

# Preface

Simulation and optimization methods, powerful tools that had their origin in Operations research, have for several decades been successfully applied to the field of water management. But only in a few rare cases has there been any feedback mechanism or direct cooperation with scientists in other areas of applied mathematics. Generally speaking, it has turned out that solving of real problems along narrow disciplinary lines has hardly led to significant innovations. Pushing the limits of these boundaries could provide a greatly widened scope for synergy and innovation. The development of complex model systems and their use for deriving optimal decisions in water management is taking place at a rapid pace. Still, simultaneous discoveries and developments in mathematics continue to be under-utilized due to the lack of coordination among disciplines. It is obvious that this failure of coordination and integration is a hindrance to expansion of progress in application of the knowledge we have gained. The purpose of this book is to help in overcoming this widespread condition.

The research and development measures described in this book comprise a practical reference for working in the area of water supply and sewage disposal. They have been carried out in cooperation with utility and disposal companies, infrastructure providers and planning authorities. Developments in the area of sewage disposal are of particular relevance, since optimal decisions on additional economically relevant investments for water pollution control will be made and implemented by national and regional guidelines and laws in the current and following years.

Water supply and drainage systems, as well as mixed water channel systems, are networks whose high dynamic is determined and/or affected by consumer habits with respect to drinking water, on the one hand, and by climate conditions, in particular rainfall, on the other hand. Besides the usual fluctuations in demand, water supply networks are subject to failures in supply and mains operation. The drains in mixed water channel systems, which usually run together in a wastewater treatment plant, consist of the drains of the water supply network, the so-called dry weather flow and the inflows from randomly occurring rainfalls. The ratio of the combined sewage flow vs. dry weather flow frequently is more than one hundred. From the point of view of hydrology and modeling, there is a need to work on both the flows

in open channels and the streams in closed tubes, within partially lumped systems as well as within algebraic, and/or differential-algebraic equations. In particular it is about hyperbolic systems of conservation equations with source terms that have found their way into the literature as Saint-Venant equations. It may happen though that, under certain flow conditions, equations in open channels and stream equations degenerate into parabolic and kinetic wave equations, and/or are presented as dead-time equations in a purely laminar case or as delay-differential equations. Status inquiries and/or estimates have to be applied when deciding on which model should be chosen in network operations.

This corresponds to a trust-region approach for hierarchic models. In the course of this approach, the optimization and control are supposed, on the basis of such dynamic systems, to be imposed on networks. The control strategy will thus show a closed loop character as well as an open loop character and will contain a continuous physical process shared along with discrete decision variables. They are thus complex multi-physical, multiscale, hybridically controlled and optimal dynamic systems on continuous networks.

The focus herewith is on two points. On one hand the dynamic is to be optimized with respect to practical relevant objectives and subject to state conditions. On the other hand, the technology developed in the first Step is supposed to locate maximum pressure in fresh water systems, and in sewage systems to determine, if necessary, transitions from running water to press water. These transition into sewage systems imply that channels fill up and the required and/or permitted discharge will be impeded. Pressure peaks in the fresh water system indicate a poor/faulty layout of the network. In both cases an attempt should be made, appropriate for the given topology, to avoid the occurrence of such indicators by means of control/steering on one hand and, if this doesn't prove to be satisfying, a design optimization on the other hand.

According to their size, water networks consist of hundreds or thousands of system elements. The processes that run continuously within the elements are one- to two-dimensional. Specific structures such as reservoirs or overflow valves make it even harder to describe such systems with mathematical models. In order to make these systems computable, the systems themselves must be aggregated and the physical fundamental equations needed to achieve efficient solution processes, must be simplified. While there are rather clearly structured, even though numerically challenging, equation systems for water supply networks, the models for simulation of combined sewers consist of a collection of extremely variable algorithms in terms of their physical properties and mathematical structure. The Saint-Venant-equations are a system of non-linear partial differential equations and are primarily based on the preservation of mass and impulse. The entire system is completed by appropriate initial- and boundary values, as well as by node conditions. The complexity of such systems rapidly reaches astronomic dimensions which can only be computationally treated by means of appropriate adaptive solution strategies. Adaptivity here takes effect on different levels: it includes individually adapted physical models with adaptive control of the spatial dimension and an efficient division of the network into areas of high and low dynamic, as well as control of spatial and temporal multi-scale

effects in numerical discretization by means of multilevel-methods. Corresponding decisions on models or their accuracy are determined by sensitivities that are being described by the corresponding adjoint systems.

