

Preface

In q -calculus we are looking for q -analogues of mathematical objects that have the original object as limits when q tends to 1. There are two types of q -addition, the Nalli-Ward-Al-Salam q -addition (NWA) and the Jackson-Hahn-Cigler q -addition (JHC). The first one is commutative and associative, while the second one is neither.

This is one of the reasons why sometimes more than one q -analogue exists. The two operators above form the basis of the method which unities hypergeometric series and q -hypergeometric series and which gives many formulas of q -calculus a natural form reminding directly of their classical origin. The method is reminiscent of Eduard Heine (1821–1881), who mentioned the case where one parameter in a q -hypergeometric series is $+\infty$. The q -addition is the natural way to extend addition to the q -case, as can be seen when restating addition formulas for q -trigonometric functions.

The history of q -calculus (and q -hypergeometric functions) dates back to the eighteenth century. It can in fact be taken as far back as Leonhard Euler (1707–1783), who first introduced the q in his *Introductio* [190] in the tracks of Newton's infinite series.

The formal power series were introduced by Christoph Gudermann (1798–1852) and Karl Weierstraß (1815–1897). In England, Oliver Heaviside (1850–1925) made yet another contribution to this subject.

In recent years the interest in the subject has exploded. Hardly a week goes by without a new paper on q . This is of course due to the fact that q -analysis has proved itself extremely fruitful in various fields and today has wide-ranging applications in such vital areas as computer science and particle physics, and also acts as an important tool for researchers working with analytic number theory or in theoretical physics.

The book has several aims. One is to give the student of q a basic insight and training in q -calculus or its equivalents, elliptic functions and theta functions. Another is to present the tools and methods that q -analysis requires and recount the history that has shaped the course of the q -discipline. Readers will find here a historical background, which has hitherto not been known to large parts of the mathematics and physics communities, since the treatments and theses containing the

early q -history from the 18th and 19th centuries were written in Latin, German and French. An example is the book *Theory of Finite Differences* by Nørlund (1924) [403] which includes Bernoulli and Euler polynomials. The book is written in German and thus not easily accessible to modern English-oriented scientists. Some of these results can therefore be said to be rediscovered and represented as important foundations of the early q -calculus.

Our book is furthermore an attempt to bring the history and the overall insight into q -analysis and calculus up-to-date and hopefully into the future also. Various Schools in q -analysis have sprouted over the last decades, e.g. and foremost the Watson School and the Austrian School, making the subject somewhat confusing for a ‘beginner’ to study, and also making it difficult to find and define a ‘common denominator’ and a normal nomenclature for q . ‘*A comprehensive treatment of q -calculus*’ (and later issues) therefore contains treatments and discussions of the very latest results and discoveries in the field of q , and furthermore presents a new and hopefully unifying logarithmic notation and an umbral method for studying q -hypergeometric series.



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