

Contents

1	Historical Overview	1
1.1	The Early Proofs of the CBHD Theorem	1
1.1.1	The Origin of the Problem	2
1.1.2	Schur, Poincaré and Pascal	9
1.1.3	Campbell, Baker, Hausdorff, Dynkin	19
1.2	The “Modern Era” of the CBHD Theorem	37
1.3	The “Name of the Game”	44
 Part I Algebraic Proofs of the Theorem of Campbell, Baker, Hausdorff and Dynkin		
2	Background Algebra	49
2.1	Free Vector Spaces, Algebras and Tensor Products	49
2.1.1	Vector Spaces and Free Vector Spaces	49
2.1.2	Magmas, Algebras and (Unital) Associative Algebras	56
2.1.3	Tensor Product and Tensor Algebra	72
2.2	Free Lie Algebras	87
2.3	Completions of Graded Topological Algebras	93
2.3.1	Topology on Some Classes of Algebras	94
2.3.2	Completions of Graded Topological Algebras	98
2.3.3	Formal Power Series	101
2.3.4	Some More Notation on Formal Power Series	106
2.4	The Universal Enveloping Algebra	108
3	The Main Proof of the CBHD Theorem	115
3.1	Exponential and Logarithm	117
3.1.1	Exponentials and Logarithms	119
3.1.2	The Statement of Our Main CBHD Theorem	124
3.1.3	The Operation \blacklozenge on $\widehat{\mathcal{T}}_+(V)$	126
3.1.4	The Operation \diamond on $\widehat{\mathcal{T}}_+(V)$	128

3.2	The Campbell, Baker, Hausdorff Theorem	132
3.2.1	Friedrichs's Characterization of Lie Elements	133
3.2.2	The Campbell, Baker, Hausdorff Theorem	139
3.2.3	The Hausdorff Group	142
3.3	Dynkin's Formula	145
3.3.1	The Lemma of Dynkin, Specht, Wever	145
3.3.2	Dynkin's Formula	151
3.3.3	The Final Proof of the CBHD Theorem	154
3.3.4	Some "Finite" Identities Arising from the Equality Between \blacklozenge and \blacklozenge	156
3.4	Résumé: The "Spine" of the Proof of the CBHD Theorem ...	158
3.5	A Few Summands of the Dynkin Series	159
3.6	Further Reading: Hopf Algebras	162
4	Some "Short" Proofs of the CBHD Theorem	173
4.1	Statement of the CBHD Theorem for Formal Power Series in Two Indeterminates	178
4.2	Eichler's Proof	187
4.2.1	Eichler's Inductive Argument	189
4.3	Djokovic's Proof	199
4.3.1	Polynomials and Series in t over a UA Algebra	199
4.3.2	Background of Djoković's Proof	205
4.3.3	Djoković's Argument	210
4.4	The "Spine" of the Proof	216
4.4.1	Yet Another Proof with Formal Power Series	220
4.5	Varadarajan's Proof	223
4.5.1	A Recursion Formula for the CBHD Series	227
4.5.2	Another Recursion Formula	229
4.6	Reutenauer's Proof	231
4.7	Cartier's Proof	253
4.7.1	Some Important Maps	254
4.7.2	A New Characterization of Lie Elements	258
5	Convergence of the CBHD Series and Associativity of the CBHD Operation	265
5.1	"Finite" Identities Obtained from the CBHD Theorem	268
5.2	Convergence of the CBHD Series	277
5.2.1	The Case of Finite Dimensional Lie Algebras	278
5.2.2	The Case of Banach-Lie Algebras	292
5.2.3	An Improved Domain of Convergence	301
5.3	Associativity of the CBHD Operation	305
5.3.1	"Finite" Identities from the Associativity of \blacklozenge	306
5.3.2	Associativity for Banach-Lie Algebras	312

5.4	Nilpotent Lie Algebras and the Third Theorem of Lie.....	320
5.4.1	Associativity for Nilpotent Lie Algebras	320
5.4.2	The Global Third Theorem of Lie for Nilpotent Lie Algebras	328
5.5	The CBHD Operation and Series in Banach Algebras	337
5.5.1	An Alternative Approach Using Analytic Functions	347
5.6	An Example of Non-convergence of the CBHD Series	354
5.7	Further References	359
6	Relationship Between the CBHD Theorem, the PBW Theorem and the Free Lie Algebras	371
6.1	Proving PBW by Means of CBHD	375
6.1.1	Some Preliminaries	375
6.1.2	Cartier's Proof of PBW via CBHD	383

Part II Proofs of the Algebraic Prerequisites

7	Proofs of the Algebraic Prerequisites.....	393
7.1	Proofs of Sect. 2.1.1	393
7.2	Proofs of Sect. 2.1.2	396
7.3	Proofs of Sect. 2.1.3	396
7.4	Proofs of Sect. 2.3.1	407
7.5	Proofs of Sect. 2.3.2	417
7.6	Proofs of Sect. 2.3.3	428
7.7	Proofs of Sect. 2.4	435
7.8	Miscellanea of Proofs	445
8	Construction of Free Lie Algebras	459
8.1	Construction of Free Lie Algebras Continued	459
8.1.1	Free Lie Algebras over a Set.....	463
8.2	Free Nilpotent Lie Algebra Generated by a Set	469
9	Formal Power Series in One Indeterminate	479
9.1	Operations on Formal Power Series in One Indeterminate	480
9.1.1	The Cauchy Product of Formal Power Series	480
9.1.2	Substitution of Formal Power Series.....	481
9.1.3	The Derivation Operator on Formal Power Series	486
9.1.4	The Relation Between the exp and the log Series	488
9.2	Bernoulli Numbers	494

10 Symmetric Algebra	501
10.1 The Symmetric Algebra and the Symmetric Tensor Space...	501
10.1.1 Basis for the Symmetric Algebra	512
10.2 Proofs of Sect. 10.1	514
A List of the Basic Notation	523
References	529
Index	537

Topics in Noncommutative Algebra

The Theorem of Campbell, Baker, Hausdorff and Dynkin

Bonfiglioli, A.; Fulci, R.

2012, XXII, 539 p. 5 illus., Softcover

ISBN: 978-3-642-22596-3