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Design of a high school science-fair electro-mechanical robot

2.1 THE ROBOT-KICKER SCIENCE FAIR PROJECT

A student's project for the high school science fair is to design, build, and operate a moving electro-mechanical robot that will be an active "player" in a local robot soccer tournament. His primary goal is to design the device so that it can trap a close-by soccer ball, and then rapidly kick it past a defending goalie into the net of the defending team. His physics teacher suggests that as a first step he should consider mathematically modeling the performance of a light-weight kicking device powered by a rapidly-acting linear solenoid actuator. The initial goal is to determine the basic design dimensions and the required kicking speed/force and stored magnetic energy required for the robot kicker. His teacher recommends that he use simple physics-based Back-of-the-Envelope methods to calculate the dependence of the kicked-soccer-ball speed on such design parameters as the mass and cross-sectional area of the solenoid plunger, and the electric current required to operate the solenoid actuator. His advisor points out that the analysis needs to determine the height above the ground where the "solenoid toe" of the kicker should strike the soccer ball in order to have the best chance of scoring a goal.

Sections 2.2.1 and 2.2.2 present the basic BotE mathematical analysis for the robot kicking device, along with several plots of the derived solutions for a range of design parameters.

2.2 BACK-OF-THE-ENVELOPE MODEL AND ANALYSIS FOR A SOLENOID KICKING DEVICE

To initiate the modeling and analysis effort we (as observers representing the student) are lead to ask the following question: Can physics-based Back-of-the-Envelope modeling provide a simple credible estimation of: (a) the required initial

ball velocity after the impact kick, (b) the total kinetic energy required for the ball, taking into account both its linear and rolling motion, (c) the dependence of the speed of the kicked ball on such important system parameters as the electric current supplied to the solenoid actuator and the mass of the steel solenoid plunger, and (d) the vertical height on the soccer ball where the “solenoid toe” should impulsively strike in order to have the best chance of scoring a goal? As we will demonstrate, the answer to each of these questions is: yes!

To calculate such detailed performance measures, one would require as input, to the baseline mathematical model, certain quantitative measures that uniquely characterize the soccer ball and the basic characteristics of a linear-solenoid actuator. Specifically, the analysis that follows shows the need for the following inputs: (1) the mass (m) and radius (R) of the soccer ball, (2) the initial solenoid plunger position (or gap) and the mass and diameter of the plunger within the solenoid, (3) the nominal maximum work output required by the actuator, as well as the “force vs stroke” characteristics of the mathematical model emulating the performance of a similar industrial solenoid actuator, and (4) the required time scale for the actuator to complete a single kick and the duty cycle for its operation (in other words, the number of kicks per second required for it to be competitive). In addition, we require a basic sketch of the solenoid actuator device required to initiate the BotE analysis.

In Section 2.2.1 we define the basic dimensions of the soccer field, the size and mass of the soccer ball, and then determine the “natural roll” velocity along the ground required for the ball to successfully get past the goalie. In fact, this “goal-scoring” velocity will be a key design requirement for the solenoid-kicker system.

2.2.1 Defining basic dimensions and required soccer ball velocity

We first define the dimensions of the robot playing field (e.g. 12 m wide by 18 m long for a “Robocup” middle-size robot league), the ball’s diameter (0.111 m) and the mass of a competition-sized soccer ball (0.45 kg) [1]. The soccer ball is modeled as a thin-walled spherical shell.

2.2.1.1 Estimating the required soccer ball velocity

Assume that the ball is positioned in front of the robot at a distance $x = 3$ m from the goal line with a goal net width of 2 m. If the robot goalie is positioned halfway between the edges of the net and is able to move laterally at a speed of 2.0 m/s, it will take him about 0.5 s to reach the edge of the 2 m wide net and block our robot’s kick to the edge of the net. If the ball is kicked from the penalty spot; $x = 3$ m and $y = 1$ m (measured from the far edge of the net), as shown in Figure 2.1, then the ball must travel a distance of 3.16 m.

The required average ball velocity must be $\geq 3.16 \text{ m}/(0.5 \text{ s})$. Therefore we set our minimum velocity requirement as

$$V_{\text{required}} = 6.32 \text{ m/s (or } 22.7 \text{ km/h)}$$

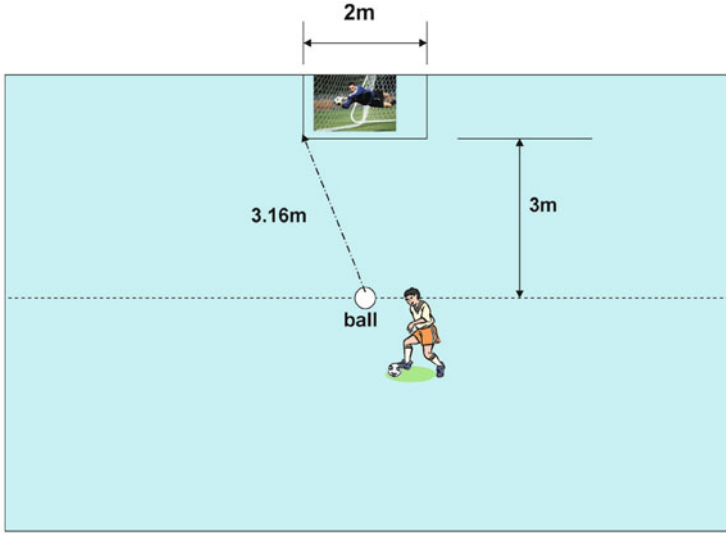


Figure 2.1. Schematic of model soccer field with player attempting to score a goal. Goal net is 2 m in width.

We assume, following the initial actuator kick, that V_{required} is the velocity after the ball of radius R achieves a “natural roll” for which the horizontal velocity is related to the angular rate of rotation of the ball, ω , by the expression, $V_{\text{natural-roll}} = \omega R$.

2.2.2 Setting up a BotE model for the solenoid kicking soccer ball problem

When the ball is kicked, the imparted momentum produces a ball velocity at ground level with the direction of the velocity vector set by the direction of the applied force vector generated by the kicker’s toe. Only horizontal impacts along the vertical plane of symmetry of the ball are considered. There is no side-spin or lift of the ball in our simple scenario. Think of this problem as a horizontal pool cue (or stick) hitting a cue-ball along the ball’s vertical plane of symmetry. If the soccer ball is kicked at a height above or below the centerline of the ball, rotation or “spin” of the ball will accompany the forward motion (See the analysis below). A skidding ball, with a low spin rate, generates a frictional force at the ground contact point that acts to increase the spin until the “natural roll” (or zero friction) condition that we noted in Section 2.2.1 is obtained. This ground-contact-induced frictional force also slows down the forward motion of the ball. The solution to the post-kick spin and roll problem is presented in the following section.

2.2.2.1 Model for dynamics of a rolling ball struck by a thin plunger

As depicted in Figure 2.2, a hollow thin-shell spherical soccer ball at rest is struck by a thin “red” plunger moving at speed v_0 at a height h above the centerline of the ball,

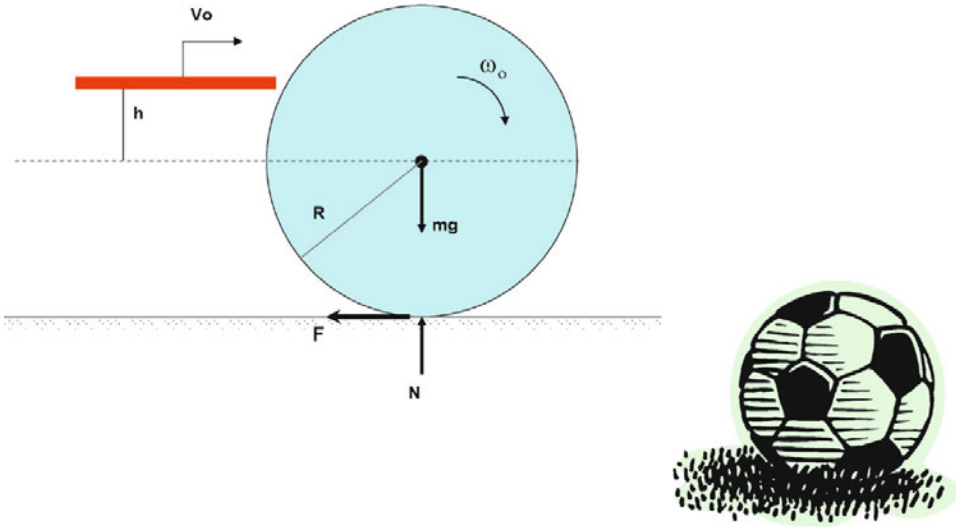


Figure 2.2. Forces acting on a linearly translating spherical ball with velocity V_b that is spinning at angular velocity ω_0 , following impact by a thin red plunger moving initially at velocity v_0 . F represents the backwards-pointing frictional force (positive in the negative x direction). N is the normal force at the point of contact needed to balance the weight of the ball.

imparting a horizontal impulse, I , due to the collision force X caused by the plunger acting for a short period of time on the ball.

This initial modeling problem requires us to solve the governing equations of motion to determine the plunger velocity required for the ball to achieve a particular final natural roll velocity V_{NR} .

We first develop a closed-form solution for the required plunger velocity as a function of both normalized impact height (h/R), and the ratio of the mass of the ball to the mass of the plunger (m_b/m_p). The conservation of momentum and energy equations are then used to calculate the specific plunger velocity, v_0 , that yields a required natural-roll velocity of 6.32 m/s (This is the velocity required to kick the ball past the goalie in our simulated *Robocup* match; as calculated in Section 2.2.1.1).

In solving for the initial spin on the ball immediately after being struck by the plunger, we note that the law of conservation of angular momentum is *not* valid here since there is a net impulsive torque applied to the ball by the plunger. This impulse abruptly changes the ball's angular momentum. We ignore the effects of sliding friction during the short duration plunger-induced impulse. We follow the derivation presented in the solution to a comparable MIT physics class problem entitled "Rotation and Translation" [2].

The impact imparts a horizontal momentum, or Impulse, arising from the collision force X acting on the soccer ball that accelerates the center of mass m_b . So now let's define the velocity of the ball after impact $\equiv V_b$. The mass of the ball is

$\equiv m_b$. The resulting expression for the applied impulse is

$$\text{Impulse} = \int X \, dt = m_b V_b \quad (2.1)$$

Note that in Equation 2.1 we ignore the effects of ground-contact friction during the short impulse time.

The short-duration collision force X , applied to the ball by the plunger at the height h above the center-line also exerts an *external torque*, $\tau = X \cdot h$, on the ball. The corresponding angular impulse A imparted to the ball is

$$A = \int \tau \, dt = h \cdot (\text{Impulse}) = h m_b V_b \quad (2.2)$$

Newton's second law of motion for a rigid body rotating about a fixed axis subject to a net torque τ (or sum of torques) is given by

$$\tau = I_{\text{cm}} \frac{d\omega}{dt} \quad (2.3)$$

where I_{cm} is the moment of inertia of the body (our spherical ball) about its center of mass and ω = the ball's angular velocity (radians/s).

