Preface

Why the "royal road"? The great geometer who today is known as *Euclid of Alexandria* was invited to come to that city by Ptolemy 2., possibly from Athens.[1] Ptolemy was a son of Ptolemy 1., one of Alexander's generals who assumed the title *King of Egypt* and founded the great museum and library in Alexander's city by the delta of the Nile. Euclid and his collaborators assembled practically the entire body of geometry known to them in thirteen books, the fruit of the towering and impressive effort of Greek mathematics. This work is known today as *Euclid's Elements*.

There is an historical anecdote to the effect that when the king wanted to learn some geometry, he found the Elements too long and too hard to read and understand, and asked Euclid if there really existed no simpler way of learning about geometry. Euclid is said to have answered, somewhat arrogantly perhaps, that "*there is no Royal Road to geometry*".

The Persian Royal Road is described by the *Father of History* Herodotus of Halicarnassus in the fifth Century BC. His text can be found in English translation in [19], 5.52–53, page 129 in the book listed in the present reference. He writes as follows:

> *Here is what the road is like. There are royal way stations and fine inns all along the way, and the whole road runs through safe, inhabited territory. There are twenty way stations on the three-hundred-seventy-seven-and-a-half-mile stretch from Lydia to Phrygia. The Halys river is on the Phrygian border. Gates stand at the river crossing, and it is absolutely necessary to pass through them to pass the well-guarded river. [...]*

[1]However, some believe that Euclid of Alexandria never existed, but that the name "Euclid" was taken from Plato's dialogue *Theaetetus*. In the dialogue there appears a minor and insignificant character, totally fictitious, called *Euclid*. As the events related are supposed to find place in the city of Megara, this character is known today as *Euclid of Megara*. Now one theory goes that this name was used as a pseudonym for a group of geometers working in Alexandria. Much as the name *N. Bourbaki* was used by a group of French mathematicians in modern times.

Herodotus then continues to describe the route in great detail, including the number of rest houses and inns along the way, and information on where one needs to cross rivers by boat. Then he concludes by explaining that The Royal Road has been measured to 14400 furlongs. A furlong, later called a *stade* by the Romans, was about 200 meters long, so the total length of the Road should be 2880 kilometers. He estimates that it is reasonable to travel 164 furlongs a day, that is about 33 kilometers. So the whole journey should take roughly 90 days.

Thus the term *A Royal Road* was intended to mean *a simple way* by Euclid himself, perhaps too simple. In that sense one might argue that there is not, or should not be, a Royal Road to Algebraic Geometry either. King Ptolemy was a competent ruler, who probably understood that geometry as such can not be made easy. But he might have wanted to learn the important *ideas* behind the proofs, without going into what we would call the *axiomatic formal deductions*. This is what Archimedes called the *Analysis*. A proof can not be obtained in this way, Archimedes says. But the analysis of the problem provides information on the problem, so it may be easier to find an actual proof.

When you tread the road of algebraic geometry, you tread in the footsteps of queens and kings of mathematics. And if you manage to study four or five pages of this book each day, stopping at some of the resting places, you complete the journey in ninety days by a comfortable margin. But that means you will be travelling along, viewing the landscape and learning the geography. You will, however, not pitch camp to go fishing or hunting from time to time. In more serious terms, I present a number of important theorems in algebraic geometry with only *comments* on how the basic idea of the real proofs work. But references to where the reader may find such complete proofs are always provided, and I try to limit myself to using relatively few, well known and readily available texts. Thus for instance the important result of *Serre duality* is stated but not proved here, the reference I have chosen being to Hartshorne's book [18]. The same applies to intersection theory. Intersection theory, especially in the singular case, is just shown the traveller in the distance, from one of the smaller hills we have climbed.

Thus it should be possible to teach a rapid course from the present text in one term. A course of this type could be useful for students or other young (or older) mathematicians who have specialized in slightly different subjects, and would like to read up on modern algebraic geometry after first gaining some knowledge about the subject.

A more in-depth treatment, with complete proofs taken from the references, with exercises and with more examples, would at least be a full one year graduate course. The term *algebraic geometry* is used in the traditional sense, founded in the fundamental and important works by *André Weil* [41] and *Alexander Grothendieck* [15]. Thus I make no mention of such themes as "*tropical*" *geometry*, for example, nor do I deal with the so called "*non commutative algebraic geometry*". Some experts of these new areas would therefore have preferred the term *classical algebraic geometry* in the title of the present treatment. But for several reasons the author finds that term to be somewhat misleading. The term "*classical*" is ambiguous, and in the present context it should be stretched, at the very most, to cover the subject roughly up to the first quarter of the 20th century. A suitable title for a treatment including the above mentioned new developments, might be simply "*Algebraic Geometry in the 21th century*". Perhaps followed by a question mark for the time being.

The book is divided into two parts. Part I, on *Curves*, introduces the basic concepts of algebraic geometry in the context of projective curves. The treatment here is quite simple, and leads up to the intersection theory in a simplified setting, as well as to the statement of the classical Riemann Roch Theorem and the concept of the *dual curve*. Part I is also intended as a preparation and motivation for an introduction to the Grothendieck theory of *schemes*, given in Part II. But Part I could well be used as a text for an undergraduate course on curves, less ambitious but providing more of an overview than the standard texts available. A good supplement would be parts of Fulton's very nice book [10]. For example, our treatment of intersection theory for curves in $\mathbb{P}^2_k$ in Chap. 4 follows this source. Hartshorne's book [18] is another excellent text, more advanced and perfectly suited for a follow up course to one based on the present text. Both of [10] and [18] have a number of very good exercises.

Some of the material in this book was surveyed several years ago, partly in Spanish, in publications of the UNAM in Mexico, [21] and [22]. The same applies to the papers [23] and [24], as well as [25], [29] and [30]. Chapter 22 contains the main part of [26].

Also, parts of the material presented in Sect. 21.4, were developed during work on the joint paper with *Joel Roberts* [31], while I was visiting the University of Minneapolis about twelve years ago. Some of this work has not been published before, and as it now appears here I take the opportunity to thank Joel for so many stimulating conversations on this and related mathematics, back then.

Acknowledgment The author was supported by a grant from the Norwegian non-fiction Literary Fund under the NFF.

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http://www.springer.com/978-3-642-19224-1

A Royal Road to Algebraic Geometry
Holme, A.
2012, XIV, 366 p., Hardcover
ISBN: 978-3-642-19224-1