Preface

On the occasion of the *centennial birthday* of the mathematician *Dov Tamari* (1911–2006), born as the German *Bernhard Teitler*, this book commemorates his ground breaking work resulting in an *associativity theory*, with important contributions to the "word (decision) problem", as well as lattice theory and geometric combinatorics. The editors of this book invited designated researchers to present modern areas of mathematics that are related to Tamari's work.

To a monomial (word) formed from a set (of letters), one can assign different meanings by properly distributing brackets. If the bracketing expresses a binary operation on the set, associativity becomes an issue. Interpreting associativity as a (left- or rightward) substitution rule leads to what is known as a *Tamari lattice*. This partial order on a Catalan set (i.e., the number of its elements is a Catalan number) first appeared in 1951 in Dov Tamari's thesis at the Sorbonne in Paris. It turned out that these Tamari lattices possess realizations on special polytopes, called *associahedra*, which appeared in a different context in Jim Stasheff's thesis in 1961. In fact, associahedra already appeared in Tamari's thesis, but not in the part that was published. Since then these beautiful structures, and quite a number of important generalizations, have made their appearance in many publications in different areas of pure and applied mathematics, such as Algebra, Combinatorics, Computer Science, Category Theory, Geometry, Topology, and more recently also in Physics. It is this interdisciplinary nature of these structures that provides much of their fascination and value.

In the first chapter of this book, Folkert Müller-Hoissen and Hans-Otto Walther describe Tamari's extremely troubled life. When the Nazis came to power in Germany of 1933, he saw himself forced to leave Germany, losing the possibility of a smooth academic career he could have had in a less cruel political and social environment. All the obstacles along his further way luckily could not break his dedication and passion for mathematics. His uncompromising demand for honesty and fairness on all levels, including politics, surely did not make his life easier. The chapter about Dov Tamari also offers an elementary introduction to some aspects of his mathematical work. It is supplemented by Carl Maxson's reminiscences as a student of Tamari.

Jim Stasheff, whose name is firmly connected with associahedra, traces the latter back to Tamari's 1951 thesis, reviews their history, and leads the reader to modern developments. Jean-Louis Loday develops an arithmetic of (planar rooted binary) trees, a framework in which Tamari lattices find a natural place. He also reviews realizations of the latter as polytopes, the associahedra.

The further chapters in this book are of a somewhat more advanced nature. Susan Gensemer reviews the problem of extending a partially defined binary operation on a set (partial groupoid) to a completely defined associative binary operation (semigroup). This problem has been at the very roots of Tamari's mathematical research.

We grouped together articles that deal primarily with associahedra and related families of polytopes, and then those that center more around *Tamari lattices* and related families of posets.

Satyan Devadoss, Benjamin Fehrman, Timothy Heath and Aditi Vashist treat geometric and combinatorial aspects of the moduli space of "particles" on the Poincaré disk. In this framework, a generalization of associahedra shows up, the cyclohedra. Cesar Ceballos and Günter Ziegler summarize some mysteries and questions concerning realizations of the associahedra. A well-known class of polytopes, the permutahedra (also called permutohedra), and moreover polytopes obtained from Cambrian lattices, which generalize Tamari lattices, are the subject of Christophe Hohlweg's article, highlighting the role of finite reflection groups in their realizations. Further classes of polytopes, like flag nestohedra, graph-associahedra and graph-cubeahedra, arise from truncations of cubes, and their properties are described in the article by Victor Buchstaber and Vadim Volodin. Stefan Forcey reports on an extension of the Tamari order to families of polytopes called multiplihedra and composihedra, and explores the interplay between lattice structures and Hopf algebra structures.

Patrick Dehornoy presents an exhaustive study of the connection between Tamari lattices and the Thompson group F, which consists of a special class of piecewise linear homeomorphisms of the unit interval onto itself. Ross Street looks at the Tamari lattice as an example of an operad and dives into monoidal categories. Frédéric Chapoton considers the category of modules over the incidence algebra of a Tamari lattice, and also the derived category of the former. Hugh Thomas explains how the Tamari lattice arises in the context of the representation theory of quivers. Nathan Reading traces the way from Tamari lattices to Cambrian lattices, in the context of finite Coxeter groups, reviews the construction of Cambrian fans, and moreover makes contact with the important concept of cluster algebras. Filippo Disanto, Luca Ferrari, Renzo Pinzani and Simone Rinaldi present a unified setting for Dyck and Tamari lattices. Dyck lattices are a refinement of Tamari lattices. The restriction of the weak (strong) Bruhat order on permutations of a fixed length to "312-avoiding permutations" leads to the Tamari (Dyck) order. A generalization of the Tamari order to a partial order on the set of "tubings" of a simple graph is described in María Ronco's work. Jörg Rambau and Victor Reiner present a survey of higher Stasheff-Tamari orders (which first appeared in the work of Mikhail Kapranov and

Vladimir Voevodsky). These are posets defined on triangulations of cyclic polytopes and there is a relation with higher Bruhat orders (first introduced by Yuri Manin and Vadim Schechtman).

A physical realization of maximal chains of Tamari lattices in terms of tree-shaped soliton solutions of the Kadomtsev-Petviashvili (KP) equation, describing, e.g., shallow water waves, is the subject of the article by Aristophanes Dimakis and Folkert Müller-Hoissen. The analysis of KP solitons naturally leads to a reduction of higher Bruhat orders to higher Tamari orders, which is different from the relation described by Rambau and Reiner.

We hope that this book will convey to the reader a bit of the fascination that was experienced by those who contributed to the foundations and modern developments described in it.

Finally, we would like to thank Dr. Thomas Hempfling for his efficient help during the publishing process.

Göttingen, Dijon, Chapel Hill
October 2011

Folkert Müller-Hoissen
Jean Pallo
Jim Stasheff

http://www.springer.com/978-3-0348-0404-2

Associahedra, Tamari Lattices and Related Structures
Tamari Memorial Festschrift
Müller-Hoissen, F.; Pallo, J.M.; Stasheff, J. (Eds.)
2012, XX, 436 p., Hardcover
ISBN: 978-3-0348-0404-2
A product of Birkhäuser Basel