For the thin hollow sphere modeled here, the moment of inertia is given by

$$I_{\text{cm}} = \frac{2}{3} m_b R^2$$

Integrating Equation 2.3 over a short time interval Δt , yields the *angular form* of the impulse relationship given by Equation 2.1

$$A = \int \tau \, dt = I_{\text{cm}}(\Delta\omega) = I_{\text{cm}}(\omega_{\text{final}} - \omega_{\text{initial}}) \quad (2.4)$$

We assume that there is no initial spin on the soccer ball before it is hit (i.e. $\omega_{\text{initial}} = 0$). We also define the final angular velocity after impact to be $\omega_{\text{final}} \equiv \omega_b$. Substituting the expression for the torque impulse, Equation 2.2, into Equation 2.4 yields

$$h m_b V_b = I_{\text{cm}} \omega_b \quad (2.5)$$

Solving Equation 2.5 for the final angular velocity after impact

$$\omega_b = \frac{h m_b V_b}{I_{\text{cm}}} = \frac{h m_b V_b}{2/3 m_b R^2} = \frac{3}{2} \left(\frac{h}{R} \right) \left(\frac{V_b}{R} \right) \quad (2.6)$$

According to Equation 2.6, if the plunger strikes the ball at its centerline ($h = 0$), there is no imparted spin after impact; i.e. $\omega_b = 0$. As a result the ball initially skids or slides along the playing surface.

The torque produced by the frictional force F acting at a distance R relative to the center of the ball, as shown in [Figure 2.2](#), immediately produces a positive angular acceleration that leads to an increase in the ball's angular velocity (for $h > 0$). The subsequent steady state or natural roll value of ω is proportional to the final natural roll velocity, i.e. $\omega_{\text{steady state}} = V_{\text{Natural Roll}}/R$.

From Equation 2.6, if the plunger rod hits the ball at a relative height $h/R = 2/3$ then $\omega_b = \left(\frac{V_b}{R}\right)$, which is the equation for the natural roll of a ball moving at a linear velocity $= V_b$. As we will see for this natural roll case, the angular velocity remains constant for all times after impact, i.e. $\omega(t) = \text{constant} = \omega_b$, as does the translational velocity $V = \text{constant} = V_b$.

Let's consider the case where the ball is hit at some arbitrary height h . We ask: What is the velocity of the ball after impact, V_b , given the velocity of the piston before impact?

To solve for V_b we first assume that there is no energy lost during the impact (an assumption that is not strictly valid for the soccer ball materials considered here). This allows us to use both the equation for the conservation of kinetic energy and the equation for the conservation of linear momentum. These two equations express the fact that the linear momentum and the linear and rotational energies of the plunger and ball system *after* impact are equal to the momentum and energy of the plunger and ball *before* impact.

The following expressions are used to calculate the ball velocity, V_b after impact, as well as the velocity of the plunger after impact V_p [3]. Note that the ball has zero initial velocity and the plunger, prior to impact, is moving at velocity v_0 (Figure 2.2).

Conservation of momentum (in the x direction)

$$\left. \begin{aligned} \text{initial momentum} &= \text{final momentum} \\ m_p v_0 &= m_p V_p + m_b V_b \end{aligned} \right\} \quad (2.7)$$

Conservation of energy (kinetic energies of linear and rotational motion)

$$\left. \begin{aligned} \text{initial energy} &= \text{final energy} \\ \frac{1}{2} m_p (v_0)^2 &= \frac{1}{2} m_p (V_p)^2 + \frac{1}{2} m_b (V_b)^2 + \frac{1}{2} I_{cm} (\omega_b)^2 \end{aligned} \right\} \quad (2.8)$$

where $I_{cm} = \frac{2}{3} m_b R^2$.

We first solve Equation 2.7 for V_p and substitute this expression into Equation 2.8. The expression derived for the rotational velocity of the ball after impact, namely $\omega_b = \frac{3}{2} \left(\frac{h}{R}\right) \left(\frac{V_b}{R}\right)$ from Equation 2.6, is then substituted into Equation 2.8. After a little algebra we obtain the following equation for the ball velocity after impact V_b

$$V_b = \frac{2v_0}{\left[1 + \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R}\right)^2\right]} \quad (2.9)$$

Substituting Equation 2.9 into Equation 2.7 yields Equation 2.10, the companion equation for the piston velocity, V_p after impact [3], is

$$V_p = v_0 \frac{\left[1 - \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R}\right)^2\right]}{\left[1 + \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R}\right)^2\right]} \quad (2.10)$$

After the initial skid and speedup of the ball's rotational velocity due to the frictional torque at the contact point, the ball subsequently develops a "natural roll" where $\omega_{\text{steady state}} = V_{\text{natural roll}}/R$. To determine the actual magnitude of $V_{\text{natural roll}}$, we can use an equation for the conservation of angular momentum, measured relative to the contact point S between the ball and the ground.

Conservation of angular momentum

In freshman physics we learned the following definition of angular momentum. The angular momentum vector $\vec{L}_s$ associated with a particle of mass m translating at velocity $\vec{v}$ relative to a given reference point S is defined as

$$\vec{L}_s = \vec{r} \times m\vec{v} \quad (2.11)$$

where $\vec{r}$ is the position vector from S to the particle. The symbol " $\times$ " denotes the cross product.

If a body made up of a collection of particles both translates and spins, then the total angular momentum of that body is simply the sum of the translational motion of its center of mass with respect to the point S ($\vec{L}_{\text{trans}} = \vec{r}_{S,\text{cm}} \times m\vec{v}_{\text{cm}}$) and the spin of the body about its center of mass ($\vec{L}_{\text{spin}} = I_{\text{cm}}\vec{\omega}$).

For the kicked ball problem shown in Figure 2.1, our soccer ball (of radius R) moves in the x direction with scalar velocity V_b and spins with angular velocity ω_b just after impact by the plunger. If there is no net torque on a body, then the total angular momentum is conserved. After impact, the condition of zero torque will apply if we select the point S at which the ball makes contact with the ground as our reference location for evaluating the total angular momentum. There is no moment caused by the frictional force F since the moment arm is zero when it is measured from S to the contact point where the sliding frictional force is applied. The resulting conservation of angular momentum relationship for our ball, valid at any later time after impact, is then given by the following expression, Equation 2.12 [2]

$$\left. \begin{aligned} \text{angular momentum just after impact} &= \text{angular momentum at a later time} \\ m_b R V_b + I_{\text{cm}} \omega_b &= m_b R V + I_{\text{cm}} \omega \end{aligned} \right\} \quad (2.12)$$

Solving Equation 2.12 for the "later" velocity V , using $I_{\text{cm}} = \frac{2}{5} m_b R^2$, yields

$$V = V_b - \frac{2}{3} R(\omega - \omega_b) \quad (2.13)$$

When the ball achieves a natural roll, $V = V_{\text{NR}}$, and $\omega = V_{\text{NR}}/R$. Substituting these natural roll expressions into Equation 2.13 we obtain

$$V_{\text{NR}} = \frac{3}{5} V_b + \frac{2}{5} (\omega_b R) \quad (2.14)$$

A similar expression, but with different constants for a solid (as opposed to a hollow) sphere was obtained by Shepard [4].

Using the derived expression for ω_b from Equation 2.6 for a ball struck at a height h above its centerline, we obtain

$$V_{\text{NR}} = \frac{3}{5} V_b \left(1 + \frac{h}{R} \right) \quad (2.15)$$

Note from Equation 2.15 that if $h = 0$, the subsequent natural roll velocity is reduced to 60% of the ball's velocity after the plunger impact, i.e. $V_{\text{NR}} = \frac{3}{5} V_b$. However, if $h/R = 2/3$, then $V_{\text{NR}} = V_b$, which is a significant velocity increase over the $h = 0$ result. Clearly a faster natural roll velocity is obtained if the plunger hits higher up on the ball. If we consider the vertical distance measured from the ground plane, the natural roll height is $(5/6)d$, where d = diameter of the ball.

Substituting Equation 2.9 for V_b into Equation 2.15, gives the following equation for the natural roll velocity as a function of the initial plunger velocity, v_0 , and the height ratio h/R for a fixed ratio of ball to plunger mass. (It should be noted that Shepard's problem 3.11 is the corresponding solution for a solid sphere [3].)

$$V_{\text{NR}} = \frac{6}{5} \left\{ \frac{\left(1 + \frac{h}{R} \right)}{\left(1 + \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R} \right)^2 \right)} \right\} v_0 \quad (2.16)$$

This equation for the ratio of natural roll velocity to plunger velocity, V_{NR}/v_0 , is plotted vs h/R in Figure 2.3 for several values of m_b/m_p .

The plunger strike position that maximizes V_{NR}/v_0 can be determined either from Figure 2.3 or by differentiating Equation 2.16 and then setting the derivative to zero. This value of h/R also is the one that minimizes plunger velocity for a given value of the natural roll velocity. As we will see later on, this minimum v_0 solution reduces the amount of solenoid energy (and current) required by our activated solenoid kicker.

The reciprocal of the ordinate in Figure 2.3, v_0/V_{NR} , is plotted in Figure 2.4 for positive values of h/R . We use this plot to readily estimate v_0 for a given required natural roll velocity.

Let's first assume, as an example, that the mass ratio for the soccer ball and plunger $m_b/m_p = 0.75$. We can see from Figure 2.4 that for this ratio the required plunger velocity for a given V_{NR} is a minimum at $h/R \approx 0.50$. At this value of h/R , the velocity ratio $v_0/V_{\text{NR}} \approx 1.18$, i.e. the required "plunger" velocity must be about

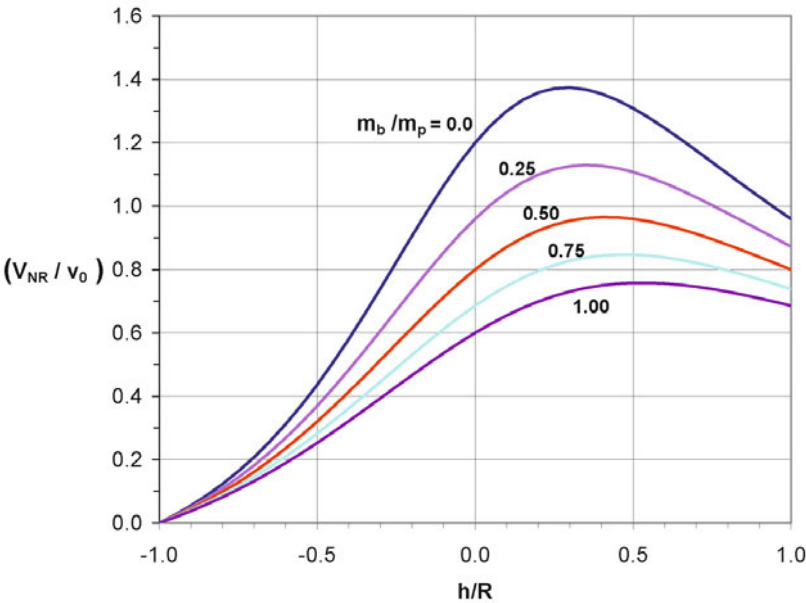


Figure 2.3. The ratio of natural-roll soccer ball velocity to initial plunger velocity, V_{NR}/v_0 as a function of the vertical height to ball radius ratio, h/R , for selected ball/plunger mass ratios, m_b/m_p .

