Chapter 2
Service Demand of Consumer with Deterministic Perceptions

2.1 Introduction

Consumers usually choose an option from a set of available options that offer the same kind of service. Such consumer choice is affected by myriad service attributes. One example that well illustrates this aspect of consumer choice might be the choice of one travel option from among a population of available options. This choice is influenced not only by prices and service times but also by relatively subjective attributes such as comfort and privacy. Moreover, different consumers prefer different options for travel services and the same consumer makes different choices at various times. Naturally, these facts extend to the choice of other services that comprise most of our consumption activities: retail, recreation, entertainment, dining, medical clinic, hotel, housing, telecommunication, and various business services.

This chapter analyzes a consumer choice of an option from a set of available options, through the application of the perception approach newly proposed in this study. The most distinctive feature of the perception approach in contrast to other approaches to consumer demand analyses consists in its ability to explicitly incorporate service quality. The perception approach quantifies the service attributes of an option using two groups of variables: group one includes quantitative variables divided into price and service time; group two consists of all qualitative attributes such as comfort, safety, cleanliness, privacy, etc. The perception approach primarily targets the service offered by multiple options of different service attributes, termed *the qualitative choice service*, but is also applicable to the service provided by a single option.

The perception approach, an application of household production theory, adopts the following basic hypothesis of household production theory: consumer utility is the function of commodities produced through the consumption of qualitative choice services. The perception approach is, however, distinguished from previous applications of the theory in that it incorporates one additional hypothesis: each qualitative attribute packed in services constitutes an independent argument of consumer utility.

D.J. Moon, *Congestion-Prone Services Under Quality Competition*, Advances in Spatial Science, DOI 10.1007/978-3-642-20189-9_2,

The additional hypothesis is incorporated into UMPs by employing the consumer (or household) production function developed under the two postulates that follow. First, each consumer reckons the magnitude of all qualitative attributes in an option, which generally differs from consumer to consumer. Second, each consumer judges the implicit price of each qualitative attribute, which also differs from consumer to consumer, but which is shared by that attribute for all available options.

Under the perception approach, the consumer production function incorporated into UMPs should be able to reasonably depict the jointness of consumer production. The input to consumer production for qualitative choice service should consist of multiple inputs that comprise all service options in the menu of options available to consumers. Also, the output should consist of multiple outcomes that include all qualitative attributes packed in services consumed.

By applying the perception approach outlined above, it is possible to construct the UMP that yields *the revealed preference condition* for the consumer choice of an option from among multiple options. The revealed preference condition identifies the option chosen by a consumer. The chosen option has the lowest implicit price. Also, the implicit price of an option depends on not only the magnitudes of all quantitative attributes in the option but also the implicit prices of all qualitative attributes. Further, as postulated above, *consumer perception for service quality*, which refers to the perceived magnitudes and implicit prices of qualitative attributes, differ from consumer to consumer. Therefore, it is natural that the choice of the best option differs among consumers.

The qualitative choice problem, the main target of demand analyses in this study, is a special subgroup of UMPs under the perception approach. Roughly speaking, the qualitative choice problem uses a special type of consumer production function. This type of production function, called *the joint homogeneous production function*, satisfies the following two conditions: first, production systems described by this function yield multiple outcomes from multiple inputs; second, the production function is mathematically homogeneous of degree one in both inputs and outputs.

By virtue of the homogeneity of consumer production functions, the qualitative choice problem yields the outcome that the implicit price of every qualitative attribute is a constant independent of the magnitude of quantitative attributes consumed. For this reason, the monetary value for all qualitative attributes packed in an option is also a constant. This property of the qualitative choice problem leads to the demand function of an option, which has a functional structure very similar to that of neoclassical consumer demand functions. Such a demand function for an option has the advantage of greatly simplifying analyses of its mathematical properties such as continuity and comparative statics with respect to the prices and service times of all options.

The qualitative choice problem for a certain qualitative choice service should be able to reflect the uniqueness of the service. This requirement can be fulfilled by employing a utility function and/or a consumer production function that appropriately formulates the uniqueness. We can, therefore, imagine that there are a large number of qualitative choice problems with different formulations. However, each qualitative choice problem requires demand analyses significantly different from other problems. For this reason, one specific example of qualitative choice problems is analyzed in this chapter, and other relevant choice problems are considered in the subsequent chapter.

This example of qualitative choice problems, termed *the basic choice problem*, is constructed under a number of additional simplifying conditions other than the postulates common to all qualitative choice problems. The most important condition is that the perceived magnitude of qualitative attributes is linearly proportional to the service time of options. This condition is applicable only to non-durable services such that the duration of benefit from the consumption of services is limited to a given service period. Such non-durable services include travel, communication, recreation, and entertainment services.

The perception approach of this chapter applies one additional important postulate such that the perceived magnitude of qualitative attributes is always deterministic, and therefore called *the deterministic perception approach*. The deterministic perception approach has the limitation of being unable to explain why a particular consumer makes different choices at particular times. For this reason, we will introduce an alternative approach that can accommodate the randomness of consumer choices. This approach hypothesizes that the perceived magnitude is an outcome of random processes affected by various uncertain factors such as ever-changing physical and emotional conditions. This alternative approach, called *the random perception approach*, will be introduced in Chap. 4.

This chapter develops and analyzes the basic choice problem according to the following plan. The following section presents a detailed procedure to construct the basic choice problem under the deterministic perception approach. Subsequently, Sect. 2.3 develops the revealed preference condition for the basic choice problem, and interprets economic implications of the developed revealed preference condition. This is followed by Sect. 2.4, which analyzes the mathematical property of service demand functions for the basic choice problem. That section also presents a more concrete definition of qualitative choice problems in connection with previous analyses of the basic choice problem.

2.2 Development of the Basic Choice Problem

2.2.1 Homogeneity of Consumer Production Functions

This subsection illustrates analyses of a UMP in the circumstance when each service traded in markets is offered by one option. This UMP is incorporated with a joint homogeneous production function.[1] Through analyses of this problem,

[1]The joint production function, in which all inputs are jointly used in producing multiple outputs, cannot be homogeneous of degree one in both inputs and outputs (Hall 1973; Pollak and Wachter 1975). However, there is a special group of joint homogeneous production functions in which only some inputs are jointly used in producing multiple outputs. One example of such a production functions is introduced in Moon and Kim (2002), and is applied to the basic choice problem considered in this chapter.

we illustrate that the UMP with a joint homogeneous production function satisfies *the consistency condition* such that the implicit price of commodities is independent of the amount of commodities consumed.

To begin, we introduce a UMP incorporated with a joint homogeneous production function. First, this UMP is used to find the commodity bundle $z \equiv (z_1, \cdots, z_K)$, which maximizes the value of a utility function $U(z)$. Second, the function $U(z)$ is concave and differentiable in z. Third, the commodity bundle z is the outcome of consumer production that uses inputs $x \equiv (x_1, \cdots, x_J)$, composed of goods and services traded in markets. Fourth, the purchasing cost of inputs satisfies the budget constraint such that $\bar{M} - \sum_j p_j x_j \geq 0$, where $\bar{M}$ is full income, and p_j is the price of input j. Fifth, the consumer production is a joint production, expressed by $F_k(x) - z_k = 0$, for all k, in which some, but not all, elements of x are jointly inputted. Sixth, the vector function $F \equiv (F_1, \cdots F_K)$ is homogeneous of degree one in both inputs x and outputs z.

The UMP specified above can be expressed as the Lagrangian L_1 such that

$$L_1(x, z, \mu, \eta; p) = \max\left\{U(z)\right\} + \sum_k \mu_k\left(F_k(x) - z_k\right) + \left(\bar{M} - \sum_j p_j x_j\right), \quad (2.1)$$

where $\mu \equiv (\mu_1, \cdots, \mu_K) \rangle 0$ and $p \equiv (p_1, \cdots, p_J)$. This UMP is too complex to directly characterize the solution of (x, z). For this reason, this UMP is decomposed into two sub-optimization problems.

