contradictory is and seems to share Avicenna's opinion that it does not have any contradictory. But as we have already noted, (1) there is no reason why one kind of proposition should not have a contradictory, (2) if we consider the disjunction as inclusive, the indefinite is no more ambiguous; therefore we could try to save this ambiguous character by considering that 'or' is exclusive so that the affirmative indefinite would mean "**A** or **I** but not both" and the negative one "**E** or **O** but not both". This "solution", however, is not quite satisfying because the indefinites would be in this case equivalent and not contraries nor subcontraries as we can show by considering their formulas.

A $\underline{\vee}$ **I** is formalized by:

$$\big[(\exists x)Sx \wedge (x)(Sx \supset Px)\big] \,\underline{\vee}\, (\exists x)(Sx \wedge Px),$$

where **A** has existential import.

E $\underline{\vee}$ **O** is formalized by:

$$\big[(\exists x)Sx \wedge (x)(Sx \supset {\sim}Px)\big] \,\underline{\vee}\, {\sim}(x)(Sx \supset Px).$$

If we consider a universe containing only two elements, that is, $\{x_1, x_2\}$, then **A** $\underline{\vee}$ **I** would be rendered thus:

$$\big\{(Sx_1 \vee Sx_2) \wedge \big[(Sx_1 \supset Px_1) \wedge (Sx_2 \supset Px_2)\big]\big\} \,\underline{\vee}\, \big[(Sx_1 \wedge Px_1) \vee (Sx_2 \wedge Px_2)\big].$$

And **E** $\underline{\vee}$ **O** is rendered thus if **E** has existential import:

$$\big\{(Sx_1 \vee Sx_2) \wedge \big[(Sx_1 \supset {\sim}Px_1) \wedge (Sx_2 \supset {\sim}Px_2)\big]\big\} \,\underline{\vee}\, {\sim}\big[(Sx_1 \supset Px_1) \wedge (Sx_2 \supset Px_2)\big].$$

If we construct the truth table of **A** $\underline{\vee}$ **I**, and that of **E** $\underline{\vee}$ **O**[10] we find that both formulas are just equivalent since the exclusive disjunction in both cases is false in all the lines except lines 2 and 3 where they are both true. This result does not correspond to Averroes' opinion and is not intuitively satisfying. If on the other hand, we consider that **A** and **E** have no existential import and the indefinites are expressed by '**A** $\vee$ **I**' and '**E** $\vee$ **O**', then we find by constructing the truth tables that the propositions are subcontraries since the inclusive disjunction is valid but no other relation expressing the square oppositions is valid, for $\supset$ is false in lines $1 + 5 + 6 + 9$, ${\sim}(\wedge)$ is false in lines $2 + 3 + 13$ to 16,

[10]The reader can check the values of this relation by himself.

and $\veebar$ is false in lines 2 + 3 + 13 to 16. Even if this interpretation is more satisfying intuitively than the preceding one, it does not correspond either to Averroes' text which says that the indefinites are either subcontrary or contrary. Here, they are only subcontrary. Furthermore, it is highly improbable that **A** does not have an import in Averroes' view, even if things are more ambiguous regarding **E**. Averroes' opinion seems then confused and not very convincing despite its plausibility.

What about the problem of existential import in Averroes' and Avicenna's theories? As we have seen earlier, their wording of **O** is indeed "Not all A are B", but Averroes equates this wording with "Some A are not B", which seems to give existential import to **O**. We can add that it happens also to Avicenna to express **O** as follows: "Some men are not writers" [14, p. 51], which raises also the problem of existential import for him; elsewhere he also says: "Not all men are writers, but rather some of them" [14, p. 54], which shows even more clearly that **O** has existential import, and seems close to the **Y** vertex defended by Blanché (see [16, p. 97]). So what Terence Parsons says about Aristotle, that is: "Aristotle's articulation of the **O** form is *not* the familiar 'Some S is not P' or one of its variants; it is rather 'Not every S is P'. With this wording Aristotle's doctrine automatically escapes the modern criticism" [23, section 2.2] does not seem to apply to our two Arabic logicians even if their wording is indeed 'Not all S are P'. According to Parsons, this wording of **O** solves all the problems about existential import. Parsons' argument is the following: if S is empty, **I** will be false, therefore, **E** will be true and then, "**O** must be true" because it is entailed by **E**; moreover, since **A** "has existential import", "if S is empty the **A** form must be false" [23, section 2.2]. This argument leads to the opinion that "*affirmatives* have existential import, and *negatives* do not" [23, section 2.2]. But (1) it is circular because it presupposes what has to be proved, that is, the validity of the relations of the square, (2) Parsons does not show how one can formalize the particular negative in a way that neutralizes its existential import. For if we formalize **A** by $(\exists x)Sx \wedge (x)(Sx \supset Px)$ [20, chapter 2, §26] and **O** by $\sim (x)(Sx \supset Px)$ (which corresponds literally to "Not all S are P") and construct a truth table by considering that there are only x_1 and x_2 in the universe, the following line of the table (where 1 means true, and 0 means false):