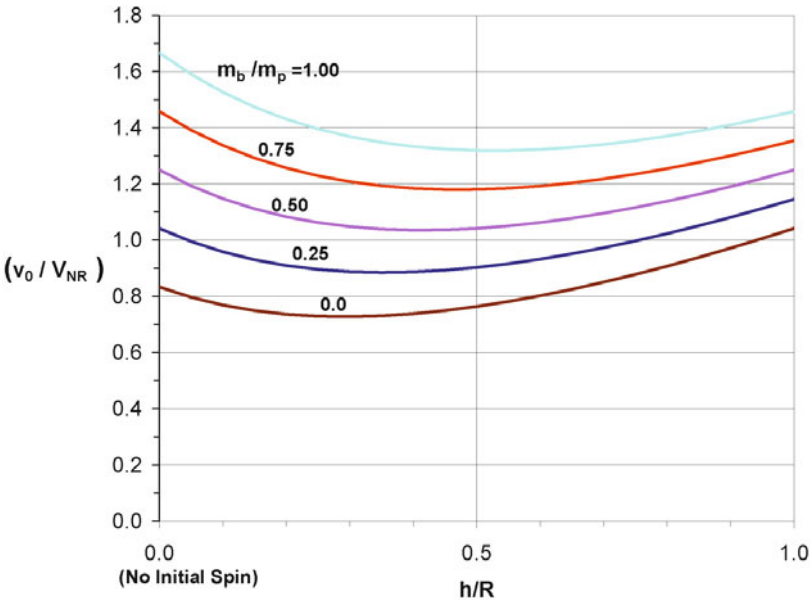


Figure 2.4. The ratio of initial plunger velocity to natural-roll soccer ball velocity v_0/V_{NR} as a function of the vertical height to ball radius ratio, h/R , for selected ball/plunger mass ratios, m_b/m_p .

20% greater than the natural soccer ball roll velocity obtained by our estimate. If the required natural roll velocity to score a goal is $V_{NR} = 6.32 \text{ m/s}$ (as calculated in Section 2.2.1.1) then the corresponding plunger velocity must be

$$v_0 = 1.18 \times 6.32 \text{ m/s} = 7.46 \text{ m/s}$$

Rounding this to the nearest 0.1 m/s, we set the *required* maximum plunger velocity to be delivered by our linear-actuator solenoid kicker design to be

$$v_{0\text{required}} = 7.5 \text{ m/s}$$

Interestingly, the minimum plunger energy solution for an impact height $h/R \approx 0.50$ is slightly *below* the ideal impact height necessary for a “natural roll”; $h/R = 2/3$.

For our minimum velocity impact solution, the solenoid plunger must strike a standard soccer ball at a height of about 3 cm (or approximately 1 inch) above the centerline of any captured soccer ball. This strike-point determines the “best height” for positioning the solenoid actuator on the moving robot platform.

2.2.3 Model for solenoid kicker work and force

2.2.3.1 Why a solenoid kicker?

The challenge set for the student is to design a simple robot kicking device that is able to kick a soccer ball with sufficient speed to send it past a defender and into the goal. We can choose from several different mechanical or electro-mechanical drive mechanisms to power the moving kicking device: elastic springs or rubber bands, pneumatic or pressure reservoirs, electric current driven linear solenoids, or a range of electric servo-motors (e.g. sim-motors). All of these mechanisms convert some form of stored potential energy (e.g. elastic spring energy, stored pressure, or stored electro-magnetic energy) into kinetic energy. The magnitude of this kinetic energy, KE (or equivalent mechanical work), is used to calculate the effective “plunger” speed $= \sqrt{2(KE/m_p)}$ needed to initiate the subsequent kicking motion.

While a compressed spring is the simplest device, it takes a considerable amount of time to reload the spring. One typically uses an auxiliary dc motor driven by a small battery to do the reloading. The basic limitation of this device for our application is that it typically takes many seconds to reload the spring. When reviewing the performance of the motor designed several years ago for Robocup competitions, the builders found that it took of the order of 5 seconds to make such a reload [5], which they considered too long a time between kicks for a fast-moving game.

Simple commercial linear-solenoids have reload times of order 0.1 seconds and can be powered by compact dc battery supplies driving a simple circuit. They are readily available and fairly cheap. Our design challenge is to determine the solenoid size that will provide the required kicking speed, recycle time, and momentum necessary for “game” conditions.

2.2.3.2 Linear-solenoid fundamentals

A solenoid linear-actuator is a long solenoid coil wound in a helical pattern, with a steel (or other ferromagnetic metal) plunger core housed within the winding. The plunger is pulled into the center of the coil when energized by an electric current. The linear solenoid actuator has many applications including: locks, doorbells, switches, and relays. When the current is passed through the coil a magnetic field is set up, with the magnetic field inside the coil much stronger than that outside the coil. When the steel plunger is placed near or within one end of the energized coil, the magnetic field causes the cylindrical plunger to become a temporary magnet of opposite polarity to that of the coil. As a result the steel plunger is virtually sucked into the center of the coil by this magnetic force and travels freely along the centerline of the coil, towards the ferromagnetic back stop. For steel or other ferromagnetic plungers the direction of the intake force on the plunger is *independent* of the direction of the current in the coil. Most solenoids include a fixed cylindrical stop that extends part way into the center bore, which improves performance. After being halted the plunger is returned to its initial gap position, usually by a modest spring, when the current is cut off. A sketch of a generic solenoid and plunger is shown in [Figure 2.5](#).

A drawing of a typical solenoid and plunger (with a “push pin” attached, as needed for a robot kicker) is shown in both [Figure 2.6](#) and [Figure 2.7](#). We might add

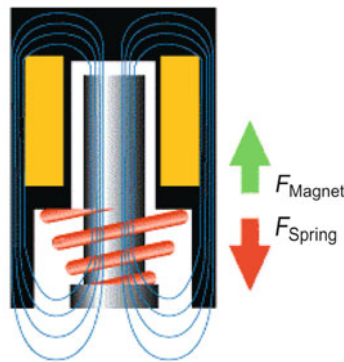


Figure 2.5. Solenoid and plunger schematic with return spring [6].

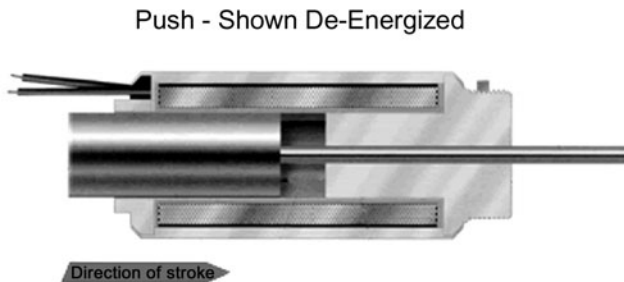


Figure 2.6. Push solenoid schematic. The push rod or plunger moves to the right when the solenoid is energized. [7, 8].

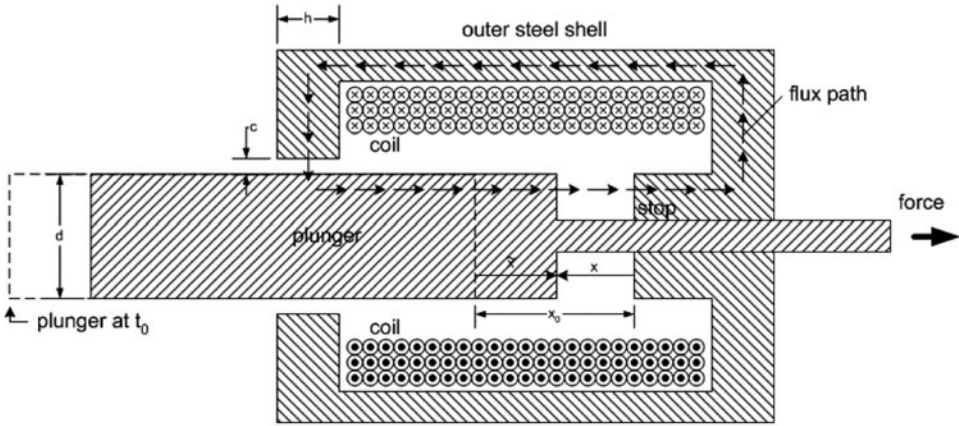


Figure 2.7. Push-pin solenoid schematic. $x(t)$ is the time-varying air gap length between the plunger face and the stop; x_0 = initial air gap at $t = 0$. Note the small annular gap, of length c , between the sliding plunger and the outer steel shell. The plunger diameter = d . $\bar{x}$ is the distance moved by the plunger (positive to the right). [Illustration courtesy of J. Jacobs]

a small disk perpendicular to the end of the push pin to assist in the transfer of the plunger force to the soccer ball—in effect, it is the kicker’s “boot”.

2.2.3.3 Linear-solenoid theory and calculation of magnetic field energy

In this section we derive an expression for the energy or work, U_B , stored in the magnetic field of a solenoid. The simple mathematical model is based on the standard freshman physics lectures that introduce the basic laws governing magnetic fields, namely Ampere’s law and Faraday’s law of induction.

Using the derived equation for the available magnetic energy produced by the solenoid, we then assume that this energy is ideally converted into mechanical work associated with the attractive force F . This force moves a steel plunger of mass m_p inside our ideal solenoid linear actuator over a distance Δx according to

$$U_B = U_{\text{mechanical}} = - \int_0^{\Delta x} F(x) dx \quad (2.17)$$

where in the absence of friction, $U_{\text{mechanical}} = \frac{1}{2} m_p V_p^2$.

Note that the minus sign in Equation 2.17 indicates that the direction of the solenoid force is opposite to the direction of increasing gap size, x , as depicted in Figure 2.7.

From Equation 2.17, we see that the force on the plunger, F , is simply the first derivative of $U_B(x)$ with respect to the variable gap distance x

$$F(x) = - \frac{dU_B}{dx} \quad (2.18)$$

The mechanical work is converted directly into a corresponding amount of kinetic

energy for the accelerating steel plunger, $\frac{1}{2}m_p V_p^2$. The final push-rod or plunger velocity, V_p , is thus calculated from the released stored magnetic energy, U_B .

2.2.3.3.1 The magnetic field of a solenoid

Ampere's law states that the line integral of the magnetic field vector $\mathbf{B}$ around a closed loop of incremental vector length $d\vec{\ell}$ is equal to the total current enclosed within, or

$$\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 NI \quad (2.19)$$

where N = number of turns of the solenoid coil

I = electric current carried in the coil (the current that pierces the closed loop). Unit is ampere or A.

μ_0 = permeability of free space = $4\pi \times 10^{-7}$ henry/m
 = $4\pi \times 10^{-7}$ newton/(ampere)²

The units for $\mathbf{B}$ are tesla or newton/ampere-m.

