Chapter 2
Special Features of Self-surface (Heat) Radiation Forming

Abstract Radiation of the hypothetical black body is considered. General notations and basic equations are presented for calculation spectral radiation of black and real bodies.

2.1 The Black Body Radiation

It is known from experiments that all matters emit constantly electromagnetic waves. The electromagnetic radiation embraces practically all ranges of wavelength. By its nature this radiation is called self heat radiation, because it arises while molecules transmit at exciting level with kinetic interaction (collisions) with consequently returning to unexciting level with quantum emitting. Thus it is understandable that the intensity of the self heat radiation is to be linked with inner energy of matter that is directly proportional to the temperature and is to depend on physical structure of matter.

Dependencies of forming the self heat radiation field allow obtaining an analytical link between quantities of energy emitted by an object at different wavelength in different directions and object's parameters. But these dependencies are simple only for ideal absorber and emitter of electromagnetic waves, which is blackbody (BB) or perfect radiator.

The blackbody is an hypothetical body that emits the maximal radiation for the temperature, does not reflect or transport the incident energy and absorbs all incident energy falling at all wavelengths and from all directions. The notation of blackbody is the key one for description of heat radiation transfer. The perfect radiator blackbody is used as an etalon for calibration of spectral instruments within spectral IR-ranges.

Max Planck (1858–1947) has assumed two presumptions concerning properties of atom oscillators in 1901 aiming theoretically explain spectral distribution of radiation emitted by heated cavity. Firstly Planck had postulated that the energy of the harmonic oscillator is expressed as $E = nhf$, where f is the oscillator frequency;

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h is the Planck's constant, and n is quantum number, that can be only integer. Later it was shown that in reality the number $n + 1/2$ is right but it does not change Planck's result.

Secondly Planck had supposed that oscillators emit energy not continuously but by portions – quants. These quants are emitted while the oscillator transmits from one quantum condition to another.

These two assumptions allowed Planck to theoretically derive a function that expresses blackbody spectral brightness; it's called now ***Planck's function*** and is very important for description of self atmosphere and surface radiation. The blackbody spectral brightness in the ranges of wave numbers v and $v + dv$ is defined by the radiant energy dE, emitted by an blackbody surface element dS during the time interval dt in the solid angle $d\omega$. The blackbody radiation obeys Lambert's law and blackbody surface is ideal diffuse thus the direction of radiation is not important.

2.2 Basic Equations

Then following to Planck's law the blackbody spectral brightness depends only on two variables the absolute temperature T and the wave number v (or equivalent characteristics λ, f)

$$B_v(T) = \frac{av^3}{\exp\left(\frac{bv}{T}\right) - 1}, \tag{2.1}$$

where $v = \frac{10000}{\lambda}$, cm^{-1} is the wave number (λ is the wavelength, μm);

T is the blackbody absolute temperature, °K;
$a = 1.19105$ W;
$b = 1.43874\ 10^{-2}$ °K/m^{-1}.

In terms of wavelength λ the Planck's function look as follows (Fig. 2.1):

$$B_\lambda(T) = \frac{(a_1\lambda^{-5})}{\exp(\frac{b_1}{\lambda T}) - 1}, \tag{2.2}$$

where $a_1 = 3.74 \cdot 10^{-16}$, W m^2, $b_1 = 1.43874 \cdot 10^{-2}$, °K · m.

In terms of the frequency f the Planck's function is written as

$$B_f(T) = \frac{a_1 c^{-4} f^3}{\exp\left(\frac{b_1 f}{cT}\right) - 1}, \tag{2.3}$$

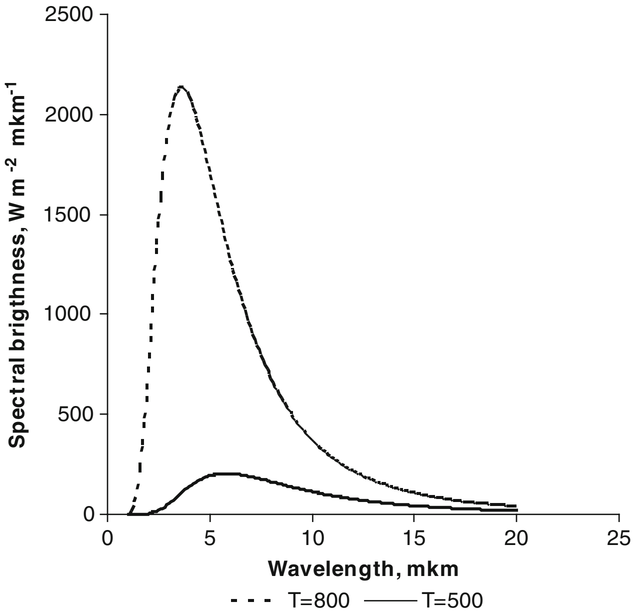

Fig. 2.1 Planck's function for two temperatures 800 and 500 °K

Where $f = c/\lambda$ is the frequency, Hz; ($c = 2.9996 \cdot 10^{14}$ μm/s is the light velocity).

