

Contents

1	NS–F Equations and Modelling: A French Touch	1
1.1	My First Contribution to the RAM Approach in Fluid Dynamics	3
1.2	My Collaboration with Jean-Pierre Guiraud in the Aerodynamics Department of ONERA	8
1.3	A Few Remarks Concerning My Preceding Books on Modelling in Newtonian Fluid Flows	10
1.4	Conclusion	14

Part I Navier–Stokes–Fourier Equations: General Main Models

2	Newtonian Fluid Dynamics as a Mathematical – Physical Science .	19
2.1	From Newton to Euler	21
2.1.1	Eulerian Elastic Fluid	21
2.1.2	From Adiabaticity to Isochoricity	23
2.2	Navier Viscous Incompressible, Constant Density Equations	25
2.3	Navier–Stokes–Fourier Equations for Viscous Compressible and Heat-Conducting Fluid Flow	28
2.3.1	The Cauchy Stress Principle	28
2.3.2	Navier–Stokes Constitutive Equations: The Cauchy–Poisson Law	29
2.3.3	Thermodynamics and Energy Equation via Fourier Constitutive Equation	32
2.3.4	Navier–Stokes–Fourier (NS–F) Equations	34
2.4	Initial and Boundary Conditions	35
2.4.1	The Problem of Initial Conditions	36
2.4.2	Unsteady Adjustment Problems	38
2.4.3	Boundary Conditions for the Velocity Vector \mathbf{u} and Temperature T	40
2.4.4	Other Types of Boundary Conditions	41

3	From NS–F Equations to General Main Model Equations	51
3.1	Non-dimensional Form of the NS–F Typical Problem	52
3.1.1	Non-dimensional NS–F Equations and Reduced Parameters	52
3.1.2	Conditions for the Unsteady NS–F Equations	57
3.2	General Main Model Equations	59
3.2.1	Some General Main Models	59
3.2.2	A Sketch of the Various General Main Models	61
3.3	Non-uniform Validity of Main Model Equations and the Local Limiting Processes	62
3.3.1	Large Reynolds Case: A First Naive Approach	63
3.3.2	Low Mach Number Case: The Navier System of Equations and Companion Acoustics Equations	65
3.3.3	Oberbeck–Boussinesq Model Equations for the Rayleigh–Bénard Shallow Thermal Convection	69
3.3.4	Stokes–Oseen Model Equations in the Case of a Low Reynolds Number	71
3.3.5	The Case of Non-linear Acoustics	76
3.3.6	A Sketch of Consistent Models Derived from NS–F Equations Relative to the Considered Reference Parameters	80
4	A Typical RAM Approach: Boussinesq Model Equations	81
4.1	Introductory Remarks Concerning Atmospheric Flow	81
4.2	Asymptotics of the Boussinesq Case	84
4.2.1	Euler Dimensionless Equations for the Thermodynamic Perturbations	85
4.2.2	Rational Derivation of Boussinesq Equations	86
4.2.3	Two Particular Cases	89
4.3	The Steady 2D Case	90
4.4	The Problem of Initial Conditions	92
4.5	A Sketch of a RAM Approach for a Boussinesq Model	94

Part II A Sketch of a Mathematical Theory of the RAM Approach

5	The Structure of Unsteady NS–F Equations at Large Reynolds Numbers	97
5.1	Introduction	97
5.2	The Emergence of the Four Regions as a Consequence of the Singular Nature of BL Equations Near $t = 0$	98
5.3	Formulation of the Unsteady NS–F Problem	100
5.4	Derivation of the Corresponding Four Model Problems	103
5.4.1	Euler–Prandtl Regular Coupling	103