Assuming a uniform magnetic field inside a solenoid with a centerline air gap length ℓ we obtain from Equation 2.19 the well known solution for a very long solenoid,

$$B = \mu_0 NI / \ell \quad (2.20)$$

2.2.3.3.2 The magnetic energy of a solenoid actuator

Using classical electromagnetic theory one can show that the magnetic energy, U_B , stored in a volume of space occupied by a magnetic field (in a vacuum or in a non-magnetic substance like air) is proportional to the square of the magnetic field magnitude B integrated over the free space volume [9]

$$U_B = \frac{1}{2\mu_0} \int B^2 d(\text{Vol}) = \frac{1}{2\mu_0} \int_0^{\ell_{\max}} B^2 A_p dl \quad (2.21)$$

For our cylindrical solenoid, the incremental volume, $d(\text{Vol})$, is equal to the plunger face cross-sectional area A_p multiplied by the incremental gap length $d\ell$. Integrating Equation 2.21, with B given by Equation 2.20, over a cylindrical volume with gap length varying between 0 and $\ell_{\max}$, yields

$$U_B = \frac{\mu_0 (NI)^2}{2} (A_p) \int_0^{\ell_{\max}} \ell^{-2} d\ell = -\frac{\mu_0 (NI)^2}{2} (A_p) \left[\frac{1}{\ell} \right]_0^{\ell_{\max}} \quad (2.22)$$

Note that if we evaluate Equation 2.22 at the lower limit, where the gap distance $\ell \rightarrow 0$, the total stored energy becomes unbounded, i.e. $U_B \rightarrow \infty$.

In real linear-actuator systems, there is always some small *additional* air gap besides the primary one along the main axis of the solenoid between the plunger and backstop. One such *additional* gap is the annular air gap between the outer steel wall (or pole) of the solenoid and the sliding plunger. This second gap in Figure 2.7 with width “ c ” makes it possible for the plunger to smoothly move in and out of the solenoid. This gap adds an additional amount of *reluctance* that reduces the level of

the magnetic field B carried by the plunger and solenoid body. You can think of reluctance as a kind of “resistance” in an ideal magnetic circuit, e.g. the flux path in [Figure 2.7](#) is analogous to the corresponding path for the current in a standard battery-driven electrical circuit. The effect of this added *reluctance* is particularly important when the centerline solenoid gap is small. Note that in this treatment the equations governing the magnetic field in a general magnetic circuit with a number of air gaps are not presented, nor utilized, thereby enabling us to use simplified equations in the solenoid kicker problem.

Following our BotE approach, we approximate this additional gap effect by adding a constant length “ a ” to the basic plunger/stop gap length ℓ in Equation 2.22. In an ideal actuator, “ a ” is proportional to the width of the clearance “ c ” as shown in [Figure 2.7](#). Hence, as an *approximation*, we amend Equation 2.22 for stored energy to include an additional *empirical gap length* “ a ” as follows

$$U_B \approx \frac{\mu_0(NI)^2}{2} (A_p) \int_0^{\ell_{\max}} (\ell + a)^{-2} d\ell = -\frac{\mu_0(NI)^2}{2} (A_p) \left[\frac{1}{\ell + a} \right]_0^{\ell_{\max}} \quad (2.23)$$

Evaluating Equation 2.23, noting that the previous singularity disappears for non-zero values of “ a ”, we obtain a *bounded* expression for the maximum energy stored in our solenoid actuator

$$U_{B_{\max}} = C \left[\frac{1}{a} - \frac{1}{(\ell_{\max} + a)} \right] \quad (2.24)$$

where

$$C = \frac{\mu_0(NI)^2}{2} (A_p).$$

Using an arbitrary gap length x (the coordinate shown in [Figure 2.7](#)) instead of a fixed gap $\ell_{\max}$, the energy solution can be written as the following function of x

$$U_B(x) = -C \left(\frac{1}{x + a} \right) + \frac{C}{a} \quad (2.25)$$

Combining the two fractions in Equation 2.24 produces a compact equation for maximum energy stored in a solenoid actuator with a given initial gap length $\ell_{\max}$

$$U_{B_{\max}} = \frac{C\ell_{\max}}{a(\ell_{\max} + a)} \quad (2.26)$$

2.2.3.3.3 The force on the plunger

Employing Equation 2.18, we see that the solenoid-driven force on the plunger $F(x)$ is given by the derivative with respect to x of $U_B(x)$ in Equation 2.25

$$F(x) = -\frac{dU_B}{dx} = -C \left(\frac{1}{x + a} \right)^2 \quad (2.27)$$

The sign of the force is negative because the force $F(x)$ on the plunger acts opposite to the direction of a positive increase in the gap dimension; the plunger is pulled into (not out of) the solenoid cavity. The force decreases with increasing gap

width x . The maximum force on the plunger, obtained at zero gap width, is obtained from Equation 2.27

$$F_{\max}(x=0) = -\frac{C}{a^2} \quad (2.28)$$

We observe that the magnitude of $F_{\max}$ is strongly sensitive to the numerical value of the auxiliary gap length parameter “ a ”. As “ a ” gets smaller, the absolute value of $F_{\max}$ grows as $1/a^2$. There is an approximate way to determine the empirical value for “ a ” to be used in our subsequent analysis. We find “ a ” by setting the magnitude of the stored energy, given by Equation 2.26, equal to the measured amount of work, $U_{B_{\max}}$, that is determined from plunger force measurements (as a function of gap distance) for a particular commercial linear actuator solenoid device. We select a commercial actuator device with an amount of energy sufficient to *potentially* meet the needs of our robot kicker.

For a particular solenoid, driven by a current I , the estimated work output of the actuator is found by numerically integrating the measured “force vs gap” curve for that device. A typical force vs stroke measurement curve is shown later in Figure 2.9. The required energy for our robot kicker problem is ≈ 17 joules which is equal to the kinetic energy of a 0.6 kg plunger moving at the required speed of 7.5 m/s, as calculated in Section 2.2.2.1.

2.2.3.3.4 The plunger velocity as a function of distance traveled

The kinetic energy imparted to the plunger as a function of the instantaneous gap spacing, x , can readily be calculated using the equations already derived for the energy, namely Equation 2.25. Let’s first consider the change in kinetic energy for a push-rod plunger accelerated from zero velocity (at an initial gap distance x_0) to a final zero-gap position at the solenoid back-stop, as shown in Figure 2.7. We write the following expression for the change in kinetic energy, ΔKE , based on the change in stored energy for the plunger at two gap positions (x_0 and x). ΔKE is chosen so that the kinetic energy = 0 when the plunger is at the initial position x_0 .

$$\Delta KE \equiv KE(x) = U_B(x_0) - U_B(x) \quad (2.29)$$

Using Equation 2.25 for the stored energy, we obtain

$$KE(x) = -C\left(\frac{1}{x_0 + a}\right) + C\left(\frac{1}{x + a}\right) = \frac{C(x_0 - x)}{[(x - x_0) + (x_0 + a)][(x_0 + a)]} \quad (2.30)$$

In order to more easily interpret Equation 2.30, let’s define (from Figure 2.7) a plunger-based distance coordinate, $\tilde{x}$ with its origin at the initial gap position of the plunger ($x = x_0$), that is positive in value for all plunger positions short of the solenoid backstop,

$$\tilde{x} = x_0 - x \quad (2.31)$$

The kinetic energy as a function of $\tilde{x}$ is then given simply by

$$KE(\tilde{x}) = \frac{1}{2}m_p V_p^2 = \frac{C\tilde{x}}{(b - \tilde{x})b} \quad (2.32)$$

where $b \equiv x_0 + a$.

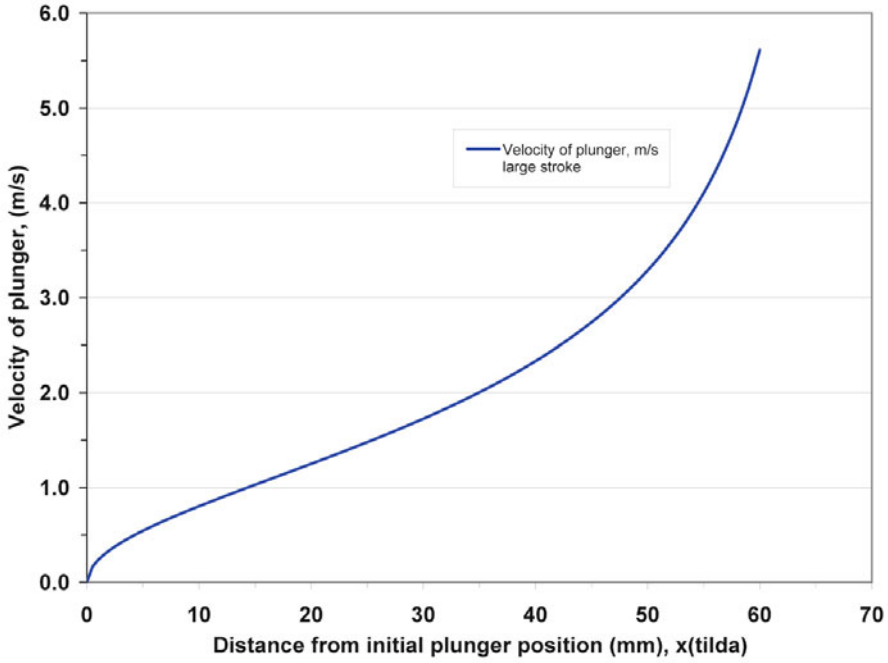


Figure 2.8. Plunger velocity (m/s) as a function of distance (mm) from the initial plunger position 60 mm from the solenoid stop. ($NI = 9,080$ ampere-turns) and plunger mass = 0.6 kg. Auxiliary gap length parameter “ a ” = 7 mm.

Solving Equation 2.32 for the corresponding plunger velocity as a function of the plunger distance $\tilde{x}$

$$V_p = \left[\frac{2C}{m_p b} \right]^{1/2} \left[\frac{\tilde{x}}{(b - \tilde{x})} \right]^{1/2} \quad (2.33)$$

where

$$C = \frac{\mu_0 (NI)^2}{2} (A_p)$$

A sample plot of this function is depicted in [Figure 2.8](#) for a set of baseline parameters.

The maximum plunger velocity, at the zero gap stop position, $\tilde{x} = x_0$ is given by the simple equation

$$V_{p_{\max}} = \left[\frac{2C}{m_p a} \right]^{1/2} \left[\frac{x_0}{(x_0 + a)} \right]^{1/2} \quad (2.34)$$

For the set of parameters used in [Figure 2.8](#), the maximum plunger velocity is 5.61 m/s, which is the maximum velocity shown at $\tilde{x} = 60$ mm. This velocity falls below the required plunger maximum velocity of 7.5 m/s required to produce a 6.3 m/s natural roll velocity for our penalty-kicked soccer ball.