Moreover, different types of decisions have to be taken in water management. The networks have to be optimized in terms of their topology and operation by targeting a variety of criteria. These criteria may for example be economic, social or ecological and may compete with each other. For this purpose practically applicable optimization methods are needed which are currently only partly available. Decisions can be of discrete or continuous nature, e.g. a decision has to be taken, if and where a system element in the network or in the planning area has to be established. In the event of a favorable decision, size, form and operation method have to be optimized. It is worth mentioning at this point that the further development, maintenance and replacement/substitution of urban drainage systems represent the most expensive civil infrastructure in Germany and is expected to continue to be a significant economical burden.

From the mathematical point of view these problems lead to network design problems including non-linear nodes and edge conditions. These problems on such a scale exceed the limits of today's methods. To solve such models the problems are approximated as partly linear or convex in such a manner that it is possible to take the qualitatively right decisions in terms of the network design (type and dimension of system elements). These approximations will also be key to further developments in choosing appropriate methods from the field of mixed-integer optimization in general and facility location problems in particular. The objective criteria relevant for optimization will be integrated into the analysis by using efficient methods of multicriterial optimization that need to be especially adapted. The disclosed solutions can be transferred to almost any water supply- or sewage system. In Germany alone, there are several thousand systems. An application on a European level or by any other industrial nation or consortium with similar standards of infrastructure, is basically also feasible.

The list of authors and participants of the project cover all relevant fields. With respect to mathematics, all necessary disciplines are available. This includes on one hand mixed-integer optimization, multi-objective and facility location optimization, numerics for cross-linked dynamic transportation systems and optimization as well as control of hybrid systems. According to flow dynamics, a long term experience in mathematical modeling of one- to three-dimensional flow processes on the basis of mixed hyperbolic-elliptical-parabolical systems exists. A close and direct connection to practical water management has been established by involving application-oriented engineering know-how out of the field of civil engineering. The interdisciplinary cooperation of partners from different scientific directions, together with people working in the industry, was the basis for directly applicable solutions of the problems that were addressed.

The industry partners came from the field of Industrial Solutions and Services, Water Technology (I&S WT) at Siemens AG, offering solutions for the control and visualization of all facilities in drinking water and waste-water systems. In close cooperation with the central goals, innovative solutions for integrated simulation of

processes and automatization are being developed. As an industry partner, Siemens provides necessary data in order to pave the way for an integration of algorithmic results into commercial solutions.

Hessenwasser provides a public drinking water supply for around 2 million people in the metropolitan area of Frankfurt/Rhine-Main. The companies core competences include integrated groundwater management, abstraction, treatment, transportation and storage of water. Related services comprise groundwater monitoring, laboratory analysis and quality consulting, artificial recharge and irrigation water supply. In times of peak demand Hessenwasser supplies more than  $400,000 \text{ m}^3$  of drinking water daily. Hessenwasser operates a distribution net of 415 km of transportation pipe which includes drinking-water reservoirs with a total volume of  $343,000 \text{ m}^3$ . Total water procurement amounts to a volume of about  $100 \text{ mil. m}^3/\text{a}$ . Sixty percent is produced in its own water works which are distributed throughout the region. Hessenwasser was founded in 2001 and is owned by the public multi-utility companies of the major cities Frankfurt, Wiesbaden and Darmstadt and of the district of Groß-Gerau.

Over recent years, Hessenwasser developed a hydraulic model for its complex cross-linked water distribution system in order to analyze the efficiency of the existing water system design and to cope with future needs. Our engineers gained a lot of expertise in advanced water distribution modeling and management. We supported the scientific approach to the project by technical advice and practical experience from our regional transportation network.

The chapters themselves cover the following topics:

In Chap. 1 the simulation of a water supply system on a mesoscale abstraction level is considered. The water network consists of storage tanks, pipes, pumps and valves. It is operated by the characteristics of the water supplier, the consumer and the pumps. For all network elements the modeling equations are given. They include mass and momentum conservation for pressurized pipe flow. For their numerical solution the method of lines is proposed. The discretization in space is based on a finite volume approach together with a local Lax-Friedrich splitting and central WENO reconstruction. Boundary and coupling conditions are implemented as algebraic equations. This leads to a system of differential-algebraic equations in time which is solved by a special Rosenbrock method. The paper ends with some typical simulation results of the network.