The first sub-optimization problem is used to estimate the optimal bundle of inputs x, which minimizes the input cost $\sum_j p_j x_j$ necessary for the production of an arbitrary commodity bundle z. This minimization problem, called the cost minimization problem for consumer production, is expressed by the following Lagrangian L_2:

$$L_2(x, \varphi; z, p) = \max\left\{\sum_j p_j x_j\right\} + \sum_k \varphi_k(z_k - F_k(x)). \quad (2.2)$$

This optimization problem gives the consumer cost function $C(p; z)$ that estimates the minimum cost necessary for the production of commodity bundle z. This cost function equals $\sum_j p_j \bar{x}_j$, in which $\bar{x}_j$ is the solution of x_j to L_2.

The second sub-optimization is used to estimate the optimal bundle of z, which maximizes the utility $U(z)$, under the budget constraint $\bar{M} - C(p, z) \geq 0$. This maximization problem is expressed as the Lagrangian L_3 such that

$$L_3(z, \eta; p) = \max\left\{U(z)\right\} + \eta\left(\bar{M} - C(p; z)\right). \quad (2.3)$$

This UMP depicts that the solution of z maximizes $U(z)$ value, under the premise that the optimal bundle of z is produced efficiently.

We are now ready to introduce the consistency condition for the implicit price of commodities. Let $(\bar{x}, \bar{\varphi})$ be the solution to the cost minimization problem L_2.

Then, by the envelop theorem for constrained optimization problems, the term $\bar{\varphi}_k$ represents the implicit price of commodity k. Also, the consistency of the implicit price φ_k implies that this price is independent of the value of z. This consistency is the direct consequence of the sixth assumption that the joint production function is homogeneous of degree one in both inputs and outputs, as proved below.[2]

Theorem 2.1. *The implicit price $\bar{\varphi}_k$, for all k, estimated from L_2, satisfies both the consistency condition and the equality $C(z;p) = \sum_k \bar{\varphi}_k z_k$.*

Proof. By the assumption that F is homogeneous of degree one in x, the function $Z_k(x, z_k) \equiv z_k - F_k(x)$ satisfies the following:

$$\sum_j \frac{\partial Z_k}{\partial x_j} = -\sum_j \frac{\partial \bar{F}_k}{\partial x_j} x_j = -\bar{F}_k = -z_k, \quad \text{all } k. \tag{2.4}$$

where $\bar{F}_k \equiv F_k(\bar{x})$. Subsequently, irrespective of homogeneity, the solution to L_2 satisfies the following first order conditions with respect to x_j:

$$\frac{\partial L_2}{\partial x_j} = p_j - \sum_k \bar{\varphi}_k \frac{\partial \bar{F}_k}{\partial x_j} = 0, \text{ all } j. \tag{2.5}$$

Equation (2.5) shows that the implicit price $\bar{\varphi}_k$ satisfies the consistency condition; that is, the solution of $\bar{\varphi}_k$ to (2.5) is independent of z. Finally, combining (2.4) and (2.5) gives the consumer cost function such that

$$C(p;z) = \sum_j p_j \bar{x}_j = \sum_j \sum_k \bar{\varphi}_k \frac{\partial \bar{F}_k}{\partial x_j} \bar{x}_j = \sum_k \bar{\varphi}_k \sum_j \frac{\partial \bar{F}_k}{\partial x_j} \bar{x}_j = \sum_k \bar{\varphi}_k z_k, \tag{2.6}$$

as claimed in the theorem. □

Theorem 2.1 shows that the consistency of implicit prices $\bar{\varphi}_k$, for all k, leads to the last equality of (2.6). Substituting this equality into L_3 gives

$$L_3(z, \eta; p) = \max \left\{ U(z) \right\} + \eta \left(\bar{M} - \sum_k \bar{\varphi}_k z_k \right). \tag{2.7}$$

This UMP has a formulation identical to that of neoclassical utility maximization problems, with the exception that the price of the latter is replaced by the consistent implicit price $\bar{\varphi}_k$. Therefore, the solution of z can be estimated in a manner identical to that used to solve neoclassical utility maximization problems.

[2] The fact that the consistency condition holds, if and only if the production function incorporated into the UMP is homogeneous of degree one in both inputs and outputs is not new. See Hall (1973) and Muellbauer (1974).

2.2.2 Quantification of Service Quality

We postulate that the factor affecting the service quality of an option comprises all non-monetary service quality attributes that refer to all quantitative and qualitative attributes, excluding price. One service quality attribute is service time. This attribute is one element of quantitative attributes, and therefore can be measured objectively. The remaining attributes consist of all qualitative attributes. These attributes reflect the subjective judgment of consumers, and therefore cannot be measured objectively. The method of quantifying these qualitative attributes in the consumer production function is explained below, with an example of non-durable service.

To begin, we introduce an index system that distinguishes one option from another. An option is denoted by two integers, m and n, so as to distinguish differences in qualitative and/or quantitative attributes among options that offer the same qualitative choice service. The first index, m, designates *a heterogeneous service group* differentiated from other groups by qualitative attributes. The second index, n, distinguishes one option from the other *homogeneous service options* belonging to the same heterogeneous service group m.

One example that clearly illustrates this index system is international air passenger services. This service can be classified into three heterogeneous service groups: first ($m = 1$), business ($m = 2$), and economy classes ($m = 3$). Each option belonging to a certain heterogeneous service group can be further classified by carrier or route, using the second index, n.

Subsequently, we introduce a method of formulating consumer production that yields two groups of commodities through the consumption of a particular service. The first group consists of one commodity, which represents one special kind of benefit specific to the service, called *the prime commodity*. The second group includes multiple commodities, each of which denotes one qualitative attribute, termed *a certain hedonic com*modity. For example, the prime commodity of a passenger service option is the service that transports a traveler from one location to another, whereas the hedonic commodities comprise all relevant qualitative attributes such as comfort and safety perceived when receiving the service.

The yield of commodities in the process of consuming the service of option mn is quantified as follows. Suppose that a consumer purchases q_{mn} units of service mn, where q_{mn} represents the number of visits to purchase services from option mn. Then, the yield of the prime commodity, denoted by y_{mn}, is estimated by

$$y_{mn} = a_m \, q_{mn}, \tag{2.8}$$

where a_m is the production coefficient of service group m for the prime commodity, and is assumed to be positive. Subsequent the yield of hedonic commodity k, expressed by z_{kmn}, is quantified as follows:

$$z_{kmn} = b_{km} \, t_{mn} q_{mn}, \tag{2.9}$$

where b_{km} is the production coefficient of option mn for commodity k and can be either positive or negative, and t_{mn} is the service time of option mn.

The production coefficient a_m in (2.8) represents the yield of the prime commodity per service offered by option mn. This coefficient a_m is common to all options belonging to heterogeneous service group m, but can differ from that of other groups. For example, the prime commodity of sightseeing trips could be enjoyment achievable at the destination, and the magnitude of enjoyment usually differs from destination to destination.

The term $b_{km}t_{mn}q_{mn}$ in (2.9) estimates the number of hedonic commodity k yielded from the service of option mn, under the condition that the service is non-durable. The yield of the hedonic commodity from a non-durable service is assumed to be linearly proportional to service time; that is, the yield of commodity k is estimated by multiplying production coefficient b_{km} by total service time $t_{mn}q_{mn}$. This coefficient b_{km} is assumed to be common to all options belonging to the same group m, and to be different from that of other service groups, b_{lm}, for all $l \neq k$.

One example that clearly explains this aspect of coefficient b_{km} is air passenger services. It is certain that the coefficient perceived by a passenger differs by seat. For example, a passenger feels that a first class seat is more comfortable than is one in the economy section of the plane. Hence, first class has a larger coefficient of comfort than does economy class. But a particular seat, e.g., economy class, has an identical coefficient across all carriers or travel routes.

Finally, there is no objective way to identify relevant kinds of qualitative attributes included in demand analyses for a particular service. The kinds of qualitative attributes relevant to demand analyses for a certain qualitative choice service depend mainly on the subjective judgments of analysts. For example, one analyst may think that qualitative attributes relevant to trip demand analyses are comfort and safety only. Another analyst could argue that privacy and cleanliness should also be considered.