5.4.2	Acoustic and Rayleigh Problems Near the Initial Time $t = 0$	107
5.5	Adjustment Processes Towards the Prandtl BL Evolution Problem	110
5.5.1	Adjustment Process Via the Acoustics/Gas Dynamics Equations	111
5.5.2	Adjustment Process Via the Rayleigh Equations	112
5.6	Some Conclusions	114
6	The Mathematics of the RAM Approach	117
6.1	Our Basic Postulate for the Realization of the RAM Approach	117
6.2	The Mathematical Nature of NS-F Equations: A Fluid-Dynamical Point of View	120
6.3	Formulation of Dimensionless Equations for Applications of the RAM Approach	124
6.3.1	Turbomachinery Fluid Flows	124
6.3.2	The G-Z “Rolled-up Vortex Sheet” Theory: Vortex Sheets and Concentrated Vorticity	127
6.3.3	The G-Z Asymptotic Approach to Non-linear Hydrodynamic Stability	130
6.3.4	A Local Atmospheric Thermal Problem: A Triple-Deck Viewpoint	135
6.3.5	Miscellanea	138
6.4	Some Key Steps for the Application of the Basic Postulate	138
6.4.1	Similarity Rules: Small Mach and Large Reynolds Numbers Flow	140
6.4.2	The Matching Principle	141
6.4.3	The Least-Degeneracy Principle and the Significant Degeneracies	144
6.4.4	The Method of Multiple Scales (MMS)	145
6.4.5	The Homogenization Analysis	149
6.4.6	Asymptotics of the Triple-Deck Theory	154

Part III Applications of the RAM Approach to Aerodynamics, Thermal Convection, and Atmospheric Motions

7	The RAM Approach in Aerodynamics	161
7.1	Derivation of a Through-Flow Model Problem for Fluid Flow in an Axial Compressor	161
7.1.1	The Veuillot Approach	161
7.1.2	The G-Z Approach	165
7.1.3	Transmission Conditions, Local Solution at the Leading/Trailing Edges, and Matching	168
7.1.4	Some Complements	169
7.2	The Flow Within a Cavity Which is Changing Its Shape and Volume with Time: Low Mach Number Limiting Case	171

7.2.1	Formulation of the Inviscid Problem	172
7.2.2	The Persistence of Acoustic Oscillations	173
7.2.3	Derivation of an Average Continuity Equation	174
7.2.4	Solution for the Fluctuations \mathbf{u}_0^* and ρ_1^*	178
7.2.5	The Second-Order Approximation	179
7.2.6	The Average System of Equations for the Slow Variation	182
7.2.7	The Long Time Evolution of the Fast Oscillations	184
7.2.8	Some Concluding Comments	187
8	The RAM Approach in the Bénard Convection Problem	193
8.1	An Introduction	193
8.2	Some Unexpected Results for the Bénard Problem of an Expansible Liquid Layer Heated from Below	194
8.2.1	The Questionable Davis (1987) Upper, Free Surface, Temperature Condition, and the Problem of Two Biot Numbers	196
8.2.2	The Mystery of the Disappearance of Influence of the Free Surface in the RB Leading-Order Shallow Thermal Convection Model	199
8.2.3	Influence of the Viscous Dissipation	201
8.3	The Marangoni Effect	203
8.3.1	The Long-Wave Approach	205
8.3.2	Towards a Lubrication Equation	208
8.4	From Deep to Shallow Thermal Convection Model Problems . . .	210
9	The RAM Approach in Atmospheric Motions	213
9.1	Some Models for the Lee-Waves Problem	213
9.1.1	First Integrals	214
9.1.2	An Equation for the Vertical Deviation $\delta(x, z)$	215
9.1.3	Dimensionless Formulation	218
9.1.4	Four Distinguished Limiting Cases	220
9.2	The Low Kibel Number Asymptotic Model	223
9.2.1	The Dissipative (Viscous and Non-adiabatic) NS–F atmospheric Equations	226
9.2.2	Hydrostatic Limiting Processes	229
9.2.3	The Dissipative Hydrostatic Equations in p -System	233
9.2.4	The Leading-Order QG Model Problem	237
9.2.5	The Second-Order Ageostrophic G–Z Model	247
9.2.6	Kibel Primitive Equations and Lee-Waves Problems as Inner and Outer Asymptotic Models	248
	Epilogue	253
	References	261
	Index	269

Navier-Stokes-Fourier Equations
A Rational Asymptotic Modelling Point of View
Zeytounian, R.K.
2012, XVI, 276 p., Hardcover
ISBN: 978-3-642-20745-7