Throughout most of the shortwave range $\lambda < 2$ μm for Earth self heat radiation the exponential term is larger than one, thus ***Wien approximation*** is obeyed by the blackbody spectral brightness:

$$B_v(T) = av^3 \exp\left(-\frac{b\,v}{T}\right) \tag{2.4}$$

For long waves ($\lambda > 100$ μm) ***Rayleigh-Jeans approximation*** is true for the blackbody spectral brightness:

$$B_v(T) = (a/b)v^2T. \tag{2.5}$$

After integrating the Planck's function over all wave numbers (wavelengths, frequencies) the blackbody *irradiance* $F(T)$ is obtained.

$$F(T) = \sigma T^4 \tag{2.6}$$

The Eq. 2.6 is called ***Stefan-Boltzmann law***, where $\sigma = 5.6693 \cdot 10^{-8}$, W/(m$^2 \cdot$ °K^4) is the Stefan-Boltzmann constant.

The wavelength of the Plancks function maximum for fixed temperature T is defined by the ***Wien displacement law***:

$$\lambda_{\max} = \frac{c_1}{T}, \quad \mu m, \tag{2.7}$$

where $c_1 = 2897.8$, μm °K.

And finally, it is possible to obtain the expression for the temperature T from the Eq. 2.1:

$$T = \frac{b\nu}{\ln\left(1 + \frac{a\nu^3}{B_\nu(T)}\right)}, \tag{2.8}$$

Characteristics of surface self heat radiation.

The spectral intensity. From the definition the blackbody radiation is the upper limit to the radiation emitted by a real substance at a given temperature. The value of the emissivity ε_ν is introduced for description the upward radiation intensity $J_\nu^\uparrow$emitted by a real surface at any wave number ν as $\varepsilon_\nu \equiv J_\nu^\uparrow / B_\nu$. It is clear that $\varepsilon_\nu < 1$ for real substances and $\varepsilon_\nu = 1$ for the blackbody. The equation $\varepsilon \equiv J^\uparrow / F(T) = J^\uparrow / \sigma T^4$ expresses ***gray body*** emissivity.

Then the spectral intensity of the self heat radiation of the surface with the temperature T_s is defined by the following expression:

$$J_\nu^\uparrow = \varepsilon_\nu B_\nu(T_s), \tag{2.9}$$

where T_s is the surface temperature; B_ν is Planck's function; ε_ν is the surface emissivity.

2.3 The Brightness Temperature

The Planck's function allows the numerical describing and conventionally illustrating the spectral distribution of the electromagnetic radiation intensity that is formed by surface or complicated system *atmosphere-surface*. It is reached by assuming that the radiation at any given wave number is formed by the black body at a certain temperature and not by a real substance with a real temperature. Such assumption provides the possibility for every value of intensity J_ν to uniquely relate to a certain value of the temperature. This temperature is not a thermodynamic value but only a convenient characteristic for one-to-one describing the spectral distribution of the radiation emitted by the system atmosphere-surface, and it is called ***brightness temperature***. This characteristic is called ***radio-brightness temperature*** at the radio wavelength ranges and ***Rayleigh-Jeans approximation*** (the Eq. 2.5) is used for calculation. The transition from spectral intensity (or brightness) of real substances to their brightness temperature allows the clear plotting of the spectrum in wide ranges and easy correlation of intensity values at different spectral diapasons. The reason is the weak spectral variability of the brightness temperature comparing with intensity. This variability disappeared in the blackbody limit case. For example the blackbody at the temperature of 7,000 K, then brightness temperature $T_\nu = 7{,}000$ K coincides with the thermodynamic and it is constant at all wavelength; the spectral brightness varies over nine orders of magnitude.

The brightness temperature of the heat radiation intensity (including the surface heat radiation) is derived from the relation below

$$B_\nu(T_\nu) \equiv J_\nu^{\uparrow} \cdot \tag{2.10}$$

The following formula for calculating the brightness temperature T_ν of the self heat radiation intensity of the surface is derived by taking into account Eqs. 2.8, 2.9 and 2.10 :

$$T_\nu = \frac{b\nu}{\ln\left[1 + \frac{a\nu^3}{J_\nu^{\uparrow}}\right]}. \tag{2.11}$$

By substituting the Eq. 2.9 it is obtained for T_ν:

$$T_\nu = \frac{b\nu}{\ln\left[1 + \frac{a\nu^3}{\varepsilon_\nu B_\nu(T_s)}\right]}. \tag{2.12}$$

It is clear that for $\varepsilon_\nu = 1$ the equality $T_\nu \equiv T_s$ is valid, hence the blackbody brightness temperature does not depend on wave number and equal to thermodynamic temperature of the blackbody surface.