Fortunately, this arbitrariness in the choice of qualitative attributes does not affect subsequent demand analyses, as briefed below. The choice of an option depends only on the sum of monetary values for all service quality attributes per service, estimated by $\sum_k \bar{\varphi}_k b_{km} t_{mn}$, where $\bar{\varphi}_k$ is the implicit price of hedonic commodity k. Therefore, it is possible to analyze the demand for service mn using the summed value $\sum_k \bar{\varphi}_k b_{km} t_{mn}$, without recourse to each $\bar{\varphi}_k$ and b_{km} values for all k. Details will be explained next.

2.2.3 Consumer Production Function

We here introduce the consumer production function that will be incorporated into the basic choice problem for a certain non-durable service. This production function is formulated to fulfill the requirement that it is homogeneous in both multiple inputs and multiple outputs. The functional form of this joint homogeneous production function and the economic implications embedded in the function are presented below.

The joint production considered here yields two groups of commodities. One commodity is a prime commodity specific to a certain non-durable service, and its quantity is denoted by y. The other group consists of multiple hedonic commodities that are the byproducts of activities in which the non-durable service is consumed, and their quantities are expressed by $z \equiv (z_1, \cdots, z_K)$. Further, the hedonic commodities can be produced through substitute productions using inputs other than the non-durable service.

The production of commodities requires three groups of inputs. The first group consists of the non-durable services offered by multiple options, each of which is denoted by mn, where $m \in \langle 1, M \rangle$ and $n \in \langle 1, N_M \rangle$. The quantities purchased from the options are expressed by $q \equiv (q_{11}, \cdots, q_{MN})$. The second group is composed of goods and services traded in markets, which are the inputs to the substitute productions of hedonic commodities. Each of these inputs is denoted by $j \in \langle 1, J \rangle$, and the quantity of these inputs to the production of commodity k is expressed by $x_k \equiv (x_{k1}, \cdots, x_{kJ})$. The third group represents consumer time. The amount of time spent in consuming one unit of service mn is expressed by t_{mn}, and the amount spent in the substitute production of commodity k is denoted by t_k.

The production function specifies the relationship between the multiple inputs and multiple outputs, as described above. This production function is expressed as a set of simultaneous equation systems, each of which identifies the functional relationship of one output with multiple inputs, as presented below.

Assumption 2.1. The production function perceived by a consumer, who follows the deterministic perception approach, satisfies the following.[3]

(a) The production function of the prime commodity, denoted by Y', is

$$Y'(q; y) = y - \sum_{mn} a_m q_{mn} = 0.$$

(b) The production function of hedonic commodity k, denoted by Z'_k, is

$$Z'_k(q, x_k, t_k; z_k) = z_k - \sum_{mn} b_{km} t_{mn}\, q_{mn} - Z_k(x_k, t_k) = 0, \text{ all } k,$$

where Z_k is the substitute production function of k.

(c) Every substitute production Z_k is twice differentiable in (x_k, t_k), and satisfies the three following additional conditions: first, each input (x_k, t_k) is non-joint to those of other productions; second, every substitute production Z_k exhibits constant returns in its input (x_k, t_k); and third, the input (x_k, t_k) for all k is positive.

[3]A production function similar to that of $Y'(q; y)$is introduced in Moses and Williamson (1963). In the study, the demand for work trip yield is specified by the formula $y = \Sigma_m q_m$, in order to analyze the mode choice behavior of commuters. Furthermore, a linear relationship similar to that of $Z'_k(q, x_k, t_k; z_k)$ is applied in Lancaster (1966).

The production function defined above is a joint production function; that is, the multiple inputs q are joint inputs to multiple commodities; one prime commodity y and multiple hedonic commodities z, as formulated in Assumption 2.1(a) and (b), respectively. This joint production function is structured so as to be homogeneous of degree one in its inputs and outputs, as proved below.

Lemma 2.1. *The joint production function defined in Assumption 2.1 is homogeneous of degree one in both inputs and outputs; that is,*

$$Y'(\alpha q; \alpha y) + \sum_k Z'_k(\alpha q, \alpha x_k, \alpha t_k; \alpha z_k) = \alpha Y'(q\,; y) + \alpha \sum_k Z'_k(q, x_k, t_k; z_k),$$

where α is a positive real number.

Proof. In light of Euler's theorem, the production function is homogeneous of degree one in outputs, if

$$y\frac{\partial Y'}{\partial y} + \sum_k z_k \frac{\partial Z'_k}{\partial z_k} = y + \sum_k z_k. \tag{2.10}$$

Further, this production function is homogeneous of degree one in inputs, if

$$\sum_{mn} q_{mn}\frac{\partial Y'}{\partial q_{mn}} + \sum_k \left(\sum_{mn} q_{mn}\frac{\partial Z'_k}{\partial q_{mn}} + \sum_j x_{kj}\frac{\partial Z'_k}{\partial x_{kj}} + t_k \frac{\partial Z'_k}{\partial t_k} \right)$$
$$= -y - \sum_k z_k. \tag{2.11}$$

Therefore, the proof can be completed through proof of (2.10) and (2.11).

Equation (2.10) can be shown through trite calculations. Equation (2.11) can readily be proved using the following equation:

$$\sum_j x_{kj}\frac{\partial Z'_k}{\partial x_{kj}} + t_k\frac{\partial Z'_k}{\partial t_k} = -\sum_j x_{kj}\frac{\partial Z_k}{\partial x_{kj}} - t_k\frac{\partial Z_k}{\partial t_k} = -Z_k, \text{ all } k. \tag{2.12}$$

Here, the last equality of (2.12) follows from Assumption 2.1(c) such that the function Z exhibits constant returns in its inputs. □

Finally, we examine the mathematical and economic implications of the conditions for the production function in Assumption 2.1. Mathematically, Assumption 2.1(a) for prime commodity depicts that the yield of the prime commodity equals $\sum_{mn} a_m q_{mn}$, under the implicit assumption that there is no substitute for that service. On the other hand, Assumption 2.1(b) for hedonic commodity k expresses that the yield z_k is the sum of the production from qualitative choice services, $\sum_{mn} b_{km} t_{mn} q_{mn}$, and the substitute production, Z_k.

The three conditions for the substitute production function Z_k in Assumption 2.1(c) are the full set of mathematical devices, which reflects the following key postulate of the perception approach: the implicit price of commodity k is a constant exogenously determined, irrespective of chosen q values, as will be confirmed in Lemma 2.2. To be specific, given that the function Z_k is homogeneous of degree one, the implicit price of commodity k for this substitute production is a constant. Further, given that $Z_k \rangle 0$, this substitute production plays the role of determining the implicit price of commodity k packed in qualitative choice services, irrespective of $\sum_{mn} b_{km} t_{mn} q_{mn}$ values, as will also be proved in Lemma 2.2.

Economically, the implications of the substitute production function Z_k can be interpreted as follows. The function Z_k estimates the yield of a consumer's substitute production for hedonic commodity k, which is identical to qualitative attribute k packed in the services of all available options. Such a substitute production uses two groups of inputs: one group is goods and services traded in markets, and the other one is consumer time.

Substitute production is illustrated with the qualitative attribute of transportation services. One relevant qualitative attribute of transportation services is comfort. One way of substituting this attribute can be to spend consumer time by resting at home, which results in a feeling of comfort. If so, the substitute function for comfort, denoted by k, can be expressed as $Z_k(x_k, t_k) = a_k t_k$, where a_k is a production coefficient. Another example is safety. One substitute for enhancing the feeling of safety from accidents might be to purchase more comprehensive insurance. Then, the substitute function can be expressed as $Z_k(x_k, t_k) = b_k x_k$ under the condition of $p_k = 1$, where b_k is a production coefficient.

2.2.4 Modeling of the Basic Choice Problem

We here formulate the basic choice problem. We first introduce a set of conditions, which is sufficient to formulate the UMP under the deterministic perception approach. The only component not included is the consumer production function explained previously.

Assumption 2.2. The decision of a consumer under the deterministic perception approach satisfies the following.

(a) Consumer utility, U, is a function of a prime commodity, y, and multiple hedonic commodities, $z \equiv (z_1, \cdots, z_K)$. It is strictly concave and twice differentiable in the relevant region of its arguments.