$$\begin{array}{l}
\{(Sx_1 \vee Sx_2) \wedge [(Sx_1 \supset Px_1) \wedge (Sx_2 \supset Px_2)]\} \veebar \sim[(Sx_1 \supset Px_1) \wedge (Sx_2 \supset Px_2)] \\
\quad 0\ 0\ 0 \quad 0 \quad 0\ 1\ 0 \quad 1 \quad 0\ 1\ 0 \qquad \mathbf{0}\ 0 \quad 0\ 1\ 0 \quad 1 \quad 0\ 1\ 0
\end{array}$$

shows that there is no contradiction since, as we can see, *there is* a case of falsity under $\veebar$ which means that $\veebar$, which is the exclusive disjunction, is not valid. This shows that "Not all S are P" should be formalized in another way since, when it is expressed as above (i.e. by $\sim(x)(Sx \supset Px)$), it is exactly equivalent to $(\exists x)(Sx \wedge \sim Px)$ and does not neutralize the existential import of **O**. So Parsons' solution would be convincing only if one gives the right formalization of **O** when it has no import.[11] Besides that, it is not obvious that the Aristotelian reading of **E** makes it free of existential import. On the contrary, many authors assume that Aristotle, as well as most traditional logicians, regards **E** as having existential import. This is, for instance, what Mark McIntire says in the following: "Classical logicians typically presupposed that universal propositions do have existential import" [22]. And this is also the opinion reported by Michael Wreen who says: "The chief difference

[11] The right reading of **O** when it is *without* import, is given by Parsons in [24]. It is the following: "Either nothing is A, or something is A that is not B" [24, p. 6].

between classical (Aristotelian) logic and modern (Russellian) logic, it's often said, is a difference of existential import. (1) In classical logic, all categorical propositions ("All S is P"; "Some S is P"; and so on) have existential import; in modern logic, particular affirmative (PA) and particular negative (PN) propositions do while universal affirmative (UA) and universal negative (UN) do not, have existential import" [26, p. 59]. According to that opinion, there is no difference between the universal affirmative and the universal negative regarding existential import since he says "*all* categorical propositions...". This interpretation of **E**-propositions is corroborated by Aristotle's text itself which says: "By universal, I mean a statement that something belongs to all or none of something; by particular that it belongs to some or not to some or not to all..." [4, I, 1, 24a, 17–19] and where he talks about **E**-propositions in the same way as about **A**-propositions: they both concern the whole of a certain class, **A** affirms something of that whole and **E** denies something of that same whole. But if **E** has existential import in Aristotelian framework, then the solution given by Parsons is not really Aristotelian as he says, but corresponds only to the Medieval theories. As we have seen, **O** without import is expressed by: "Either nothing is A, or something is A that is not B", and this wording corresponds to Ockham's and Buridan's interpretation of **O** (see [24, p. 5]).

As to the other relations, subalternation between **A** and **I** holds when **A** has an existential import, but the one between **E** and **O** does *not* hold when **O** is expressed by $\sim(x)(Sx \supset Px)$ (i.e. when **O** has existential import), as is shown by this line of the table:

$$\big[(Sx_1 \supset \sim Px_1) \wedge (Sx_2 \supset \sim Px_2)\big] \supset \sim\big[(Sx_1 \supset Px_1) \wedge (Sx_2 \supset Px_2)\big]$$

0 1 1 1 0 1 1 **0** 0 0 1 0 1 0 1 0

If **E** has an existential import, subalternation holds indeed. Regarding subcontrariety, which is expressed by: $(\exists x)(Ax \wedge Bx) \vee \sim(x)(Ax \supset Bx)$ when both propositions have existential import, things are not much better since it does not hold in that case. This means that the Aristotelian as well as the traditional views are still confused and contain some incoherencies. These incoherencies are not avoided by Avicenna and Averroes, who do not say explicitly that **E** and **O** are free of existential import and seem to assume that they do have an import.