As we will show, a 7.5 m/s plunger velocity can be obtained by increasing the

magnitude of NI (or ampere-turns) for the baseline solenoid in [Figure 2.8](#). Finally, we note that for large initial gap distances x_0 compared to “ a ”, i.e. $\frac{x_0}{a} \gg 1$, the maximum velocity is given by the following expression independent of x_0

$$V_{\text{pmax}} \rightarrow \left[\frac{2C}{m_p a} \right]^{1/2} \quad (2.35)$$

2.2.3.3.5 Estimated total plunger travel time

With the equation for plunger velocity as a function of plunger distance $\tilde{x}$ given by Equation 2.33, we can readily calculate the time required for the plunger to move from its initial gap position to the final zero gap position. We designate this time as t_{max} . Since the incremental travel time dt is equal to the incremental distance $d\tilde{x}$ divided by the instantaneous velocity, $V_p(\tilde{x})$, t_{max} is obtained from the following integral

$$t_{\text{max}} = \int_0^{x_0} \frac{d\tilde{x}}{V_p(\tilde{x})} \quad (2.36)$$

We leave it as an exercise for the reader to show that the approximate value of the integral, valid for $a/x_0 \ll 1$, is

$$t_{\text{max}} \approx \frac{\pi}{2} \left(\frac{m_p x_0^3}{2C} \right)^{1/2} \quad (2.37)$$

Equation 2.37 indicates that the total plunger travel time is proportional to the initial gap distance to the 3/2 power and inversely proportional to NI (ampere-turns) since $C = \frac{\mu_0 (NI)^2}{2} (A_p)$. Therefore the higher the current I , the shorter the travel time. This plunger travel time is important in our student’s soccer tournament, since it helps to determine how fast the robot device can repeat a kick, assuming that another soccer ball is available (The ball must be *trapped* just in front of the plunger).

2.2.3.3.6 Power-up time scales for an R-L circuit

We have not included in our repeat-time estimate: the time for the return spring on the plunger to reset the plunger gap, or the time for a resistance-inductance (R-L) circuit to bring the solenoid to full current or power once the “kick” button is hit. The power-up time scale for this circuit is of order $5L/R$ where the solenoid

inductance $L \approx \frac{\mu_0 N^2 A_p}{\ell_{\text{solenoid}}} \approx 0.01$ henry for the solenoid considered below. This

formula for the inductance L of a long solenoid is derived in Section 33-2 of the classic physics textbook by Halliday and Resnick [9]. The calculated R-L power-up time scale ($5L/R$) is of order 25 milliseconds for a coil with a 2 ohm resistance. This interval can be reduced by decreasing the number of turns N in the solenoid design, because that reduces the inductance L . Equation 2.34 shows that for a fixed plunger

velocity requirement, NI must be set at a particular level. This constant level implies that the current I must be increased if N is decreased. The increased current generates greater I^2R heating losses which have to be taken into account when determining the number of kicks in a game.

2.2.3.3.7 Calculated levels of maximum force, energy, and velocity

The following parameters are typical of a large push-type tubular solenoid such as produced by Magnetic Sensor Systems [10] with a 3.0 inch diameter and a 4.13 inch outer cylindrical shell length. The output work of this particular device, at its *nominal* NI settings, is lower, but it is still within a factor of two of our maximum energy requirement of about 17 joules.

- Maximum plunger stroke, $x_0 = 60 \text{ mm} = 0.06 \text{ m}$
- Ampere turns = $NI = 9,080$ for a 10% duty cycle for our reference case of a 3×4.13 inch push-type solenoid actuator
- Plunger diameter = 1.68 inches = 0.0426 m
- Plunger Area, $A_p = 1.425 \times 10^{-3} \text{ m}^2$
- Plunger mass, $m_p = 0.6 \text{ kg}$
- $\mu_0 = 4\pi \times 10^{-7} = 1.257 \times 10^{-6} \text{ newton}/(\text{ampere})^2$
- From a numerical integration of the measured force vs stroke or from the displacement curve of [Figure 2.9](#), the estimated measured work

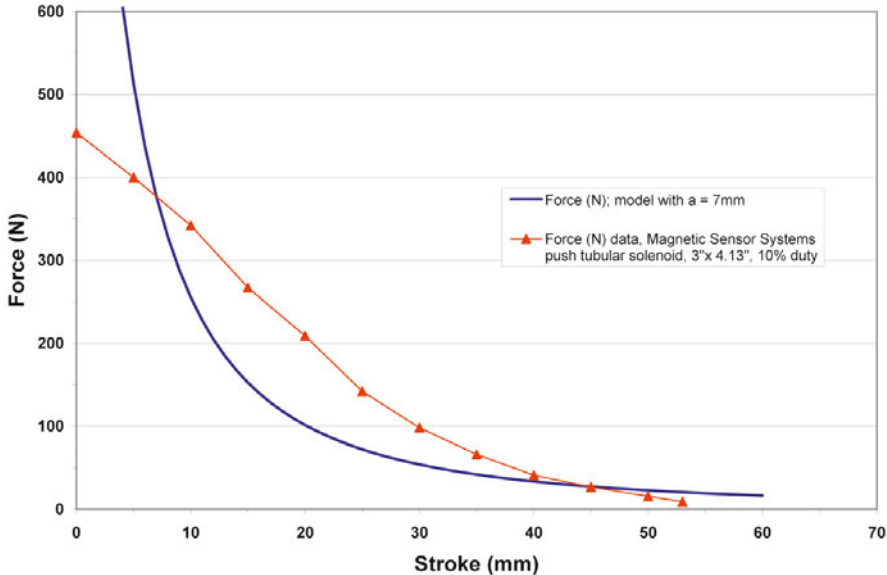


Figure 2.9. Solenoid force (newtons) as a function of stroke or gap distance (mm). Comparison of analytical force model (dark blue curve with length parameter $a = 7 \text{ mm}$) against the measured force vs stroke data (red triangles) published for the Magnetic Sensor System push tubular solenoid operating at a 10% duty cycle with $NI = 9,080$ [11].

$= \int_0^{\Delta x_0} F(x) dx \approx 9.5 \text{ J}$ (for the 3×4.13 inch push-type solenoid actuator with $NI = 9,080$ ampere-turns). From Equation 2.26 we calculate “ a ” $\approx 7 \text{ mm} = 0.007 \text{ m}$.

- Wire size (American wire gauge, AWG = 17); $d = 1.15 \text{ mm}$
- Resistance = 1.9 ohm for $N = 713$ for 115 m length; resistance/wire-length = 0.166 ohms/m
- Estimated current = 12.7 amperes for a 10% duty cycle and $NI = 9,080$ ampere-turns as recommended by Magnetic Sensor Systems
- Voltage = 24.2 volts; or two 12 volt batteries

We can now calculate typical values for the maximum force and maximum energy (or delivered work) for our model solenoid given the solenoid/plunger parameters for the reference 3×4.13 inch push-type solenoid actuator at a 10% duty cycle

$$C = \frac{\mu_0(NI)^2}{2}(A_p) = \frac{1.257 \times 10^{-6}(9,080)^2}{2}(1.425 \times 10^{-3})$$

$$= 7.38 \times 10^{-2} \text{ newton-m}^2$$

$$F_{\max}(x=0) = -\frac{C}{a^2} = -\frac{7.38 \times 10^{-2}}{(0.007)^2} = 1,506 \text{ N}$$

$$U_{B_{\max}} = \frac{C\ell_{\max}}{a(\ell_{\max} + a)} = \frac{7.38 \times 10^{-2}(0.06)}{(0.007)(0.067)} = 9.44 \text{ J}$$

The maximum velocity is calculated by directly setting the kinetic energy to the maximum stored magnetic energy of 9.44 J

$$V_{p_{\max}} = \sqrt{\frac{2U_{B_{\max}}}{m_p}} = \sqrt{\frac{2(9.44)}{0.60}} = 5.61 \text{ m/s}$$

Our simple asymptotic approximation for $V_{p_{\max}}$ (from Equation 2.35) yields about a 6% higher plunger velocity

$$V_{p_{\max}} \rightarrow \left[\frac{2C}{m_p a} \right]^{1/2} = \left[\frac{2(7.38 \times 10^{-2})}{(0.6)(0.007)} \right]^{1/2} = 5.93 \text{ m/s}$$

The total plunger travel time from Equation 2.37 is estimated to be

$$t_{\max} \approx \frac{\pi}{2} \left(\frac{m_p x_0^3}{2C} \right)^{1/2} = \frac{\pi}{2} \left(\frac{(0.60)(0.06)^3}{2(7.38 \times 10^{-2})} \right)^{1/2} = 0.047 \text{ s} = 47 \text{ millisecond}$$

If 47 millisecond was the only time to consider in our problem, the robot-kicker could theoretically kick a soccer ball over 20 times per second. If the power-up time for the solenoid circuit is of order 25 millisecond then the soccer ball could be kicked every 72 millisecond or about 14 times per second.

An additional important time scale must be considered for our solenoid. It is the time needed to cool the solenoid device to prevent damage to the electrical wires or insulation. This is the reason for the recommended low 10% duty cycle for the high current industrial solenoid considered here (i.e. 10% of the time the solenoid is energized, 90% of the time the current is turned off). If we adopt a 10% duty cycle, the ball can be kicked on average every 720 millisecond, which is about every $\frac{3}{4}$ of a second, thus allowing about 4 kicks for every 3 seconds. This might be a little slow for robot soccer. However it is possible to make a significant number of kicks rapidly within a few tenths of a second, as needed. As we show in the Appendix at the end of this chapter, it is possible to use a *100% duty cycle* if the total number of kicks made in a game is limited to a few hundred.

An additional BotE analysis needs to be undertaken in order to generate an estimate for the maximum temperatures, the heating and “cooling” time scales, and the maximum recommended number of kicks in a game for our solenoid. In the Appendix, we perform a Quick-Fire analysis of the solenoid heating problem to estimate: (1) the temperature rise produced during a single solenoid-actuator initiated kick, and (2) the increased solenoid temperatures caused by a number of rapid follow-up kicks. We conclude that heating analysis by estimating the allowable number of kicks that can be performed without causing thermal damage to the electrical insulation of the wires of our solenoid coil.

2.2.3.3.8 Comparison of estimates with solenoid actuator data

In this section we compare our modeled force vs gap (or stroke) performance curve to actual data for the industrial solenoid actuator of interest. We also estimate the level of current (or ampere-turns) needed to satisfy the required solenoid plunger speed of 7.5 m/s. Figure 2.9 shows modeled and measured solenoid force as a function of stroke or gap distance (mm).

We see that our model force curve as a function of gap has more curvature than does the measured force vs stroke data for the Magnet Sensor Systems solenoid. But we have chosen the empirical gap parameter “ a ” for our calculations to be 7 mm in order to make our theoretical work output (in Joules) equal to the measured work output for the data given in Section 2.2.3.3.7. Figure 2.9 shows that our modeled force, F , falls below the data for stroke or gap lengths in the range of 10 to 40 mm. On the other hand, Equation 2.27 produces a higher force than that of the data when the plunger approaches zero stroke.