(b) The yield of a commodity bundle,(y, z), satisfies the time constraint such that

$$t_w + \sum_{mn} t_{mn}\, q_{mn} + \sum_k t_k = T_o,$$

where t_w is the time spent at work, and T_o is the analysis period.

(c) The production activity satisfies the budget constraint such that

$$\sum_{mn} p_{mn}\, q_{mn} + \sum_{kj} p_j x_{kj} = I_o + w t_w,$$

where p_{mn} is the price of option mn, w consumer wage, p_j the price of input j, and I_o is non-labor income.

(d) The consumer can choose the amount of time, t_w, without any binding constraints.

(e) The prices of inputs other than qualitative choice services, such as I_o, w, and p_j, for all j, are fixed.

Combining the two sets of conditions in Assumptions 2.1 and 2.2 gives the basic choice problem. This choice problem, denoted by the Lagrangian L_o, is used to estimate the optimal bundle of inputs (q, x, t_w) and outputs (y, z), which maximizes the utility $U(y, z)$ under the condition that $p \equiv (p_{11}, \cdots, p_{MN})$ and $t \equiv (t_{11}, \cdots, t_{MN})$ are exogenously given:

$$L_o(q, x, t_w, y, z, \lambda, \mu, v, \eta, \phi; p, t) \equiv \max\{U(y, z)\} + \lambda\left(\sum_{mn} a_m q_{mn} - y\right)$$
$$+ \sum_k \mu_k \left(\sum_{mn} b_{km} t_{mn} q_{mn} + Z_k(x_k, t_k) - z_k\right) + v\left(T_o - t_w - \sum_{mn} t_{mn} q_m - \sum_k t_k\right)$$
$$+ \eta\left(I_o + w t_w - \sum_{mn} p_{mn} q_{mn} - \sum_{kj} p_j x_{kj}\right) + \sum_{mn} \phi_{mn} q_{mn}, \tag{2.13}$$

where $x \equiv (x_{11}, \cdots, x_{KJ}, t_1, \cdots, t_K)$ is the vector of goods and times inputted to substitute productions for K hedonic commodities, and $\lambda \rangle 0$, $\mu \equiv (\mu_1, \cdots, \mu_K) \rangle 0$, $\phi \equiv (\phi_{11}, \cdots, \phi_{MN}) \geq 0$, $v \rangle 0$, and $\eta \rangle 0$ are Lagrange multipliers. Furthermore, the term $\phi_{mn} q_{mn}$ represents the inequality constraint such that $q_{mn} \geq 0$.

Subsequently, we merge two constraints for time and budget in Assumption 2.2(b) and (c), respectively, into one constraint. The time spent at work t_w is a choice variable, as postulated in Assumption 2.1(d). Hence, it holds that

$$\frac{\partial L_o}{\partial t_w} = -v + \eta w = 0. \tag{2.14}$$

Replacing v in (2.13) with ηw and adding the two constraints in (2.13) gives *the budget constraint for the full income*:

$$\sum_{mn} (p_{mn} + w t_{mn}\, q_{mn}) + \sum_{kj} p_j x_{kj} + \sum_k w t_k = I_o + w T_o \equiv \bar{M}. \tag{2.15}$$

Here, the term $\bar{M}$ is the full income of consumers. The term v/η, which equals wage w, is called *value-of-time*.

Using the budget constraint for full income in (2.15), the basic choice problem L_o can be converted into the Lagrangian L_1 such that

$$L_1(q,x,y,z,\lambda,\mu,\eta,\phi\,;p,t) \equiv \max\{\,U(y,z)\} + \lambda\left(\sum_{mn} a_m q_{mn} - y\right)$$
$$+ \sum_k \mu_k \left(\sum_{mn} b_{km} t_{mn} q_{mn} + Z_k(x_k, t_k) - z_k\right)$$
$$+ \eta\left(\bar{M} - \sum_{mn}(p_{mn} + wt_{mn})\,q_{mn} - \sum_{kj} p_j x_{kj} - w\sum_k t_k\right) + \sum_{mn} \phi_{mn} q_{mn}. \quad (2.16)$$

This basic choice problem L_1 has an expression that is somewhat simpler than that of the original problem L_o. However, the problem L_1 is still too complex to perform subsequent demand analyses. For this reason, in the next section, we decompose the basic choice problem into two interrelated sub-optimization problems, and analyze these two problems.

2.3 Optimal Choice of Consumers

2.3.1 Consistency of Implicit Prices

The first sub-optimization problem of the basic choice problem L_1 is used to estimate the optimal input bundle that minimizes the production cost of an arbitrarily given commodity bundle. This cost minimization problem for consumer production is constructed using the production function defined in Assumption 2.1, which is homogeneous of degree one in both inputs and outputs. Because of the homogeneity, this minimization problem gives the implicit price of commodities in options, which satisfies the consistency condition, as shown below.

The cost minimization problem for consumer production, denoted by L_2, is used to search for the optimal input bundle (q,x) that gives the minimum cost required for the production of an arbitrary commodity bundle (q,x):

$$L_2(q,x,\pi,\varphi,\gamma;y,z,p,t) \equiv \min\left\{\sum_{mn}(p_{mn}+wt_{mn})q_{mn} + \sum_{kj} p_j x_{kj} + w\sum_k t_k\right\}$$
$$+\pi\left(y - \sum_{mn} a_m q_{mn}\right) + \sum_k \varphi_k\left(z_k - \sum_{mn} b_{km} t_{mn} q_{mn} - Z_k(x_k,t_k)\right)$$
$$-\sum_{mn} \gamma_{mn} q_{mn}, \quad (2.17)$$

where $\pi \rangle\ 0$, $\varphi \equiv (\varphi_1,\cdots,\varphi_K)\rangle\ 0$ and, $\gamma \equiv (\gamma_{11},\cdots,\gamma_{MN}) \geq 0$.

Let the vector $(\bar{q},\bar{x},\bar{\pi},\bar{\varphi},\bar{\gamma})$ be the saddle point of the Lagrangian L_2. According to the envelop theorem for constrained optimization problems, the Lagrange multiplier $\bar{\pi}$ is the implicit price of the prime commodity for services analyzed, and the

Lagrange multiplier $\bar{\varphi}_k$ for all k is the implicit price of the hedonic commodity k for these services. Below, these implicit prices are estimated under the condition that only one option mn is available.

Lemma 2.2. *When only one option mn is available, the cost minimization problem L_2 gives the following implicit prices of commodities.*

i. *The implicit price of the hedonic commodity k, $\bar{\varphi}_k$, is a constant estimated by*

$$\bar{\varphi}_k = p_j \Big/ \frac{\partial \bar{Z}_k}{\partial x_{kj}} = w \Big/ \frac{\partial \bar{Z}_k}{\partial t_k}, \quad all\, k,\ j.$$

ii. *The implicit price of the prime commodity for option mn, π_{mn}, is*

$$\pi_{mn}(p_{mn}, t_{mn}) = \frac{1}{a_m}(p_{mn} + v_m t_{mn}),$$

where

$$v_m = w - \sum_k \bar{\varphi}_k b_{km}.$$

iii. *Both $\bar{\varphi}_k$ and π_{mn} satisfy the consistency condition in output (y, z).*

Proof. See Appendix A.1 □

Lemma 2.2 estimates the implicit price π_{mn} and shows that this implicit price can be decomposed into three cost components: p_{mn}, wt_{mn}, and $\sum_k \bar{\varphi}_k b_{km}$. The price p_{mn} represents the money paid in purchasing one unit of service mn. The term wt_{mn} estimates the value of service time spent while consuming the service. The term $\sum_k \bar{\varphi}_k b_{km}$ quantifies the monetary value of all qualitative attributes packed in the service. The economic implications of such an implicit price π_{mn} are as below.

First, the implicit price π_{mn} differs from consumer to consumer. Of the three cost components examined above, the value of wt_{mn} is closely related to consumer wage, which usually differs from one individual to the next. Also, the value of $\sum_k \bar{\varphi}_k b_{km}$ largely depends on the subjective perception of a consumer, which is generally not identical to that of other consumers.