In Chap. 2 we consider the solution of the model equations of water supply networks and continuous optimal control tasks. We begin with the description of a simulation tool, in particular the numerical treatment of the water hammer equations. This includes a description of the implemented discretization scheme together with a stability and convergence analysis. As we will see, the applied scheme perfectly matches with the properties of the water hammer equations and thus builds a useful foundation for solution of the entire model equations as well as optimal control tasks. Then we consider the computation of sensitivity information, which is necessary for the application of gradient-based optimization techniques. Here, we follow a first-discretize approach to derive adjoint equations. Due to the special structure of the considered problems, very efficient algorithms can be applied. Finally, the chapter deals with the problem of singularities in the model equations of water supply

networks. Here, a physically motivated regularization approach is applied and also extended to be applicable in an adjoint calculus.

In Chap. 3 we introduce a mixed integer linear modeling approach for the optimization of dynamic water supply networks based on the piecewise linearization of nonlinear constraints. One advantage of applying mixed integer linear techniques is that these methods are nowadays very mature, that is, they are fast, robust, and are able to solve problems with up to a huge number of variables. The other major point is that these methods have the potential of finding globally optimal solutions or at least to provide guarantees of the solution quality. We demonstrate the applicability of this approach on examples networks.

In Chap. 4 we compare continuous nonlinear optimization with mixed integer optimization of water supply networks by means of a meso scaled network instance. We introduce a heuristic approach, which handles discrete decisions arising in water supply network optimization through penalization using nonlinear programming. We combine the continuous nonlinear and the mixed integer approach introduced in Chap. 3 to incorporate the solution quality. Finally, we show results for a real municipal water supply network.

Chapter 5 gives an overview of optimal control of sewer networks with dynamic process models. After introducing the method of model predictive control (MPC) and its requirements for optimization and process modeling, a focus is set on practical applications and the industrial viewpoint. An up-to-date sewer management system is introduced and used to illustrate industrial requirements and the mathematical challenges involved in it.

In practical application, open-channel or free-surface channel flow under the influence of gravity in sewers has traditionally been modeled with mathematical models based on one-dimensional governing equations of continuity and momentum—the so-called Saint-Venant equations. High volumetric flow rates or strong rains may lead to a transition from partial to fully filled cross sections in a sewer net, i.e. a free surface flow is no longer guaranteed. Hence the mathematical model of the Saint Venant equations loses its validity in whole or in parts of the channels and a transition occurs to the pressurized pipe equations. The main goal of Chap. 6 is to bring forward our knowledge about the process of changing the governing regime of fluid equations in the channel flow and to attempt to perform a general modeling tracking the movement of the transition interface between a free surface flow and the pressurized flow in a one-dimensional channel. Various flow cases with or without a moving transition are numerically investigated by means of the high-precision Discontinuous Galerkin Finite Element method. An exact knowledge of this event allows one to optimize the controlling of equipment and the operation in a sewer or design a new sewer correctly and effectively.

Chapter 7 introduces a software tool for MPC of sewer networks with a dynamic process model which is based on an interactive approach. A flexible optimizer, which implements local and global optimization methods, is connected to a dynamic sewer network model to evaluate the objective function values. Numerical results for a simple urban drainage network are presented, illustrating the functionality of the approach.

In Chap. 8 a hydrodynamic process model based on shallow water equations is discretized on 1D-networks with the method of finite volumes. Based on finite volumes, we replace algebraic coupling conditions by a consistent finite volume junction model. We use discrete adjoint computation for one-step Runge-Kutta schemes to generate fast and robust gradients for descent methods. We use the descent methods to generate an optimal control for an example network and discuss the computational results.

In Chap. 9 we compare the quality and generation performance of the optimal control sequence produced by the software frameworks BlueM.MPC and Lamatto.

In Chap. 10 we consider the goals and objectives arising in wastewater management in the context of a multiobjective analysis. This allows, among others, the individual consideration of (1) the overflow volume (i.e., the total amount of released water), (2) the pollution load in the released water, and (3) the cost of the generated control. Given a specific sewage network and data of typical inflow scenarios, a multiobjective offline analysis of the problem and, in particular, of the trade-off between the different goals, provides the decision maker with valuable information about the problem characteristics. This information can then be used to specify a suitable scalarized, single-objective optimization problem for the real-time optimal control that represents the decision maker's preferences in a best possible way. If an efficient solver for such scalarizations is available (which is the case for the problems considered here), this leads to an efficient online procedure that is justified by an extensive offline problem analysis. Even though the methods presented in this chapter were tailored for wastewater management problems, they are also applicable in the context of the other applications mentioned in this volume.

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