Industrial-built solenoids are usually designed to reduce the maximum load on the “stop face” at the zero gap point by using design modifications to “flatten” the overall force vs stroke curve while still maintaining maximum power output. One of the design changes involves changing the shape of the plunger face away from a simple flat configuration to that of a truncated cone as well as shaping the receiving fixed center pin [12]. Such detailed design modifications are beyond the scope of our simple BotE estimates.

Figure 2.10 shows the dependence of plunger maximum velocity and the maximum work output from our modeled solenoid actuator as a function of NI ,

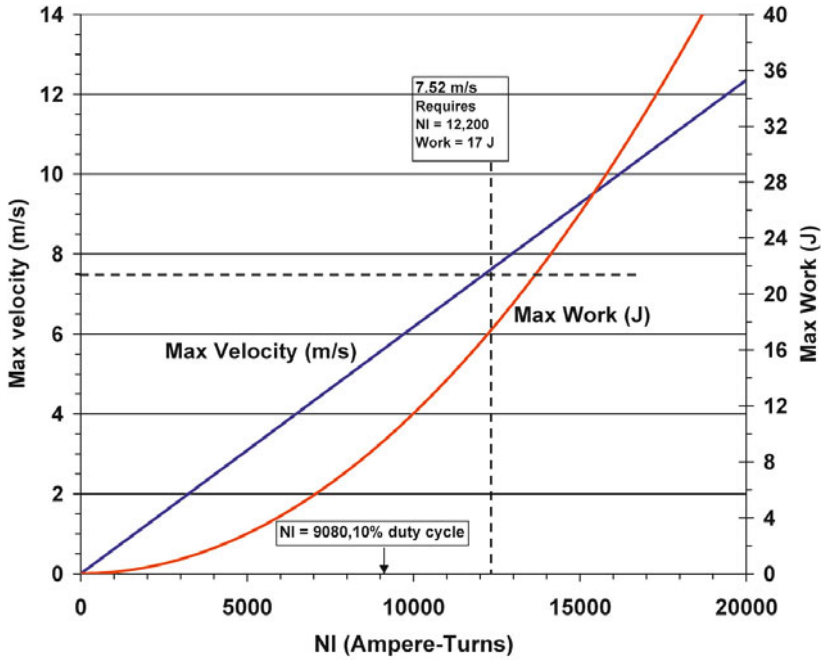


Figure 2.10. Theoretical maximum final velocity and maximum work as a function of NI , the product of applied current and the number of solenoid turns for a given solenoid plunger mass of 0.6 kg. Initial plunger position (or initial gap = 60 mm).

the ampere-turns for the activated solenoid, as derived from Equations 2.26 and 2.33.

Remember that our solenoid actuator must be capable of kicking a soccer ball to a required final natural roll velocity of 6.3 m/s to score a competitive goal. We previously determined that the maximum plunger velocity required to deliver a 6.3 m/s natural roll of a soccer ball is approximately equal to 7.5 m/s (as given by Equation 2.16 and plotted in non-dimensional form in Figure 2.3). For a plunger velocity of 7.5 m/s, Figure 2.10 shows that the required level of NI needed to produce this velocity is 12,200 ampere-turns. The solenoid actuator output work for this NI is $U_{B-\max} = 17$ Joules. From Section 2.2.3.3.7 and the data for the Magnetic Sensor System solenoid being considered, the voltage for a 1.9 ohm solenoid operating at a 10% duty cycle is 24.2 volts. Ohm's law gives a current of 12.7 amperes to operate the reference solenoid with a 1.9 ohm resistance. If $NI = 9,080$ ampere-turns, as recommended for our reference commercial solenoid, then N must be 713 turns.

Note that for the plunger velocity requirement of 7.5 m/s, our solution of $NI = 12,200$ ampere-turns results in a higher nominal current of $I = NI/713 = 17.1$ A if we maintain N at 713 turns

$$(I_{\text{required}} \approx 17 \text{ A for } N = 713 \text{ turns})$$

The corresponding required voltage is then $E = I \cdot R = (17.1)(1.9) = 32.5 \text{ V}$
 $(E_{\text{required}} \approx 33 \text{ volts; or 3 twelve volt batteries})$

This higher current requirement 17 A leads to an 80% higher Ohmic heating rate for our solenoid system compared to that of the reference push-tube solenoid if we keep the number of solenoid-turns the same as for our reference solenoid at 713 and also use 17 gauge solenoid wire.

For the required number of ampere-turns estimated above at 12,200, the calculated total time to carry out a single kick is approximately 60 millisecon. This is based on estimates for both the travel time of the plunger and the solenoid power-up time. (For calculation details see Table 2.2, Section 3.3 below). For a 10% duty cycle, the time between kicks will then be about 600 millisecon, corresponding to a little under 2 kicks per second based on industrial *near continuous* operational heat load and cooling considerations.

In real solenoid actuator systems the packing volume, namely how close the electrical components are packed in a small space, is also a design consideration, particularly when the solenoid is subjected to high heat loads [12].

When all the above issues are taken into consideration, some alteration of the design parameters may be needed in order to satisfy all the competing system constraints for our robot system. However, our estimate appears to be a good *first-cut* at the problem.

2.2.4 Final design requirements for linear-actuator solenoid and supporting electrical system

The following tables summarize the key BotE equations used in the preceding sections. Numerical results are given for important solenoid-kicker parameters, in particular the calculated magnitudes of ball and plunger velocity, maximum available work required, and the number of solenoid ampere-turns for our modeled kicker. For a nominal resistance, based on 713 turns of 17 gauge wire, we also calculate the required current and applied voltage.

Table 2.1 presents the key BotE results for the dynamics of a rolling soccer ball struck by a thin plunger. The required solenoid piston velocity is calculated for the minimum required input energy solution needed to produce a given natural roll velocity to readily score a goal.

Table 2.2 presents the BotE results for a basic solenoid actuator design that achieves a plunger velocity given by the results shown in Table 2.1. The primary calculated solenoid parameters are: (a) the required solenoid-stored magnetic energy (joules); (b) the required solenoid ampere-turns, NI ; (c) the estimated solenoid current I and required voltage for a given solenoid wire size or gauge; and (d) the estimated kicker repetition time, based on estimated plunger travel times and the estimated R-L circuit power-up time scale.

Figure 2.11 (thanks to J. Jacobs, personal communication, October, 2010) is a candidate circuit schematic that shows how the 17 ampere current required to drive our pulsed solenoid can be turned “on” and “off” using a moderately priced com-

mercial electronic solenoid driver. As per Table 2.2 the voltage applied to the driver is supplied by three 12 volt batteries. The solenoid driver is cued by a low (0 to 5 volt) digital control voltage signal supplied directly from the robot itself. The initiation signal is radioed to the robot from the student team that remotely operates the robot.

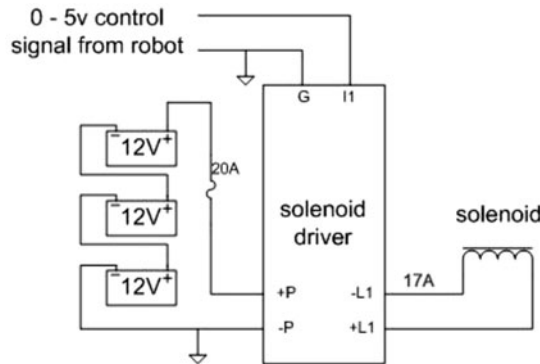


Figure 2.11. The schematic shown is based on the Model Si5SD1-50V-20A control board. This single, 50 V, 20 A solenoid driver (based on a 60 A MOSFET power semiconductor) is sold with an integrated heat sink by Signal Consulting on their website <http://signallic.com/products/Si5SD1-50V-20A.html>.

2.3 APPENDIX: MODELING OF THE TEMPERATURE RISE PRODUCED BY OHMIC HEATING FROM SINGLE OR MULTIPLE SOLENOID-ACTUATOR KICKS

To illustrate how it is possible to obtain a BotE estimate quickly using a minimum amount of technical information we take as a sample assignment the problem of estimating the temperature rise produced by the current applied during one or more solenoid-actuator kicks.

2.3.1 Quick-Fire problem approach

As in Chapter 1, in the Quick-Fire approach to a BotE problem we develop our estimates by systematically following, in sequence, the following five steps:

- Define and/or conceptualize the problem using a sketch or brief mathematical description.
- Select a model or approach, either mathematical or empirical that describes the basic physics of the problem.
- Determine the input data parameters and their magnitudes as required to solve the problem, either from data sources or by scaling values by analogy, or simply by using an educated guess.

Table 2.1. BotE input parameters and summary of the calculations of the dynamics of a rolling soccer ball struck by a thin plunger (see Figure 2.2) to establish the requirements for our linear-actuator solenoid kicker.

<i>Required soccer ball and plunger velocity estimates</i>	
	<i>Given</i>
	<i>Data reference</i>
Soccer ball mass = m_b (kg)	0.45 Section 3.1
Ball diameter = $2R = d(m)$	0.111 Section 3.1
Goal net width = $w(m)$	2.0 Section 3.1.1
Distance to goal-net edge X_{gn} (m)	3.16 Section 3.1.1
Lateral speed of goalie V_{goalie} (m/s)	2.0 Section 3.1.1
	<i>Calculation</i>
Minimum required natural roll ball velocity to score a goal = $V_{required} = V_{NR}$ (m/s)	$V_{NR} = \left(\frac{X_{gn}}{w/2}\right) \cdot V_{goalie} = \left(\frac{3.16}{1.0}\right) \cdot 2.0 = 6.32 \text{ m/s}$
Initial ball spin after plunger hit at height h , ω_b (radian/s); initial ball velocity V_b is unknown	$\omega_b = \frac{3}{2} \left(\frac{h}{R}\right) \left(\frac{V_b}{R}\right)$ [Equation 2.6 based on piston impact on ball, Figure 2.2]
Velocity of the ball after impact, V_b , for a given piston velocity before impact, v_0 (m/s)	$V_b = \frac{2v_0}{\left[1 + \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R}\right)^2\right]}$ [Equation 2.9; from conservation of momentum and energy, both linear and rotational]

Natural roll velocity in terms of V_b , for a given piston impact height, h		$V_{NR} = \frac{3}{5} V_b \left(1 + \frac{h}{R} \right)$ (Equation 2.15 using $\omega = V_{NR}/R$)
Piston velocity v_0 for a given natural roll velocity		$v_0 = \frac{5}{6} \left\{ \frac{\left(1 + \frac{m_b}{m_p} + \frac{3}{2} \left(\frac{h}{R} \right)^2 \right)}{\left(1 + \frac{h}{R} \right)} \right\} V_{NR}$ [from the inverse of Equation 2.16; as in Figure 2.4]
Calculated piston velocity given $m_p = 4/3 m_b \approx 0.6$ kg for $h/R \approx 0.50$ necessary to obtain a minimum v_0 ; per Figure 2.4. For $h/R = 0.5$ the impact height $h = 0.5(0.111 \text{ m}/2) = 2.78$ cm above the ball's centerline		From above equation: $v_0 = \frac{5}{6} \left\{ \frac{\left(1 + \frac{3}{4} + \frac{3}{2} (0.50)^2 \right)}{(1 + 0.50)} \right\} (6.32 \text{ m/s}) = 7.46 \text{ m/s}$ Rounding to nearest 0.1 m/s 7.5 m/s is the estimated maximum velocity required of the push-type plunger in our linear-actuator solenoid (see Table 2.2).

