Preface

"*Probability does not exist*", such a statement is at the beginning of the book by *B. de Finetti* dedicated to the foundation of probability, nowadays called "*probability theory*". We believe that such a statement is *overdone*. The target of the reference is made very clear, namely the analysis of the *random effects* or of a random experiment. The result of an experiment like throwing dice or a LOTTO outcome is *not* predictable. Such an *indeterminism* is characteristic analysis of an experiment. In general, we analyze here *nonlinear relations* between *random effects*, maybe stochastic or not. Obviously, the center of interest is on linear models, which are able to approximate nonlinear relations. There are typical nonlinear models, which *cannot* be linearized. Our experimental world is divided into linear or linearized *and* nonlinear models. Sometimes, nonlinear models can be transformed into *polynomial equations*, also called *algebraic*, which allow a rigorous solution set. A prominent example is the *resection problem of overdetermined type*. If we are able to express the characteristic rotation matrix between reference frames by *quaternions*, an algebraic equation of known high order arrives. But, in contrast to linear models, there exist a whole world of solutions, which have to be discussed. *P. Lohse* presented in his Ph.D thesis wonderful *examples* of which we refer.

A.N. Kolmogorov founded the *axiomatic (mathematical) theory* of probability. Typical for the axiomatic treatment, there are *special rules* relating to *measure theory*. Here, we only refer to excellent introductions of measure theory, for instance *H. Bauer (1992), J. Elstrodt (2005)* or *W. Rudin (1987)*. Of course, there are very good textbooks on the theory of probability, for instance *H. Bauer (2002), K.L. Chung (2001), O. Kallenberg (2002), M. Loeve (1977)*, and *A.N. Shiryaev (1996)*.

It is important to inform our readers about these many subjects, which are not treated here: robust estimation, Bayes statistics (only examples), stochastic integration, *ITÔ* stochastic processes, stochastic signal processing, martingales, Markov processes, limit theorems for associated random fields, modeling, measuring, and managing risks, genetic algorithms, chaos, turbulence, fractals, neural networks, wavelets, Gibbs field, Monte Carlo, and many, indeed *very important subjects*. This statement refers to an overcritical reviewer who complained of the subject, which we did *not* treat. *Unfortunately* we had to strongly restrict our subjects. Our *contribution* *is only an atom in the* 10^8 *atoms*.

What will be treated?

It is the task of the observer or the designer to decide: what is the best fit between a linear or nonlinear model or the "real world" of the observer. Both types of analysis have to make an a priori statement about a lot of facts.

- "Are the *observations* random or not, a stochastic effect *or* a fixed effect?"
- "What are the *parameters* in the model space or would you prefer a 'model free' concept?"
- "What is the nature of the *parameters, random effects* or *fixed effects*?"
- "Could you find the *conditions* between the observations or between the parameters? What is the nature of the conditions, *linear or nonlinear, random or not*?"
- "What is the bias between the model, and its *estimation or prediction*?"

There are various criteria, which characterize the observation space or the parameter space:

> linear or nonlinear, estimation theory, linear and nonlinear distributions, l^r *– optimality*, (LESS: l^2), restrictions, mixed models of fixed and random effects, zero-, first-, and second order design, nonlinear statistics on curved manifolds, minimal bias estimation, translational invariance, total least squares, generalized least squares, minimal distance estimation, optimal design, and optimal prediction.

Of key interest is to find *equivalence lemmas*, for instance: Assume direct observations. Then a least squares fit (*LESS*) is equivalent to a best linear uniformly unbiased estimation (*BLUUE*).

The Chap. 1 deals with the *first problem* of algebraic regression, the consistent system of linear observational equations or a system of *underdetermined linear equations*. From the first page, examples onwards, we treat MINOS and the "horizontal rank partitioning". In detail, we give the *eigenvalue decomposition of weighted minimum norm solution*. Examples are *FOURIER series, FOURIER-LEGENDRE series* including the *NYQUIST frequency* for special data. Special *nonlinear models* with *datum defects* in terms of *Taylor polynomials* and the *generalized Newton iterations* are introduced.