But the medieval formalization of **O**, given by Parsons in [24], which could be expressed in the modern symbolism by: "$\sim(\exists x)Sx \vee \sim(x)(Sx \supset Px)$" or equivalently "$\sim(\exists x)Sx \vee (\exists x)(Sx \wedge \sim Px)$, even if it is a good way to render **O** when it has *no* import and appears to solve all the relations of the square, could be also challenged. This formalization is equivalent by De Morgan's law to the following one:[12] $\sim[(\exists x)Sx \wedge (x)(Sx \supset Px)]$, which says simply: it is not the case that there are S and that all these S are P. It neutralizes the existential import of **O** since what it says is just $\sim$**A** when **A** has existential import. With this formalization, and if **A** has existential import, **E** has no import and **I** has existential import, the relations of the square, i.e. $\sim(\mathbf{A} \wedge \mathbf{E})$, $\mathbf{A} \veebar \mathbf{O}$, $\mathbf{E} \veebar \mathbf{I}$, $\mathbf{A} \supset \mathbf{I}$, $\mathbf{E} \supset \mathbf{O}$, $\mathbf{O} \vee \mathbf{I}$ are all valid as the truth-tables show very clearly. To see this, let us take $\mathbf{A} \veebar \mathbf{O}$ and $\mathbf{O} \vee \mathbf{I}$ when **O** is formalized in that way, we have the following formulas where we consider a universe of only one element $\{x_1\}$ (but even with two elements, the relation is valid):

$\mathbf{A} : (\exists x)Sx \wedge (x)(Sx \supset Px) = Sx_1 \wedge (Sx_1 \supset Px_1)$ (with one element)
$\mathbf{O} : \sim[(\exists x)Sx \wedge (Sx \supset Px)] = \sim[Sx_1 \wedge (Sx_1 \supset Px_1)]$ (under the same conditions).

[12]This formalization of **O** comes from a suggestion made by Fabien Schang in an informal discussion. I thank him for fruitful discussion about this topic.

The exclusive disjunction is expressed by: $[Sx_1 \wedge (Sx_1 \supset Px_1)] \veebar \sim[Sx_1 \wedge (Sx_1 \supset Px_1)]$ and is without any doubt valid. If we formalize **I** by: $(\exists x)(Sx \wedge Px)$, subcontrariety is expressed by the following: $\sim[Sx_1 \wedge (Sx_1 \supset Px_1)] \vee (Sx_1 \wedge Px_1)$.[13] This formula is also valid. In the same way all the other relations of the square are valid, which means that these formalizations are the ones that show the validity of Parsons' and Buridan's solution. But we can show that this solution is not the only one that saves the relations of the square, since there are other alternatives that have the same effect, i.e. make all the relations of the square valid. These alternatives may be exhibited by a systematic examination[14] and they do not all require **O** to lack an import.

Now, as we have said earlier, this solution may be challenged, since the aforementioned formalization of **O** might seem to some people counter-intuitive. As a matter of fact, **O** may have an existential import in some cases. An example of such cases is mentioned by an anonymous referee who considers the following sentence: "Some politicians do not tell the truth". He rightly notes that if we translate **O** in the way Parsons and the medieval logicians translate it, this would lead to the following formulas: $(\exists x)Sx \supset \sim(x)(Sx \supset Px)$, which means: "If there are politicians, then not all of them tell the truth". This in turn leads to its equivalent formula by contraposition, that is: $(x)(Sx \supset Px) \supset \sim(\exists x)Sx$, which means: "If all the politicians tell the truth, then there are no politicians". This last formula seems very counter-intuitive, so the proposed formalization of **O** seems to be unacceptable, because of its undesirable consequences. We can answer by saying that this counter-intuitive character is related to the way the sentence is expressed, since that sentence has the following structure: "Some S are not P". And obviously, when one expresses **O** in that way one gives an import to it. But the formula we have given is not supposed to express **O** *with* import, on the contrary, it expresses an **O** which *does not have* an import. Our formula, as well as Terence Parsons' one is indeed a *reformulation* of **O** by "Not all S are P", which is supposed to account for an **O** *without* import. So the consequences seem in that case quite natural: they reflect the lack of import of that kind of negative particular. But this criticism shows above all that **O** should not be taken to always lack an import, and this is quite right, since in many cases, **O** does have an import. Anyway, this solution does not seem to be Averroes' and Avicenna's one, since what they say about the existential import of **O** is not as clear as what the medieval logicians say. Their wording of **O** does not mean that they do not give it an import.

What about their opinion about conversion? Let us start by Averroes, since his opinion is clearer than Avicenna's one. According to Averroes, **E-**conversion holds as well as **I**-conversion. **E**-conversion is stated very clearly in the following quotation: "As to the absolute universal premises, the negative converts in a way that preserves its quantity" ([11, p. 70], [9, p. 144]) and, **I**-conversion is also stated in the following: "As to the particular affirmative, I say also that it converts to a particular" ([9, p. 145], [11, p. 72]). So, if we consider that, despite the fact that Averroes does not talk about subalternation, he admits the same truth conditions for the different propositions as Avicenna, then we could say that he defends SQUARE as well as [SQUARE]. For these truth conditions state just the values that make true the implication between **A** and **I** on the one hand, and between **E** and **O** on the other. So we can say that subalternation holds indeed in his system even if it

[13]The reader may check the validity of this relation and all the others by constructing truth tables with the given formulas.