Second, the value of $\sum_k \bar{\varphi}_k b_{km}$ is a constant. In this term, the implicit price $\bar{\varphi}_k$ is a constant determined by the substitute production Z_k, which is independent of the consumption amount q_{mn}. Furthermore, the production coefficient b_{mk} is a constant representing the subjective consumer perception, under the deterministic perception approach.

Third, the term v_m, called *the net-value-of-time of group m*, is smaller than the value-of-time w, by the margin $\sum_k \bar{\varphi}_k b_{km}$. This margin represents the opportunity cost of K byproducts that amount to $(b_{1m}, \cdots, b_{Km})$, where b_{km} is the yield of commodity k. This margin therefore estimates cost savings accrued by the reduction in substitute productions of K hedonic commodities. Therefore, the net-value-of-time v_m is the portion of the value-of-time w, and can be sorted into the net cost required to produce one unit of the prime commodity.

Fourth, the net-value-of-time v_m has a linear relationship with value-of-time w. The net-value-of-time equals $w - \sum_k \bar{\varphi}_k b_{km}$. The implicit price $\bar{\varphi}_k$ in the v_m value equals $w/(\partial \bar{Z}_k/\partial t_k)$, and therefore is linearly proportional to w. Hence, it follows that the net-value-of-time v_m is linearly increasing in value-of-time w. This implies that a consumer having a larger wage tends to perceive a larger net-value-of-time than does a consumer with smaller wage who receives the same service, unless the values of $\partial \bar{Z}_k/\partial t_k$ and b_{km} are not significantly different among consumers.

Fifth, the term $v_m t_{mn}$, called *the net-service-time-value of option mn*, is a measure that quantifies the quality of service mn. The net-service-time-value represents the total non-monetary cost experienced in consuming one unit of service mn. This net-service-time-value becomes larger, as service time t_{mn} increases, and as the production coefficient b_{km}, for all k, decreases. Moreover, a larger service time and a smaller production coefficient imply lower service quality from the standpoint of quantitative and qualitative attributes, respectively. It can therefore be said that net-service-time-value increases as service quality decreases.

Sixth, the implicit price of a prime commodity, denoted by π_{mn} and estimated by $(p_{mn} + v_m t_{mn})/a_m$, is inversely proportional to the yield of prime commodities per service, denoted by a_m. The factor a_m expresses the quantity of prime commodities packed in option mn. Therefore, the implicit price π_{mn} represents the implicit price of a prime commodity in service mn, but not the implicit price of service mn, estimated by $p_{mn} + v_m t_m$.

Subsequently, we extend the analysis for the case when only one option is available to the case when multiple options are available. The analysis for this generalized case is presented below, focusing on characterizing the option chosen in consumer production.

Theorem 2.2. *The solution to L_2 for the case when multiple options are available satisfies the following.*

i. *The implicit price of prime commodity, $\bar{\pi}$, is*

$$\bar{\pi}(p,t) = \min_{mn} \{\pi_{mn}(p_{mn}, t_{mn})\},$$

and satisfies the consistency condition in output (y, z)

ii. *The solution $\bar{q}_{mn}$ satisfies the equality such that*

$$a_m \bar{q}_{mn} \begin{cases} = y, & \text{if } \bar{\pi} = \pi_{mn} \langle \pi_{m'n'}, \text{ all } m'n' \neq mn \\ = 0, & \text{if } \bar{\pi} \langle \pi_{mn} \\ \leqslant y, & \text{if } \bar{\pi} = \pi_{mn} = \pi_{m'n'}, \text{ some } m'n' \neq mn. \end{cases}$$

Proof. See Appendix A.1 □

Theorem 2.2 introduces a criterion for the choice of one option from among multiple options that offer the same service. This criterion can be interpreted as follows. First, the implicit price of prime commodities in Theorem 2.1.i, denoted by $\bar{\pi}$, is equal to the implicit price of the prime commodities for the option that charges the lowest implicit price. Second, Theorem 2.1.ii shows that the consumer chooses the option that requires the lowest implicit price; that is, only the option that has the lowest implicit price for its prime commodity will have a positive demand.

Finally, the consistency of π_{mn} implies that the chosen option mn has an implicit price π_{mn} that is lower than those of other available options, irrespective of (y, z) values. This consistency condition can, of course, apply to the optimal bundle that solves the original basic choice problem L_1. Therefore, the option chosen by a utility maximizer can be identified by simply comparing the implicit prices of available options, as confirmed subsequently.

2.3.2 Revealed Preference Condition

Here we consider the second sub-optimization problem of the original basic choice problem L_1, called the *reduced form of the basic choice problem* (*reduced form*, for short). This reduced form is used to estimate the optimal commodity bundle that maximizes consumer utility, which is identical that of the original basic choice problem. Using this reduced form, we determine the revealed preference condition that characterizes a decision to choose a certain option.

The first step in constructing the reduced form is to develop the consumer cost function $C(p, t\,; y, z)$. This estimates the minimum production cost of an arbitrary bundle (y, z):

$$C(p, t; y, z) \equiv \sum_{mn} (p_{mn} + wt_{mn})\, \bar{q}_{mn} + \sum_{kj} p_j \bar{x}_{kj} + w \sum_{k} \bar{t}_k, \tag{2.18}$$

where $\bar{q}_{mn}$, $\bar{x}_{kj}$, and $\bar{t}_k$ are the solutions to L_2. By virtue of the consistency of all implicit prices, the cost function is simplified as shown below.

Lemma 2.3. *The consumer cost function for L_2 satisfies the following equality:*

$$C(p, t; y, z) = \bar{\pi}(p, t)\, y + \sum_{k} \bar{\varphi}_k z_k.$$

Proof. This lemma is an extension of Theorem 2.1 for a simple example of joint homogeneous production functions to the production function of Assumption 2.1. The details of the proof are presented in Appendix A.2. □

Substituting the cost function in Lemma 2.3 into the basic choice problem L_1 in (2.16) gives the reduced form, denoted by L_3, such that

$$L_3(y,z,\eta;\bar{\pi}(p,t)) \equiv \max\{U(y,z)\} + \eta\left(\bar{M} - \bar{\pi}(p,t)\,y - \sum_k \bar{\varphi}_k z_k\right). \quad (2.19)$$

The solution to L_3 is identical to the solution to L_1 in (2.16), as shown below.

Lemma 2.4. *Let $(\bar{q},\bar{x},\bar{y},\bar{z},\,\cdot\,)$ and $(\hat{y},\hat{z},\hat{\eta})$ denote the solutions to the basic choice problem L_1 and its reduced form L_3, respectively. When the minimization problem L_2 takes option mn, it holds that*

$$(\bar{y},\bar{z}) = (\hat{y},\hat{z}),\ \text{and}\ \bar{y} = a_m\bar{q}_{mn}.$$

Proof. See Appendix A.3 □

Theorem 2.2 indicates that, if option mn satisfies the inequality $\pi_{mn} \langle\ \pi_{m'n'}$ for an arbitrary bundle of (y,z), this option satisfies the same inequality for the entire bundle of (y,z). On the other hand, Lemma 2.4 shows that the solution of (y,z) to the original basic choice problem L_1 equals the solution to the reduced form L_3. By combining these two findings, we characterize consumer choice behaviors, as below.

Theorem 2.3. *Suppose that the cost minimization problem L_2 results in the choice of option mn. Then, the solution to the basic choice problem L_1 satisfies the revealed preference condition such that*

$$\bar{\pi} = \pi_{mn} \ \langle\ \pi_{m'n'} \Leftrightarrow U(\bar{y},\bar{z}) = U(\bar{y}_{mn},\bar{z}_{mn})\ \rangle\ U(\bar{y}_{m'n'},\bar{z}_{m'n'}),\ \text{all}\ m'n' \neq mn,$$

where $(\bar{y},\bar{z})$ is the optimal solution of (y,z) when all the multiple options are available, whereas $(\bar{y}_{mn},\bar{z}_{mn})$ is that of (y,z) when only one option mn is available.