The Chap. 2 is based on the special *first problem*, the *bias problem* expressed by the Equivalence Theorem of the adjusted minimum norm solution (G_x-MINOS) and linear uniformly minimum biased estimator (S-LUMBE).

The *second problem* of algebraic regression, namely solving an *inconsistent system of linear observational equations* as an *overdetermined* system of linear equations is extensively treated in Chap. 3. We start with a front example for the *Least Squares Solution* (LESS). Indeed we discuss alternative solutions depending on the *metric of the observation space* like Second Order Design, the *Taylor-Karman* structure, an *optimal choice of the weight matrix* and the *Fuzzy sets*.

By an eiqenvalue decomposition of G_y-LESS, we introduce "*canonical LESS*". Our case studies range from partial redundancies, latent conditions, the theory of high leverage points versus breaking points, direct and inverse Grassman coordinates and PLÜCKER coordinates. We conclude with historical notes on *C.F. Gauss*, *A.M. Legendre* and generalizations.

In another extensive review, we concentrate on the second problem of probabilistic regression, namely the *special Gauss–Markov model without datum defect of* Chap. 5. We set up the *best, linear, uniformly unbiased estimator* (BLUUE) for moments of the *first order* and the *best, quadratically uniformly unbiased estimator for the central moments of the second order* (BIQUUE). We depart again from a detailed example BLUUE and BIQUUE. We set up Σ_y-BLUUE and note the *Equivalence Theorem* of G_y-LESS *and* Σ_y-BLUUE. For BIQUUE, we study the *block partitioning of the dispersion matrix*, the invariant quadratic estimation of types IQE, IQUUE, HIQUUE versus HIQE according to *F.R. Helmert*, variance-component estimation, simultaneous estimation of the first moments and the second central moments, the so called E-D correspondence, *inhomogeneous multilinear estimation, and BAYES design of moments estimation*.

The *third problem* of *algebraic regression* of Chap. 5 is defined as the problem to solve *inconsistent systems of linear observation equations with datum defect* as an overdetermined and indetermined system of *linear equations*. In the introduction, we begin with the front page example, subject to the minimum norm- least squares solution of the front page example, namely by the technique of *additive rank* partitioning as well as *multiplicative rank partitioning*. In more detail, we present MINOS in the second section including the weights of the MINOS as well as LESS. Especially, we interpret G_y, G_x-MINOS in terms of the *generalized inverse*. By *eigenvalue decomposition*, we find the solution to G_y, G_x-MINOS. A special section is devoted to *total least squares* in terms of α-*weighted hybrid approximate solution* (α-HAPS) within *Tykhonov-Phillips regularization*.

The *third problem of probabilistic regression* as a *special Gauss–Markov* model with *datum problem* is treated in Chap. 6. We set up *best linear minimum unbiased estimators* of type *BLUMBE* and best linear estimators of type *hom BLE, hom S-BLE and hom-α-BLE* depending on the weight assumption for the *bias*. For example, we introduce *continuous networks* of first and second derivatives. Finally, we discuss the limit process of discrete network into continuum.

The *overdetermined system of nonlinear equations on curved manifolds in* Chap. 7 is presented for inconsistent system of directional observation equations, namely by minimal *geodesic distance* (MINGEODISC). A special example is the directional observation from circular normal distribution of type *von MISES-FISHER* with a note of *angular metric*.

Chapter 8 is relating to the fourth probabilistic regression as a setup of type BLIP and VIP for the *central moments of first order*. We begin the definition of a *random effect model* according to our *magic triangle*. Three examples relate to

(i) *Nonlinear error propagation* with random effect components
(ii) *Nonlinear vector-valued error propagation* with random effect in *Geoinformatics*

(iii) *Nonlinear vector-valued error propagation* for *distance measurements* including *HESSIANS*.

The *fifth problem* of *algebraic regression* in Chap. 9 is defined as solving *conditional equations of type homogeneous and inhomogeneous*.