Table 2.2. BotE results for a basic solenoid actuator design that achieves a plunger velocity of 7.5 m/s given by Table 2.1. The calculated solenoid parameters are: (a) the required solenoid-stored magnetic energy (joules); (b) the required ampere-turns, NI ; (c) the estimated solenoid current I and required voltage for a given solenoid wire gauge; and (d) the estimated kicker repetition time based on estimated plunger travel times and R-L circuit power-up time scales.

<i>Required solenoid energy, ampere-turns, current etc. to meet given plunger velocity requirement</i>		
	<i>Given</i>	<i>Data source: [10, 11].</i> Solenoid geometrical parameters are typical of a large Push Type Tubular Solenoid (3.0 inch diameter by 4.13 inch in outer cylindrical shell length) produced by Magnetic Sensor Systems [www.solenoidcity.com].
Plunger mass, m_p	0.6 kg	Section 2.2.3.3.7
Plunger diameter = 1.68 inch	0.0426 m	Section 2.2.3.3.7
Plunger Area, A_p	$1.425 \times 10^{-3} \text{ m}^2$	Section 2.2.3.3.7
Maximum plunger stroke, $x_0 = 60 \text{ mm}$	0.06 m	Section 2.2.3.3.7
Estimated empirical length scale “ a ” for the baseline large push-type tubular solenoid; $a \approx 7 \text{ mm}$ [10, 11]	0.007 m	Section 2.2.3.3.7 The empirical length scale “ a ” is defined as additional gap scale length scaled to the force vs gap (or stroke) data for commercial solenoid actuators. One typical contributor to “ a ” in commercial actuators is the annular air gap between the outer steel wall of the solenoid and the steel plunger itself.
Resistance for AWG 17 wire with 115 m length	1.9 ohm or resistance per length = 0.166 ohms/m	Section 2.2.3.3.7

$\mu_0 = 4\pi \times 10^{-7}$ newton/(ampere) ²	1.257×10^{-6}	Permeability of free space
		<i>Calculation</i>
Maximum energy, U_B , stored in a solenoid actuator with a given initial gap length, x_0		$U_{B_{\max}} = \frac{C x_0}{a(x_0 + a)}$ (Eq. 2.26), Section 3.2.2.2 where $C = \frac{\mu_0 (NI)^2}{2} (A_p)$ and NI = the number of ampere-turns. The quantitative value of NI for our model design is determined below to meet the plunger velocity requirement
Theoretical Maximum plunger velocity		$V_{p_{\max}} = \left[\frac{2C}{m_p a} \right]^{1/2} \left[\frac{x_0}{(x_0 + a)} \right]^{1/2}$ (Eq. 2.34); Section 3.2.2.4
Theoretical value of NI needed to produce a given plunger velocity		$NI = V_{p_{\max}} \left[\frac{m_p}{\mu_0 A_p} \right]^{1/2} \left[\frac{a(x_0 + a)}{x_0} \right]^{1/2}$ (Solve Equation 2.34 for C and then take the square-root)
Numerical value of NI needed to produce a given plunger velocity		$NI = 7.5 \left[\frac{1.257 \times 10^{-6} (1.43 \times 10^{-3})}{0.6} \right]^{1/2} \left[\frac{0.007(.067)}{.06} \right]^{1/2}$ $NI = 12,150$ ampere-turns See max velocity curve in Figure 2.10.
Maximum energy, U_B , stored in a solenoid actuator with NI = value needed to produce a given plunger velocity (see calculation above)		$U_{B_{\max}} = \frac{\mu_0 (NI)^2 (A_p) x_0}{2a(x_0 + a)}$ $= \frac{1.257 \times 10^{-6} (12,150)^2 (1.43 \times 10^{-3}) .06}{2(.007)(.067)}$ $= 17.0 \text{ J}$

(continued)

Table 2.2 (cont.)

Required current, I and voltage, E assuming that the number of turns $N = 713$ and the resistance $= 1.9$ ohms for a wire size set at 17 gauge (AWG) as per the large push-type tubular solenoid with a 10% duty cycle	$I = \frac{NI}{N} = \frac{12,150}{713} = 17.1 \text{ A}$	
Estimated required supply voltage, E	$E = I \cdot R = 17.1 \text{ A} \cdot (1.9 \text{ ohm}) = 32.5 \text{ volts}$ Ohm's law. Use 3 twelve volt batteries.	
Dissipated power due to Ohmic heating, P (watts)	$P = I^2 R = E \cdot I$ $P = (32.5)(17.1) \approx 555 \text{ W}$ the power dissipated by more than five 100 W bulbs (Ohmic heating law)	
Total plunger travel time (sec)	$t_{\max} \approx \frac{\pi \left(m_p x_0^3 \right)^{1/2}}{2C}$ where $C = \frac{\mu_0 (NI)^2}{2} (A_p) = 0.132 \text{ newton-m}^2$ $t_{\max} \approx \frac{\pi \left(m_p x_0^3 \right)^{1/2}}{2C} = 1.57 \left(\frac{0.6(06)^3}{2(.132)} \right)^{1/2} = 34.8 \text{ millisecond}$ (Equation 2.37) Section 3.2.2.5	
Power up time scale, t_{RL} , for an R-L circuit; about $5L/R$ sec. Solenoid length is estimated to be 4 inch $= 0.1$ m for 4.13 inch push-type	$L \approx \frac{\mu_0 N^2 A_p}{\ell_{\text{solenoid}}} = \frac{1.257 \times 10^{-6} (713^2) 1.43 \times 10^{-3}}{0.1}$ Inductance $L = 0.914 \times 10^{-3}$ henry $t_{RL} = 5L/R = 5(9.14 \times 10^{-3} / 1.9) = 24 \text{ millisecond}$	
Total time required for solenoid power-up plus the plunger travel time	$T_{\text{total}} = t_{\max} + t_{RL}$ $= 35 + 24 = 59 \text{ millisecond} \approx 0.06 \text{ s}$	
Estimated kicks rate	● About 17 kicks per second for solenoid kicker if cooling not a factor (100% duty cycle) ● Less than 2 kicks per second for a 10% duty cycle	

- (d) Substitute the input data into the model and compute a value or range of values for the estimate.
- (e) Present the results in a simple form and provide a brief interpretation of their meaning.

We demonstrate here the use of the Quick-Fire approach in establishing the key thermal measures for our solenoid-actuator problem.

2.3.2 Problem definition and sketch

We pose the following set of questions for our solenoid thermal problem:

- Determine a simple equation for the temperature increase in a solenoid coil contained within a steel cylindrical shell as a function of total current activation time. Consider the small time period before reaching a steady state temperature. The solenoid is assumed to be convectively cooled by an air stream perpendicular to the long axis of our solenoid-actuator.
- Find the temperature increase per “kick” and estimate the peak coil temperature for a typical robot soccer game.
- Determine the maximum number of powered solenoid cycles (i.e. the number of kicks) that the robot can initiate without the coil temperature reaching some specified thermal limit based primarily on the failure point of the coil’s electrical insulation.

2.3.3 The baseline mathematical model

We first write down a thermal energy balance equation based on [Figure 2.12](#). In order to reduce the complexity of the problem, we use the method of “lumped capacitance” to determine the spatially averaged temperature, $T(t)$, within the solenoid-actuator cylinder boundaries. This temperature is a function of time (but not space) because we do not account for heat conduction gradients within the solenoid components; in other words, we do not account for differences in the internal temperatures in the copper wire, steel, and air regions.

With this model, the internal temperature, $T(t)$, begins to increase with time as the solenoid current, i , starts to flow in the coil with resistance, R . The initial time rate of change of temperature is initially determined by a simple balance between the heat storage per unit time in the copper coil

$$\frac{d}{dt}[mC_p(T - T_\infty)]$$

and the rate of Ohmic or i^2R heating (measured in watts). Note that based on our assumptions, m and C_p are respectively the mass and specific heat of the copper wire only. We also assume that the solenoid is cooled slowly by air convection where the heat out of the cylinder is proportional to the temperature difference (i.e. $T(t) - T_\infty$)

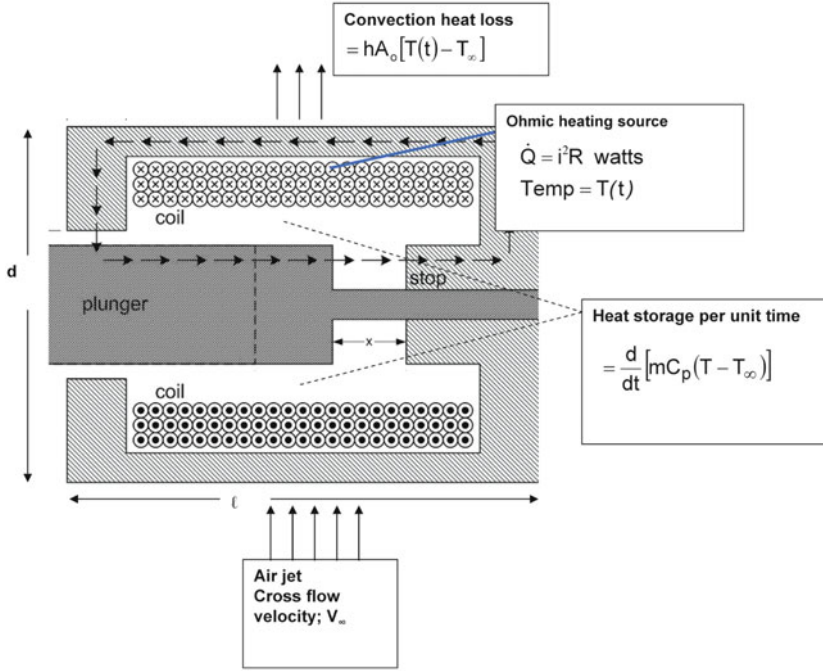


Figure 2.12. Sketch of cylindrical solenoid. Thermal balance “lumped capacitance” components: (a) Ohmic heating by the applied current flowing through the coil with a prescribed resistance per unit length, (b) heat storage in the coil mass, i.e. the thermal capacity, and (c) convective air jet cooling of the external cylinder. [Illustration courtesy of J. Jacobs]

multiplied by the product of the convective heat coefficient “ h ” in units of $\left[\frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}}\right]$ and the outer surface area for convection A_0 . We presume T_∞ is a constant ambient reference temperature for the stream of air flowing normal to the axis of the cylinder. We determine h empirically using well-known curve fits to laboratory experiments for flow over heated cylinders. We balance these three energy components, according to the first law of thermodynamics, to produce the following energy balance differential equation describing the time varying internal temperature $T(t)$,

$$\underbrace{\frac{d}{dt}[mC_p(T - T_\infty)]}_{\text{heat storage change}} = \underbrace{i^2R}_{\text{ohmic heating}} - \underbrace{hA_0(T - T_\infty)}_{\text{convective heat loss}} \quad (\text{A.1})$$

We assume that the solenoid’s internal temperature T is equal to T_∞ (i.e. the ambient air temperature) at $t = 0$ when the current “ i ” immediately jumps from a value of zero to its steady state dc value of 17 amperes chosen for our design solution.