[14]This examination is made in another article written with Fabien Schang, which is under consideration.

is not really an opposition. Since he admits both conversions, we can say that he defends [SQUARE] too.

But things are more complex with Avicenna. We have seen that he holds SQUARE for the propositions he talks about in *al-Ibāra*. He also admits **I**-conversion for assertoric propositions. However, his opinion about **E**-conversion and about assertoric propositions in general is not the same as those defended by Aristotle and Averroes, since the analysis he presents in *al-Qiyās* distinguishes between two readings of this kind of propositions, and invalidates **E**-conversion in one of these two readings. To understand this, let us consider those readings. In [13], Avicenna defines the assertoric by saying that "it is more general than the necessary" [13, p. 28] because an assertoric sentence does not contain any modal word, so that what is important in that kind of sentences it does not require any necessity or non necessity in the relation between the subject and the predicate [13, p. 26]. Even if its matter is necessary, its necessity is different from perpetuity. For instance, the sentence "Every man is an animal" does not mean "Every man is perpetually an animal" (as when we say that "God is perpetually existent") but rather that "All men are animals as long as they exist" [*mā' dāma ḍātuhu wa jawharuhu mawjūdan*] [13, pp. 21–22]. In other absolute (or assertoric) sentences, we could have 'at some times' instead of 'as long as they exist' as in the following example: "All who wake sleep, (at some times)" [13, p. 23] (i.e. not necessarily 'as long as they exist'). This reading is called the ḍātī reading and is translated by Tony Street by the word 'substantial' [25, p. 551]. It is different from the waṣfī reading (translated as the "descriptional" reading [25, p. 551] which says the following: "All what is white is visible, as long as it is white" [13, p. 22] (one can also say: "while white" (see [25, p. 551]). Now in the ḍātī reading, the negative universal absolute does *not* convert, for if we take the following **A**-sentence: "All men are laughing (at some times)", which is an absolute one, and is true since it happens to everyone to laugh from time to time, we may have a corresponding absolute **E**-sentence, which would say: "No man is laughing, (at some times)", which means that no man could be said to laugh all the time, so that one can say that it happens to everyone not to laugh, at some times. This seems to be the way Avicenna understands this sentence since he says: "the predicate 'is laughing' can be negated from every man, at some times" [13, p. 82]. This sentence does not convert, because its conversion would lead to the following sentence: "No laughing thing is a man, (at some times)", which cannot be true, since "it is impossible to negate the predicate 'man' from what is laughing in effect" [13, p. 82] because a laughing thing cannot be said not to be a man. So for the ḍātī reading of absolute sentences, [SQUARE] does not hold, because conversion does not hold. Note that even SQUARE does not hold for this reading, since as we can see from the examples given (that is: "All men are laughing (at some times)" and "No man is laughing (at some times)") **A** and **E** may be true together, so they are no more contraries. But for the waṣfī reading, conversion may hold when we express **E**-sentences in the following way: "No As are Bs, while As". This may convert as: "No Bs are As, while Bs" [25, p. 551]. For instance: "Nothing that sleeps, wakes, while sleeping", which converts as "Nothing that wakes, sleeps while awake" [25, p. 551]. So, in this interpretation of absolute sentences [SQUARE] would hold as well as SQUARE, since when we say "All men are laughing, while men" and "No man is laughing, while being a man", both sentences are false together, and we can say that **A** and **E**, in this waṣfī reading, are contraries.

5 Conclusion

It follows from the above that Avicenna's and Averroes' treatment of the notion of opposition and of the opposed propositions are quite different from each other and different from Aristotle's treatment too. The notion of opposition appears to be stronger in Aristotle's view, it is considered as plural in Averroes' and Avicenna's views, but these authors differ in that Avicenna distinguishes between a strong opposition which is contradiction and other species which are less strong and seem to be partial oppositions while Averroes considers that the difference of quality is fundamental to define opposition and does not admit for this reason subalternation although he does admit subcontrariety unlike Aristotle. Their analysis seems also to prefigure the medieval distinctions and classifications and it is based on a method which we could characterize as semantic since it relies on a distribution of truth-values which follows itself from the meanings of the sentences. They seem to give an existential import to the quantified propositions, so their theories do not escape the modern criticisms. Nevertheless, each of them defends, in his own way, what Terence Parsons calls SQUARE and [SQUARE]. Avicenna's analysis is more subtle and complex than the two others, since it shows that for the substantive reading of the absolute propositions (the so-called ḍātī reading), neither SQUARE, nor [SQUARE] hold. But for the descriptive (or waṣfī) reading of the assertoric **E**-propositions, he defends both SQUARE and [SQUARE].

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