From Eqs. 2.9 and 2.1 the expression for calculating the surface temperature T_s (real thermodynamic value) from measured heat intensity $J_\nu^{\uparrow}$ is obtained. The assumption of atmosphere absence is taking (for example the temperature of the Moon surface):

$$T_s = \frac{b\nu}{\ln\left[1 + \varepsilon_\nu \frac{a\nu^3}{J_\nu^{\uparrow}}\right]}. \tag{2.13}$$

The Eq. 2.13 provides the result of remote retrieval of the surface temperature T_s from measuring the surface self heat radiation intensity $J_\nu^{\uparrow}$. It is necessary a priori knowing the surface emissivity ε_ν and absence of gaseous substance between the surface and instrument. Comparison of Eqs. 2.12 and 2.13 allows understanding the difference between brightness and thermodynamic temperatures.

The sensitivity function. Let us introduce the function S_ν for numerical estimation of the sensitivity of the Planck's function to the temperature variability:

$$S_\nu = \frac{\partial B_\nu[T]}{\partial T}. \tag{2.14}$$

The differentiating of the Eq. 2.1 over the temperature gives the expression for the sensitivity function:

$$S_\nu = \frac{ab\nu^4}{\left[\exp\left(\frac{b\nu}{T}\right) - 1\right]^2}\left[\exp\left(\frac{b\nu}{T}\right)\right]\frac{1}{T}. \tag{2.15}$$

2.4 Practice 1

2.4.1 Objectives

1. Studying the blackbody spectral distribution at different temperatures.
2. Studying the spectral distribution of the Planck's function derivative at different blackbody temperatures.
3. Studying the spectral distribution of the safe heat radiation intensity of real body at different temperatures with numerical simulating spectral variations of its emissivity.
4. Studying the spectral distribution of the brightness temperature of the surface with Planck's function derivative at different blackbody temperatures numerical simulating spectral variations of the surface emissivity.

2.4.2 Software and Set of Input Parameters

1. Computer programs "F_PLANCK", "I_RADT", setup at the directory *dz*-2006*Lab*1.
2. The set of input parameters for programs (Table 2.1).

Table 2.1 Variants of parameter values

Number of the variant	λ_{min}, µm	λ_{max}, µm	T_1, °K	T_2, °K	T_3, °K
1	1	20	550	770	840
2	1	30	440	560	610
3	15	40	260	270	320
4	10	60	200	300	350
5	20	50	150	250	350
6	2	80	100	200	300
7	15	50	270	290	310
8	10	50	300	350	600
9	20	100	230	260	340
10	20	80	300	400	500

2.4.3 Test Questions

1. What is definition of the blackbody?
2. What is spectral distribution of the blackbody brightness?
3. What defines the place of spectral brightness maximum?
4. What is the formula for defining the blackbody spectral brightness dependence on wave number and temperature?
5. Does the value of blackbody brightness temperature change on wavelength?
6. Do maximums of spectral brightness and Planck's function derivative over temperature coincide?
7. What parameters does the intensity of the surface heat radiation depend on?
8. Might the surface brightness temperature be equal to its thermodynamic temperature?
9. Derive the formula for the Planck's function derivative over temperature.
10. Demonstrate the validity of the Eq. 2.6 using differentiation of the Eq. 2.3 over wavelength.

2.4.4 Sequential Steps of the Exercise Implementation

1. To study the theory with using additional books, pointed in reference list.
2. To take the three variants of input parameters from the Table 2.1 for computer programs "F_PLANCK.exe", "I_RADT.exe". You can use another set of input parameters. Take in mind that the program can operate with wavelength $\lambda \geq 1.0$ μm.
3. To analyze
   - the Planck's function and its derivative over the temperature with using computer program "F_PLANCK.exe" in chosen spectral ranges;
   - the variation of spectral dependence of mentioned functions on temperature;

   Results are demonstrated on the screen and output in file "*fplanck*".
4. To create plots of Planck's functions and its derivative of the temperature (with using Excel)
5. To plot the modeling presentation of the emissivity versus wavelength $\varepsilon(\lambda)$ in Excel in chosen wavelength ranges. The emissivity varies within ranges 0.50−0.97. It is necessary to create the table containing not less 25 values, to plot the emissivity, to approximate the curve with using the polynomial 3rd order trend line, output the corresponded equation at the plot and to fix values of polynomial coefficients for using them in the computer program "I_RADT.exe". (Call attention to the inverse order of the polynomial and input in program "I_RADT.exe" coefficients.).

6. To examine the spectral dependence of surface self heat radiation with using values of the emissivity approximation coefficients obtained in item five. Call special attention to spectral dependence of the surface brightness temperature
7. To plot calculated spectral values, for putting into the final report.

2.4.5 Requirements to the Report

Compile the final concise report with elements of theory, resulting pictures, and conclusions that reflects main stages of the work.

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