

Foreword to the English edition

It is natural that authors should be delighted when their works are translated into other languages, and even more so when the translation is done very nicely – as it is the case with the present translation of our notes on plane algebraic curves by John Stillwell. On the other hand, it is also true that as time goes by one gets more aware of the defects of one's work. One of our friends has criticized us for writing a heavy volume on such an elementary subject, and we have to admit that this criticism is not totally unjustified. However, we would like to say in our defence that a number of people who are now doing research on singularities found the book quite useful as a first introduction, and so it is our hope that readers of the English edition will have the same experience.

We would like to point out that there is now a more effective approach to iterated torus knots than the one presented in this book. This is developed in the beautiful new book by David Eisenbud and Walter Neumann, "Three-dimensional link theory and invariants of plane curve singularities", and in the forthcoming work of Françoise Michel and Claude Weber.

We would like to thank John Stillwell for all his work. We realize that in some instances translating such a book must have been a really difficult task. We feel that he has succeeded very well.

Egbert Brieskorn and Horst Knörrer

Bonn, April 1986

"Es ist die Freude an der Gestalt
in einem höheren Sinne, die den
Geometer ausmacht."

(Clebsch, in memory of Julius
Plücker, Göttinger Abh. Bd. 15).

Foreword

In the summer of 1976 and winter 1975/76 I gave an introductory course on plane algebraic curves to undergraduate students. I wrote a manuscript of the course for them. Since I took some trouble over it, and some colleagues have shown interest in this manuscript, I have now allowed it to be reproduced, in the hope that others may find it useful.

In this foreword I should like to explain what I wanted to achieve with this course. I wanted – above all – to show, by means of beautiful, simple and concrete examples of curves in the complex projective plane, the interplay between algebraic, analytic and topological methods in the investigation of these geometric objects. I did not succeed in developing the theory of algebraic curves as far as is possible – I must even say that in some places the course stops where the theory is just beginning. Rather, I aimed to allow the listeners to develop as much familiarity as possible with the new objects, and the best possible intuition. For this reason I almost always used the most elementary and concrete methods. Also for this reason, I have taken the trouble to make a great number of drawings. I once read a remark of Felix Klein to the effect that what a geometer values in his science is that he sees what he thinks.

Another principle which I have tried to put into effect in this course is that of breaking through the formal lecture style – the style which replaces the development of ideas by a staccato of definitions, theorems and proofs. Thus heuristic, historical and methodological

considerations took up a substantial portion of the course, which they likewise occupy in the manuscript. I am well aware of the disadvantages of this method for the reader, and the resulting lack of formal precision, conciseness, clarity and elegance is an annoyance to me too. However, I have accepted this annoyance in order to be able to develop the ideas in a natural way and to promote understanding and thought.

I have developed no new scientific ideas in this course, but have drawn much from other sources. Thus in Chapter II I have depended heavily on notes of a course by R. Remmert on algebraic curves which brought me into contact with algebraic geometry for the first time as a student, and on the book by Walker. I have also used the introduction to algebraic geometry by van der Waerden. In the historical remarks I have relied a lot on the corresponding Enzyklopädie articles and the books of Smith and Struik. My aim was not historical refinement but to give students a picture of the beginnings from which the theory has developed. The whole later history – from the second half of the 19th century onwards – was not so important for this pedagogical purpose, and for it I refer to the new book of Dieudonné or the beginning of the book by Shafarevich. For the local investigations of topology and resolution of singularities I have depended on lecture notes of F. Pham and H. Hironaka as well as original work of A'Campo. Finally, I have used many other sources, which I perhaps have not always acknowledged. I hope the authors will forgive me.

What is lacking in the course? It lacks a chapter on the deformation of singularities, in which I would have liked to introduce the beautiful results of A'Campo and Gusein-Zade on the computation of the monodromy groups of plane curves. For this I refer to the report of A'Campo to the International Congress of Mathematicians in Vancouver in 1974. I have tried to admit the deformation viewpoint at least implicitly in sections 8.5, 9.2 and 9.3. What is not lacking? There is no lack of introductions to modern algebraic geometry. This course is not intended to be such an introduction. For this purpose there are now the beautiful books of D. Mumford, I.R. Shafarevich, P.A. Griffiths and R. Hartshorne. The purpose of the course is to familiarise students, in a natural, intuitive and concrete way, with the various methods for the investigation of singularities, and to lead in this way to my own field of work. I believe that it has reached this goal. This is shown by a series of beautiful Diplomarbeiten which have come

into being in the meantime. If the notes presented here can also serve others similarly, then they have fulfilled their purpose, even if only as a source of suggestions or as a collection of material.

In conclusion, I should like to thank all of those who have helped in the production of this extensive piece of work : diploma mathematician Mr. Ebeling and above all Dr. Knörrer for working out some lectures in Chapter III, and for a critical inspection and proof-reading of the manuscript, and help with assembling the references, the three secretaries Mrs. Schmirler, Mrs. Weiss and Mrs. Eligehausen for the laborious and alienating work of typing the text, and the printers at the Mathematische Institut, Mr. Vogt and Mr. Popp, for printing the almost one thousand pages. But above all I should like to thank two students, Mr. Koch and Mr. Scholz, for taking on the enormous job of producing the manuscript, proof-reading, making the index and redrawing more nicely many of the figures. Without them the manuscript would never have been completed. Once again : many thanks!

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