The outer surface area A_0 for radial heat transport from within the cylinder

enclosing the solenoid is calculated using the cylinder diameter d and cylinder length ℓ by the expression

$$A_0 = \pi d\ell$$

By defining $\theta \equiv (T - T_\infty)$ and $\dot{Q} \equiv i^2 R$, we can rewrite this energy balance differential equation in the following compact form

$$\left. \begin{aligned} (mC_p) \frac{d\theta}{dt} &= \dot{Q} - (hA_0)\theta \\ \text{initial condition } \theta(t=0) &= 0 \end{aligned} \right\} \quad (\text{A.2})$$

2.3.4 Physical parameters and data

Estimated copper wire mass [14]:

$$m = (\text{wire density } \rho, \text{ mass per unit length}) \times (\text{wire length}) = \rho L$$

For 17 gauge wire, with $L = 150$ m, wire mass

$$m = (0.92 \times 10^{-2} \text{ kg/m})(150 \text{ m}) = 1.38 \text{ kg}$$

$$\text{Specific heat of copper: } C_p = 390 \frac{\text{J}}{\text{kg } ^\circ\text{C}}$$

Solenoid current:

$$i = 17 \text{ A [see Section 2.2.3.3.8, Figure 2.10]}$$

Resistance R (ohms) for about 150 m length of 17 gauge solenoid wire [14]:

$$R = \frac{(\text{copper resistivity})(L)}{(\text{wire} \times \text{sectional area})} = \frac{(1.678 \times 10^{-8} \text{ ohm} \cdot \text{m})(150 \text{ m})}{(1.1 \times 10^{-6} \text{ m}^2)} = 2.3 \text{ ohm} \approx 2 \Omega$$

Heating rate at full current levels:

$$\dot{Q} = i^2 R \approx 578 \text{ W}$$

Heat transfer coefficient, h , for cross flow of air over a cylinder of diameter d :

The value of $h \left[\frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right]$ is found from experimental curve fits of the related non-dimensional heat transfer parameter, called the Nusselt number, $Nu_d = hd/k$ plotted as a function of the flow Reynolds number. These fits of the data, Nu_d vs Re_d , are based on laboratory experiments that measure the heat loss from a heated cylinder in cross flow. In this expression, k is the thermal conductivity of the convective fluid (air in our case) measured far from the cylinder. “ k ” has units of $\left[\frac{\text{W}}{\text{m} \cdot ^\circ\text{C}} \right]$. Reynolds number, the ratio of kinematic forces to viscous forces, $Re_d = \frac{V_\infty \cdot d}{v_\infty}$ is directly proportional to flow velocity V_∞ and inversely proportional to the kinematic viscosity of air; $v_\infty \approx 10^{-6} \text{ m}^2/\text{s}$. In general the empirical curve fits are of the

form $Nu_d = (C)(Re_d)^m$, where d is the diameter of the cylinder. The constant C is estimated (Table 9-1 of [15]) to be $C = 0.024$ and the exponent $m = 0.805$ for high Reynolds numbers air flows in the following range of Reynolds numbers: $Re_d = 4 \times 10^4$ to 4×10^5 . Let's pick a low air-cooling velocity that might be produced by a fan operating in that Reynolds number range

$$V_\infty = 4 \text{ m/s (or 8.8 miles/hr)}$$

The corresponding Reynolds number for a 3 inch diameter cylinder is

$$Re_d = \frac{4 \cdot (0.076)}{10^{-6}} = 3.04 \times 10^5$$

Using the curve-fit coefficients, the Nusselt number is

$$Nu_d = 0.024(3.04 \times 10^5)^{0.805} = 622$$

and so the corresponding numerical value of the heat transfer coefficient h is

$$h = \frac{(Nu_d)(k_{\text{air},\infty})}{d} = \frac{(622) \cdot (0.026)}{(0.076)} = 213 \left[\frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right]$$

The area A_0 that scales the convective heat transfer from the outer surface of the cylinder encasing our solenoid coil (diameter $d = 0.076 \text{ m}$ and length $\ell = 0.1 \text{ m}$) is

$$A_0 = \pi d\ell = \pi(0.076)(0.1) = 2.39 \times 10^{-2} \text{ m}^2$$

We now have calculated, or selected from our references, all the constants we require to estimate the temperature from our “lumped capacitance” solenoid model, Equation A.1. The values for $T(t)$ can be calculated directly from the approximate or exact solutions to the model energy balance differential equation Equation A.2.

2.3.5 Numerical results

We first set forth a simple approximate solution to our model differential equation, valid for small heating times, $t \ll \tau$.

The time scale τ is a characteristic time measure where the heat storage term in Equation A.2 is comparable in magnitude to the convective cooling term, and can readily be calculated using solenoid system and heat transfer parameters from the solution to Equation A.2. The following is the exact solution of this differential equation. It should be well known to calculus and electrical engineering students, and can be checked by direct substitution in Equation A.2.

The solution for θ , the temperature difference relative to the ambient, is

$$\theta = \left(\frac{\dot{Q}}{hA_0} \right) [1 - \exp(-t/\tau)] \quad (\text{A.3})$$

where $\tau = mC_p/hA_0$.

Based on the parameters presented in Section 3.4, we find that τ is of order of 100 seconds for air cooling at 4 m/s (It is a longer time scale if we reduce the amount

of air cooling). The exact value is given by

$$\tau = mC_p/hA_0 = (1.38)(390)/[(213)(0.0239)] = 106 \text{ s}$$

For heating times much greater than 100 s (i.e. large values of t/τ), Equation A.3 approaches the following steady state temperature determined by setting the outgoing cooling rate equal to the Ohmic heating rate; $\theta_{\text{steady}} \rightarrow \left(\frac{\dot{Q}}{hA_0}\right)$. For our solenoid pulse-heating problem we will be concerned primarily with much shorter times (indeed times less than 1 s) where the effects of convective cooling on the temperature change are quite small.

Short time solution:

When $t \ll \tau$, the heat storage term is much larger than the convective cooling term. In this case, Equation A.2 reduces to the simple expression

$$(mC_p) \frac{d\theta}{dt} = \dot{Q} \quad (\text{A.4})$$

Integrating Equation A.4 over time t and applying the initial condition $\theta(t=0) = 0$ gives us the following simple result

$$\theta = \left(\frac{\dot{Q}}{mC_p}\right)t \quad t \ll \tau \quad (\text{A.5})$$

Equation A.5 establishes that the predicted temperature change is directly proportional to t for short “constant current” heating times.

Note that this result can also be obtained from the Equation A.3 by taking its limit as $t/\tau \rightarrow 0$; simply expand the exponential term for small values of t/τ .

Using the constants $\dot{Q}, m, C_p$ determined in section 3.4, we calculate the proportionality constant in Equation A.5 and find that the short time temperature increase is given by the linear equation

$$T - T_\infty = \left(\frac{578}{(1.38)(390)}\right)t = (1.07^\circ\text{C/s})t \quad t \ll \tau \quad (\text{A.6})$$

Let's assume that we want to calculate the temperature change for a single “firing” of the solenoid actuator. As previously shown, the duration of this single firing is approximately 0.06 s in total. That is, we assumed for this Quick-Fire calculation that the current is approximately constant for all 60 millisecc, which isn't quite correct because the current rises rapidly and then levels off for large times during the estimated 24 millisecc power-up R-L time period for the solenoid circuit.

For a “pulse” lasting $t = 0.06 \text{ s}$ (that is for 60 milliseconds of Ohmic heating)

$$T - T_\infty \equiv \Delta T_1 = 0.064^\circ\text{C} \quad \text{single pulse} \quad (\text{A.7})$$

2.3.6 Interpretation of results

In a given robot soccer game of 30 minutes duration, one can crudely estimate that, at most our solenoid actuator would kick the ball 3 to 4 times per minute, or about 100 kicks in all. We can estimate the *highest* solenoid temperature *during a game* by considering a worst case scenario; we will take as a given that there is absolutely *no cooling* between pulses. This is the same as a 100% duty cycle with no convective cooling at all, i.e. zero air speed. The estimated temperature increase for the game, ΔT_{game} , is then 100 times the value of the single pulse temperature increase ΔT_1

$$\Delta T_{\text{game}} \approx 100 \cdot (0.064^\circ\text{C}/\text{pulse}) = 6.4^\circ\text{C} \quad (\text{A.8})$$

If the temperature of the solenoid actuator was initially at a room temperature of 20°C , the final peak solenoid temperature would be 80°F

$$\underbrace{T_{\text{game,peak}}}_{\text{no cooling}} \approx 26.4^\circ\text{C or about } 80^\circ\text{F} \quad (\text{A.9})$$

which is reasonably low.

Estimate of total allowable kicks per game (no cooling)

For our simple thermal solution in the absence of cooling, the coil temperature increases by 64 millidegrees every time the ball is kicked. We must therefore ask how many kicks it would require to cause some degree of failure of the electrical system due to the higher temperatures?

Ledex corporation, a leading manufacturer of solenoid actuators states that wire temperatures should be no higher than 130°C to prevent failure of class B electrical insulation and no higher than 180°C for Class H insulation [16]. If we set 130°C , for the poorest of these two insulation classes, as an upper limit for our solenoid kicker, then $\Delta T_{\text{game}} = 110^\circ\text{C}$. The total allowable kicks per game N is

$$N_{\text{upper limit}} = 110^\circ\text{C}/0.064^\circ\text{C per kick} = 1,719 \text{ kicks} \quad (\text{A.10})$$

The total time required by the solenoid actuator to perform 1,719 pulses without cooling between pulses is about 103 s, or a little less than 2 minutes of *continuous* operation. As 103 seconds is about equal to the time scale τ , the cooling effects should also begin to be taken into consideration as our kicker exceeds the 1,000 kick mark. We conclude that it would be unwise to run our solenoid to make over 1,700 kicks because this may cause serious damage to our electrical system.

From this worst case BotE *Quick-Fire* analysis it is clear that our robot kicker will be able to easily handle the temperature rise due to Ohmic heating if we keep the number of game kicks at or below several hundred kicks. Our analysis and calculations have provided the estimates required to address the Quick-Fire heating problem that we initially set for our robot kicking device.

But what if we had a more stressful solenoid system designed for a different use; a solenoid not limited to just a few hundred pulses, but one that might run for a very long time? Various industrial applications, like fuel injection systems or industrial

hydraulic actuators operate continuously for significant periods of time. They therefore require active cooling, large heat sinks, or the use of *low* duty cycles to keep them from reaching potential high damaging temperatures. We don't have time to analyze these more stressing industrial applications, but the Back-of-the-Envelope approach could certainly be used to predict the long term temperatures for these industrial type solenoid-actuators.

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