Proof. See Appendix A.4 □

Theorem 2.3 depicts the revealed preference condition of a utility maximizer as follows. "The left side implies the right" indicates that, if a consumer perceives that option *mn* has the lowest implicit price of a prime commodity, the choice of the option attains a higher level of the consumer utility than is attained by the choice of other options. Conversely, "the right side implies the left" shows that, if option *mn* chosen by a utility maximizer, the option requires the consumers to pay the minimum implicit price for the prime commodity.

2.3.3 An Illustration of Consumer Choices: Travel Choices

The previous consumer demand analyses based on the perception approach can provide plausible explanations for consumer choice behaviors that cannot be adequately explained within the context of neoclassical consumer demand theory. This advantage of demand analyses based on the perception approach is illustrated below, with travel choice problems for trip mode and destination.

To begin, we consider the following mode choice problem of a trip-maker. First, no matter what mode choice is made, the trip purpose remains the same; this means that service output a_m is common to all options, irrespective of their qualitative attributes. Second, each mode has qualitative attributes that differ from the others; in other words, each mode is designated as a different heterogeneous option $m \in \langle 1, M \rangle$. Third, each mode is assumed to have only one travel route; this implies that $N = 1$.

In this case, the production function of Assumption 2.1 can be amended as follows:

$$Y'(q; y) = y - \sum_m q_m = 0, \tag{2.20}$$

$$Z'_k(q, x_k, t_k; z_k) = z_k - \sum_m b_{km} t_m q_m - Z_k(x_k, t_k) = 0, \text{ all } k, \tag{2.21}$$

where t_m is the travel time of mode m.

The production function defined above is actually identical to the production function of Assumption 2.1, and therefore is homogeneous of degree one. Hence, it follows from Theorem 2.3 that the chosen mode m satisfies the following revealed preference condition:

$$\pi_m = p_m + v_m t_m \, \langle \, \pi_{m'} = p_{m'} + v_{m'} t_{m'}, \text{ all } m' \neq m, \tag{2.22}$$

where v_m is the net-value-of-travel-time of m, and p_m is the fare of option m.

One example that can successfully illustrate the advantage of the above mode choice criterion involves an air passenger who buys a first class ticket, rather than a business or economy class ticket on the same airplane. In this example, all options have the same travel time, denoted by t. Therefore, the full price of an option can be estimated by $p_m + w\,t$, whereas the implicit price of prime commodity for option m is $p_m + v_m t$.

It is certain that the full price for a first class seat is more costly than that for other options, since its price is more expensive, while their service times are identical. Therefore, there is no reason for the traveler who prefers the cheapest option when paying full price to choose a first class ticket. In contrast, the choice criterion (2.22) that compares the implicit price provides the following plausible explanation: the choice of the first class ticket is the outcome of a judgment that the additional fare does not exceed the additional value assignable to the better services.

Importantly, the above analysis can be applied to show that a consumer who receives a larger wage has a greater probability of choosing a more expensive seat when the seat offers higher-quality service. Equation (2.22) shows that the difference in the implicit price of a passenger service between a first and business class seats is equal to $p_2 - p_1 + t(v_2 - v_1)$, where 1 and 2 denote first and business class seats, respectively. Moreover, that equation implies that, if a consumer perceives this difference is positive, the consumer chooses a first class seat. In other words, if a consumer judges that $t(v_2 - v_1)$ is larger than $p_1 - p_2$, the consumer chooses a first class seat.

To be specific, the first class seat provides higher-quality service than is offered by the business class. This connotes that $v_2 - v_1$ is positive. Further, the fact that the former is more expensive implies that $p_1 - p_2$ is also positive. On the other hand, Lemma 2.2 shows that $v_2 - v_1$ equals $\sum_k \bar{\varphi}_k(b_{k1} - b_{k2})$, in which $b_{k1} - b_{k2}$ should be positive. Further, the fourth comment for Lemma 2.2 indicates that the implicit price $\bar{\varphi}_k$ tends to be greater as the consumer wage is larger. Therefore, as the wage of a consumer is larger, it is highly probable that the perceived value of $\sum_k \bar{\varphi}_k(b_{k1} - b_{k2})$ and thus of $v_2 - v_1$ is greater. Hence, it can be concluded that a consumer with a larger wage has a stronger preference for a first class seat.

Subsequently, we consider the trip destination choice problem, using the example of short-term sightseeing trips. Suppose that a traveler has already decided the trip mode and the travel period. Suppose, also, that the prime commodity of sightseeing trips is enjoyment achievable at a destination, and that the yield of the prime commodity differs by location. Suppose, however, that the yields of qualitative attributes per unit of travel time, including access and egress time, exhibit no significant difference among options.

In this special case, the production function for the prime commodity of trips must reflect the difference among options in the yield of the prime commodity, as expressed by a_m. Hence, the production function for prime commodities can be expressed by

$$Y'(q; y) = y - \sum_m a_m q_m = 0. \tag{2.23}$$

On the other hand, the production function for hedonic commodities can be constructed in such a manner that all options have the same production coefficient b_k:

$$Z'_k(q, x_k, t_k; z_k) = z_k - \sum_m b_k t_m q_m - Z_k(x_k, t_k) = 0, \text{ all } k, \tag{2.24}$$

where t_m is the sum of on-site and travel times for option m.

The production function defined above is homogeneous of degree one. Hence, it follows from Theorem 2.3 that the chosen option m satisfies the following:

$$\pi_m = \frac{1}{a_m}(p_m + v\, t_m) \langle\ \pi_{m'} = \frac{1}{a_{m'}}(p_{m'} + v\, t_{m'}), \text{ all } m' \neq m \tag{2.25}$$

where v is the net-value-of-time common to all options, and p_m is the sum of in-site expenditure and travel cost of option m.

The above choice criterion can be applied to explain why a traveler chooses a location more distant than other locations. The chosen option m might require higher in-site expenditure and/or a longer travel time than is required by other available options. Therefore, we cannot exclude the possibility that the full price of trip, $p_m + w\,t_m$, and/or the implicit price of trip, $p_m + v\,t_m$, are larger than those for other options. It is, however, certain that the chosen option m requires the minimum implicit cost per unit of enjoyment, estimated by $(p_m + v\,t_m)/a_m$.

Finally, the revealed preference condition of Theorem 2.3 is graphically illustrated using the following simple example of a mode choice problem. First, a shopper frequently makes trips to a certain store. Second, the decision-making components of the shopper are trip frequency and mode, and two trip modes are available: auto and transit, denoted by 1 and 2, respectively. Third, the outputs of consumer productions are composed only of two commodities: one prime commodity referred to as shopping, and one hedonic commodity defined as comfort.

In these circumstances, suppose that the consumer chooses mode 1. By the inequality on the left side of Theorem 2.3, the choice of mode 1 implies the following inequality: $\pi_1 = p_1 + v_1 t_1 \langle \pi_2 = p_2 + v_2\,t_2$, where p_m, t_m, and v_m are travel cost, travel time, and net-value-of-time of mode m. Here, the net-value-of-times v_1 and v_2 are distinct constants, since the qualitative attributes of the two modes differ.

We next consider the problem of graphically representing the budget constraint of the reduced form L_3 for the hypothetical case when only one mode is available. By Lemma 2.3, the cost function C_m for this hypothetical case is $C_m(y,z) = \pi_m\,y + \bar{\varphi}z$, for $m = 1, 2$, where $\bar{\varphi}$ is the implicit price of comfort. Therefore, the budget constraint of the reduced form L_3 can be expressed as $\bar{M} = \pi_m\,y + \bar{\varphi}\,z$, $m = 1, 2$. This budget line for each mode equals the production possibility frontier of y and z for consumer productions, under the condition that only that mode is available, as depicted in Fig. 2.1.