Alternatively, the *fifth problem* of *probabilistic regression in* Chap. 10 is treated as *inhomogeneous general linear GAUSS–MARKOV model including fixed and random effects*. In an explicit representation of errors in the *general GAUSS–MARKOV model with mixed effects*, we present an example, "*collocation*". Our comments relate to the *KOLMOGOROV–WIENER prediction* and more *up-to-date* literature.

The *sixth problem of probabilistic regression* in Chap. 11 is defined as a special random effect model called "*error-variables*". Here, we assume with respect to the *second order statistics* the *first order design matrix to be random*. Another name for this is the *Total Least Squares* as an algebraic analogue. At first, we relate the *Total Least Squares* to the nonlinear system "error-in-variables". Secondly, we introduce the models SIMEX and SYMEX from the approach of *Carroll–Cook–Stefanski–Polzehl–Zwanzig* depending on the variance-covariance matrix of the random effects.

As *special problem of nonlinearity*, we treat the popular *3d-datum transformation* and the *PROCRUSTES Algorithm in* Chap. 12.

As the *sixth problem* of *generalized algebraic regression*, we introduce finally the *GAUSS–HELMERT problem as a system of conditional equations with unknowns*. One part is the variance-covariance estimation $D\{y\}$, second the variance-covariance model of conditional equations, and finally the complete conditional equations with unknown in the third model $\mathbf{Ax} + \mathbf{By} = \mathbf{C}$. As a result of Chap. 13, we present the *block-structure of the dispersion matrix*.

As special problems of algebraic regression *as well as* stochastic estimation, we treat in Chap. 14 the *multivariate GAUSS–MARKOV model*, especially the *n-way classification model* and dynamical systems.

We conclude with *algebraic solution of systems of equations of type* linear, bilinear, quadratic and so on referring to *combinatoric subsets*, the *GROEBNER basis method*, and the *Multipolynomial resultants* in Chap. 15.

A very important part of our contribution is included in the appendices. Appendix A is an introduction to tensor algebra, linear algebra, matrix algebra *as well as* multilinear algebra. Topics are multilinear functions and their decomposition, matrix algebra and the Hodge star operator including self duality. But the major parts in *linear algebra* are "Ass", "Uni", and "Comm", Rings, spaces, division algebra, Lie algebra, *de Witt algebra*, composition algebra, generalized inverse, special matrix like *Helmert, Hankel, Vandemonte*, scalar measures of matrices, complex algebra, quaternion algebra, octonian algebra, and *Clifford algebra*.

A short introduction is given on *sampling distribution* and their uses in *confidence intervals and confidence region* in *Appendix B*. Sampling distribution for the *GAUSS–LAPLACE normal distribution* and its generalization to the *multidimensional GAUSS–LAPLACE normal distribution* as well as the distribution of the *sample mean* and *variance-covariance matrix* including the *correlation coefficients* are reviewed.

An introduction into statistical notions, random events and stochastic processes in *Appendix C* range from the *moment representation* of a probability, the *GAUSS–LAPLACE and other* normal distributions, to *error propagation*, scalar-, vector- and tensor valued stochastic processes of one- and multiparameter systems. Simple examples of one parameter include (i) cosine oscillation with random amplitudes and phases (ii) superposition of two uncorrelated random functions (iii) pulse modulation, (iv) random sequences with constant and moving averages of type ARMA (r,s), (v) *WIENER processes*, (vi) special analysis of the parameter *stationary* and *non-stationary stochastic processes* with discrete and continuous spectrum white and coloured noise with band limitation, (vii) *multiparameter systems homogeneous and isotropic* multipoint systems are given. Examples are (i) *two-dimensional EUCLIDEAN networks*, (ii) criterion matrices, (iii) *space gravity spectroscopy based on the TAYLOR-KARMAN structures*, (iv) *nonlinear prediction* and (v) nonlocal time series analysis.

Appendix D is devoted to GROEBNER basis algebra, the BUCHBERGER Algorithm, and C.F. GAUSS combinatorial formulation.

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