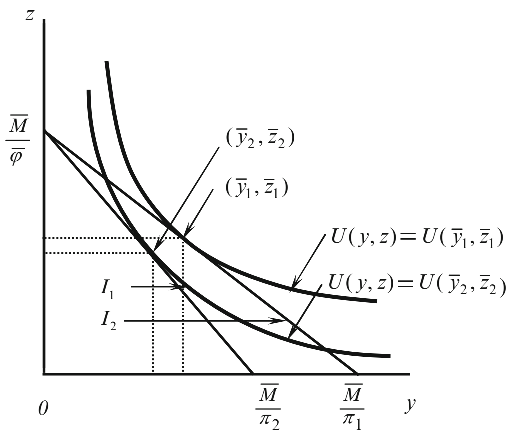

Fig. 2.1 Representation of revealed preference for the choice of a trip mode

By the inequality $\pi_1 \langle \pi_2$, the budget line for mode 1 is located atop the line for mode 2. This result leads to the right-side inequality of Theorem 2.3, such that $U(\bar{y}, \bar{z}) = U(\bar{y}_1, \bar{z}_1) \rangle U(\bar{y}_2, \bar{z}_2)$, where $(\bar{y}, \bar{z})$ is the solution in the case when the two modes are available, and $(\bar{y}_m, \bar{z}_m)$ is the solution in the case when only mode m is available.

2.4 Other Topics for Demand Analyses

2.4.1 Service Demand Functions

This subsection is concerned with the mathematical properties of demand function for option mn, denoted by $\bar{q}_{mn}$, for two independent vectors, $p \equiv (p_{11}, \cdots, p_{MN})$ and $t \equiv (t_{11}, \cdots, t_{MN})$. We first develop the function $\bar{q}_{mn}$ from the reduced form L_3, which has a formulation very similar to those of neoclassical utility maximization problems. We subsequently analyze the mathematical properties of $\bar{q}_{mn}$ by applying the well-known mathematical properties of neoclassical consumer demand functions.

Firstly, we develop the demand function $\bar{q}_{mn}$ from the basic choice problem L_1. The solution to the problem L_1 is the function of quantitative attribute vectors (p, t). Therefore, the basic choice problem L_1 can be expressed by the following:

$$\begin{aligned} &L_1(q(p,t), x(p,t), y(p,t), z(p,t), \lambda(p,t), \mu(p,t), \eta(p,t), \phi(p,t); p, t), \\ &\qquad \equiv \max\{U(r(p,t))\} + \sum_i \kappa_i(p,t) g_i(r(p,t); p, t), \end{aligned} \tag{2.26}$$

where $r \equiv (q, x, y, z)$ and $\kappa \equiv (\lambda, \mu, \eta, \phi)$.

Under this convention, the Lagrangian of the basic choice problem L_1 at the saddle point can be expressed as follows:

$$L_1(\bar{r}(p,t), \bar{\kappa}(p,t); p, t) \equiv U(\bar{r}(p,t)) + \sum_i \bar{\kappa}_i(p,t) g_i(\bar{r}(p,t); p, t), \tag{2.27}$$

where $(\bar{r}(p,t), \bar{\kappa}(p,t))$ is the saddle point of L_1.

Theoretically, the saddle point for a certain value of (p,t) can be estimated from the Kuhn-Tucker conditions for that value. Further, it is clear that the saddle point is the function of (p, t). This means that the demand function $\bar{q}_{mn}(p, t)$, an element of $\bar{r}(p, t)$, is sensitive to the independent vector (p, t).

Secondly, we convert the expression of L_1 at the saddle point into that of the reduced form L_3. The solution of (y, z) to L_3 is a function of $\bar{\pi}(p, t)$ that equals $\min_{mn}\{\pi_{mn}(p_{mn}, t_{mn})\}$, as shown in (2.19). Hence, the Lagrangian L_3 at the saddle point can be expressed as follows:

$$L_3(\hat{y}(\bar{\pi}), \hat{z}(\bar{\pi}), \hat{\eta}(\bar{\pi}); \bar{\pi}) \equiv U(\hat{y}(\bar{\pi}), \hat{z}(\bar{\pi})) + \hat{\eta}(\bar{\pi})\left(\bar{M} - \bar{\pi}\,\hat{y}(\bar{\pi}) - \sum_k \bar{\varphi}_k \hat{z}_k(\bar{\pi})\right). \tag{2.28}$$

This reduced form has a formulation identical to that of neoclassical utility maximization problems, except that $\bar{\pi}$ is an independent vector but the vector function of (p,t).

Thirdly, we identity the relationship between the function $\bar{q}_{mn}$ estimated from (2.27) and the function $\hat{y}(\bar{\pi})$ developed from (2.28). Functions $\bar{q}_{mn}$ and $\bar{\pi}$ have common explanatory vectors (p,t). Moreover, the two functions $\bar{q}_{mn}$ and $\hat{y}(\bar{\pi})$ satisfy the relationship presented below.

Theorem 2.4. *The solution $\bar{q}_{mn}$ to the basic choice problem L_1 and the solution $\hat{y}$ to the reduced form L_3 satisfy the following relationship:*

$$a_m \bar{q}_{mn}(p,t) \begin{cases} = \hat{y}\left(\bar{\pi}(p,t)\right), & \textit{if} \;\; \bar{\pi} = \pi_{mn} \langle\, \pi_{m'n'}, \quad \textit{all } m'n' \neq mn \\ = 0, & \textit{if} \;\; \bar{\pi} \langle\, \pi_{mn} \\ \leqslant \hat{y}\left(\bar{\pi}(p,t)\right), & \textit{if} \;\; \bar{\pi} = \pi_{mn} = \pi_{m'n'}, \quad \textit{some } m'n' \neq mn. \end{cases}$$

Proof. By Lemma 2.4, it holds that $\bar{y}\,(p,t) = \hat{y}\,(\bar{\pi}(p,t))$. Substituting this equality into Theorem 2.2.ii gives the equation presented above. □

Theorem 2.4 shows that, when option mn satisfies the revealed preference condition for the choice of this option, it holds that

$$\bar{q}_{mn}(p,t) = \frac{1}{a_m}\hat{y}\left(\frac{1}{a_m}(p_{mn} + v_m\, t_{mn}\right) \tag{2.29}$$

$$\bar{q}_{m'n'}(p,t) = 0, \text{ all } m'n' \neq mn. \tag{2.30}$$

These two equations show the following. First, the demand for option mn has a positive value only when the implicit price π_{mn} satisfies the revealed preference condition such that $\pi_{mn} \leqslant \pi_{m'n'}$, for all $m'n' \neq mn$. Second, when the demand for an option is positive, the demand for that option depends solely on the implicit price of the prime commodity, which is a function of price p_{mn} and service time t_{mn} only.

Finally, we analyze the continuity and comparative statics of demand function $\bar{q}_{mn}$ with respect to (p,t). The function $\bar{q}_{mn}$ equals $\hat{y}(\bar{\pi})/a_m$ in the range of (p,t), which satisfies the condition that $\pi_{mn} \langle\, \pi_{m'n'}$, for all $m'n' \neq mn$. On the other hand, the function $\hat{y}$ is developed from the reduced form L_3, which has a formulation identical to that of neoclassical utility maximization problems. Hence, it is possible to analyze the continuity and comparative statics of $\hat{y}$ with respect to $\bar{\pi}$ by applying the well-known properties of neoclassical consumer demand functions. Through the use of this property of $\hat{y}$, we below deduce the mathematical properties of $\bar{q}_{mn}$.

Theorem 2.5. *The demand function of prime commodity, $\hat{y}(\bar{\pi})$, and the demand function of option mn, $\bar{q}_{mn}$, satisfy the following.*

i. *The function $\hat{y}(\bar{\pi})$ is continuous in every variable of (p,t). However, the function $\bar{q}_{mn}$ is not continuous at the point where*

$$\bar{\pi}(p,t) = \pi_{mn}(p_{mn},t_{mn}) = \pi_{m'n'}(p_{m'n'},t_{m'n'}), \quad \textit{some} \ \ m'n' \neq mn.$$

ii. *Suppose that the function $\bar{q}_{mn}$ is positive and continuous at a certain point (p,t). Then, the function has the following comparative statics at that point:*

$$\frac{\partial \bar{q}_{mn}(p,t)}{\partial p_{mn}} = \frac{1}{v_m}\frac{\partial \bar{q}_{mn}(p,t)}{\partial t_{mn}} = \frac{\partial \hat{y}(\bar{\pi}(p,t))}{\partial \pi_{mn}} \ \langle \ 0 \tag{2.31}$$

$$\frac{\partial \bar{q}_{mn}(p,t)}{\partial p_{m'n'}} = \frac{1}{v_{m'}}\frac{\partial \bar{q}_{mn}(p,t)}{\partial t_{m'n'}} = 0, \ \textit{all}\, m'n' \neq mn. \tag{2.32}$$

iii. *Suppose now that the function $\bar{q}_{mn}$ is not continuous at a certain point (p,t), and that $\pi_{mn}(p_{mn},t_{mn}) = \pi_{m'n'}(p_{m'n'},t_{m'n'})$, for some $m'n' \neq mn$. Then, it holds that*

$$\frac{\partial \bar{q}_{mn}(p,t)}{\partial p_{mn}} = \frac{1}{v_m}\frac{\partial \bar{q}_{mn}(p,t)}{\partial t_{mn}} = -\infty \tag{2.33}$$

$$\frac{\partial \bar{q}_{mn}(p,t)}{\partial p_{m'n'}} = \frac{1}{v_{m'}}\frac{\partial \bar{q}_{mn}(p,t)}{\partial t_{m'n'}} = \infty. \tag{2.34}$$

Proof. See Appendix A.5

Theorem 2.5 shows the following. First, the demand function of option *mn* is continuous and decreasing in p_{mn} and t_{mn} in the range of (p,t), on which the demand for this option is positive. Second, the demand function of option *mn* is discontinuous at the implicit price of that option, which equals that of some other options but is less than those of other remaining options.

It should be emphasized that the comparative statics estimated in the above theorem have one critical shortcoming: they cannot properly quantify substitutability among options. In general, a change in price or service time of an option causes a demand shift to competing options, and the shifted demand is finite. Contrary to this, the partial derivative $\partial\bar{q}_{mn}/\partial p_{m'n'}$ is zero at the point (p,t) where $\bar{q}_{mn}$ is continuous, whereas the term $\partial\bar{q}_{mn}/\partial p_{m'n'}$ is infinitely large at the point where $\bar{q}_{mn}$ is not continuous.

2.4.2 Mathematical Properties of Qualitative Choice Problems

Qualitative choice problems are the main target for forthcoming demand analyses of this monograph. Qualitative choice problems refer to a subgroup of UMPs under the perception approach. These choice problems should take a reduced form similar to neoclassical utility maximization problems, as does the basic choice problem. Below, we present the implications of this requirement for qualitative choice problems and the advantage of qualitative choice problems over other UMPs under the perception approach.

To begin, we consider the requirement that qualitative choice problems should take a reduced form similar to neoclassical utility maximizations. This requirement implies that the reduced form of qualitative choice problems should have an expression identical or almost identical to the reduced form of the basic choice problem in (2.19). Moreover, the implicit price of commodities in the budget constraint for the reduced form should possess certain mathematical regularities that can greatly simplify forthcoming demand analyses. This aspect of qualitative choice problems is illustrated with the basic choice problem, an example of qualitative choice problems.

One distinctive feature of the reduced form in (2.19) is that the implicit price of all commodities fulfills two different consistency conditions that follow. First, the implicit price π_{mn} satisfies the consistency condition such that this implicit price is independent of commodity bundles (y, z). Second, the net-value-of-time v_m in π_{mn} fulfills the consistency condition such that the net-value-of-time is independent not only of commodity bundles (y, z) but also of quantitative attribute vectors (p, t).

As illustrated above with an example, all qualitative choice problems should have a reduced form that fulfills the two different consistency conditions: the consistency of implicit prices of all commodities; and that of net-value-of-time or equivalent terms representing the monetary value of qualitative attributes packed in a service. The first consistency for implicit price is fulfilled when the consumer production function of a UMP is homogeneous in both inputs and outputs, as shown in Theorem 2.1. However; it is not straightforward to identify the condition under which a UMP satisfies the second consistency for net-value-of-time. This issue will be explored in detail in the following chapter

Subsequently, we introduce the advantages of qualitative choice problems over other UMPs under the perception approach. The advantages of qualitative choice problems consist in that only these choice problems can yield analytical outcomes that are crucial inputs to forthcoming economic analyses of service markets in this study. These advantages, all of which stem from the two different kinds of consistency introduced above, are as below.

First, only qualitative choice problems have their stochastic versions for the random perception approach considered in Chaps. 4 and 5. Mathematically, only a parameter in optimization problems can be designated as a random variable, but a function cannot be. On the other hand, the random perception approach hypothesizes that a net-value-of-time is a random variable that reflects uncertainty for the yield of hedonic commodities in consumer production. For this reason, the hypothesis of randomness can apply only to the net-value-of-time that satisfies the second consistency condition such that it is independent of q and (p, t).

Second, only qualitative choice problems yield the demand functions $\bar{q}_{mn}$ and $\bar{y}$ for which comparative statistics with respect to (p, t) can be estimated through relatively simple analyses. Since qualitative choice problems take net-value-times satisfying the consistency requirement, it is feasible to identify the relationship between $\bar{q}_{mn}/\partial p_{mn}$ and $\bar{q}_{mn}/\partial t_{mn}$ and between $\bar{q}_{mn}/\partial p_{m'n'}$ and $\bar{q}_{mn}/\partial t_{m'n'}$ in a manner shown in (2.31)~(2.34). Such a property of qualitative choice problems greatly simplifies analyses of comparative statistics, the main theme of Chap. 5.

We illustrate such an advantage of qualitative choice problems with an example. Suppose that a UMP under the perception approach gives the outcome that satisfies the equality of Theorem 2.4, but that gives the v_m value sensitive to (p_m, t_m). Then, (2.31) can be amended as follows:

$$\begin{aligned}\frac{\partial\,\bar{q}_{mn}(p,t)}{\partial\,p_{mn}} &= \frac{\partial\,\hat{y}\,(\bar{\pi}(p,t))}{\partial\,\pi_{mn}}\,\frac{\partial\pi_{mn}}{\partial p_{mn}} = \frac{\partial\,\hat{y}\,(\bar{\pi}(p,t))}{\partial\,\pi_{mn}}\left(1+\frac{\partial v_{mn}}{\partial p_{mn}}t_{mn}\right)\\ &\neq \frac{1}{v_m}\,\frac{\partial\,\bar{q}_{mn}(p,t)}{\partial\,t_{mn}} = \frac{1}{v_m}\,\frac{\partial\,\hat{y}\,(\bar{\pi}(p,t))}{\partial\,\pi_{mn}}\left(1+\frac{\partial v_{mn}}{\partial t_{mn}}t_{mn}\right).\end{aligned} \tag{2.35}$$

This equation shows that $v_m\bar{q}_{mn}/\partial p_{mn} \neq \bar{q}_{mn}/\partial t_{mn}$. This means that it is not simple to identify the relationship between $\bar{q}_{mn}/\partial p_{mn}$ and $\bar{q}_{mn}/\partial t_{mn}$, which is a very important input to the forthcoming analyses of this study. Two choice problems similar to this example will be introduced in Sect. 3.4.

Third, qualitative choice problems can provide the revealed preference condition amenable to statistical estimations. The revealed preference condition that identifies chosen options compares implicit prices π_m for all mn. On the other hand, in statistical estimations of the revealed preference condition, the variables unknown to statistical analysts are confined to v_m values for all m. These v_m values are statistically identifiable only when they are independent of dependent variables q and explanatory variables (p,t); the v_m values should fulfill the consistency condition.

It should be noted that statistical estimations of $v_m (= w - \sum_k \bar{\varphi}_k\, b_{km})$ values are free from the choice of qualitative attributes included in the formulation of qualitative choice problems. As noted in Subsect. 2.2.2, it is impossible to objectively identify all relevant qualitative attributes included in consumer production functions for a particular service. However, statistical estimations of v_m values target the summed value of $\sum_k \bar{\varphi}_k\, b_{km}$, but not the value of $\bar{\varphi}_k\, b_{km}$ for each commodity k. For this reason, the difficulty of objectively identifying relevant qualitative attributes does not cause any bias or error in statistical estimations.

http://www.springer.com/978-3-642-20188-2

Congestion-Prone Services under Quality Competition
A Microeconomic Analysis
Moon, D.-J.
2012, XVIII, 426 p., Hardcover
ISBN: 978-3-642-20188-2