Preface to the Second Edition

Everything the Power of the World
does is done in a circle. The sky is
round and I have heard that the earth
is round like a ball and so are all the stars.
The wind, in its greatest power, whirls.
Birds make their nests in circles,
for theirs is the same religion as ours.
The sun comes forth and goes down
again in a circle. The moon does the
same and both are round. Even the
seasons form a great circle in their
changing and always come back again
to where they were. The life of a man
is a circle from childhood to childhood.
And so it is everything where power moves.

Black Elk (1863–1950)

Nonlinear phenomena represent intriguing and captivating manifestations of nature. The nonlinear behavior is responsible for the existence of complex systems, catastrophes, vortex structures, cyclic reactions, bifurcations, spontaneous phenomena, phase transitions, localized patterns and signals, and many others. The importance of studying nonlinearities has increased over the decades, and has found more and more fields of application ranging from elementary particles, nuclear physics, biology, wave dynamics at any scale, fluids, plasmas to astrophysics. The soliton is the central character of this 167-year-old story. A soliton is a localized pulse traveling without spreading and having particle-like properties plus an infinite number of conservation laws associated to its dynamics. In general, solitons arise as exact solutions of approximative models. There are different explanation, at different levels, for the existence of solitons. From the experimentalist point of view, solitons can be created if the propagation configuration is long enough, narrow enough (like long and shallow channels, fiber optics, electric lines, etc.), and the surrounding medium has an appropriate nonlinear response providing a certain type of balance between nonlinearity and dispersion. From the numerical calculations point of view, solitons are localized structures with very high stability, even against collisions between themselves. From the theory of differential equations point of view, solitons are cross-sections in the jet bundle associated to a bi-Hamiltonian evolution equation (here Hamiltonian pairs are requested in connection to the existence of an infinite collection of conservation laws in involution). From the geometry point of view, soliton equations are compatibility conditions for the existence of a Lie group. From the physicist point of view, solitons are solutions of an exactly solvable model having isospectral properties carrying out an infinite number of nonobvious and counterintuitive constants of motion.

The progress in the theory of solitons and integrable systems has allowed the study of many nonlinear problems in mathematics and physics: nonlocal interactions, collective excitations in heavy nuclei, Bose–Einstein condensates in atomic physics, propagation of nervous pulses, swimming of motile cells, nonlinear oscillations of liquid drops, bubbles, and shells, vortices in plasma and in atmosphere, tides in neutron stars, only to enumerate few of possible applications. A number of other applications of soliton theory also lead to the study of the dynamics of boundaries. In that, the last three decades have seen the completion of the foundation for what today we call nonlinear *contour dynamics*. The subsequent stage of development along this topic was connected with the consideration of an almost *incompressible* systems, where the boundary (contour or surface) plays the major role.

Many of the integrable nonlinear systems have equivalent representations in terms of differential geometry of curves and surfaces in space. Such geometric realizations provide new insight into the structure of integrable equations, as well as new physical interpretations. That is why the theory of motions of curves and surfaces, including here filaments and vortices, represents an important emerging field for mathematics, engineering and physics.

The first problem about such compact systems is that shape solitons, which usually exist in infinite long and shallow propagation media, cannot survive on a circle or sphere. That is because such compact manifolds cannot offer the requested type of environment (long and narrow), even by the introduction of shallow layers and rigid cores. However, there is another basic idea which supports, in a natural way, the existence of nonlinear solutions on compact spaces. Because of its high localization, a soliton is not a unique solution for the partial differential system. Its position in space is undetermined because, far away from its center, the excitation is practically zero. On the other hand, all linear equations provide uniqueness properties for their solutions. It results that strongly localized solutions, and almost compact supported solutions can be generated only within nonlinear equations. There is an exception here: the finite difference equations with their compact supported wavelet solutions, but in some sense a finite difference equation is similar to a nonlinear differential one.

Despite the many applications and publications on nonlinear equations on compact domains, there are still no books introducing this theory, except for several sets of lecture notes. One reason for this may be that the field is still undergoing a major development and has not yet reached the perfection of a systematic theory. Another reason is that a fairly deep knowledge of integrable systems on compact manifolds has been required for the understanding of solitons on closed curves and compact surfaces.

The goal of the second edition of this book is to analyze the existence and describe the behavior of solitons traveling on closed, compact surfaces or curves. The approach of the physical problems ranging from nuclear to astrophysical scales is made in the language of differential geometry. The text is rather intended to be an introduction to the physics of solitons on compact systems like filaments, loops, drops, etc., for students, mathematicians, physicists, and engineers. The author assumes that the reader has some previous knowledge about solitons and nonlinearity in general. The book provide the reader examples of systems and models where the interaction between nonlinearities and the compact boundaries is essential for the existence and the dynamics of solitons.

We focused on interesting and recent aspects of relations between integrable systems and their solutions and differential geometry, mainly on compact manifolds. The book consists of 17 chapters, a mathematical annex, and a bibliography. First part contains the fundamental differential geometry and analysis approach. To render this book accessible to students in science and engineering, Chap. 2 recalls some basic elements of topology with emphasis on the concept of being compact. In Chap. 3 we review the representation formulas for different dimensions. The formulas express how a lot of information about the evolution of differentiable forms and fields inside a compact domain can be recovered only from its boundary. Chapter 4 introduces the reader to the calculus on differentiable manifolds, vector fields, forms, and various types of derivatives. We take the reader from map all the way to the Poincaré lemma. Next we introduce different types of fiber bundles, including the Cartan theory of frames, and the theory of connection and mixed covariant derivative (for immersions). Without always presenting the proofs, we tried though to keep a high level of rigorousness (relying on classical mathematical textbooks) all across the text while we still introduce intuitive comments for each definition or affirmation. Chapter 5 lays the basis for the differential geometry of curves in $\mathbf{R}_3$. We devote here special sections to closed curves and curves lying on surfaces. Complementary, in Chap. 6 we introduce elements of differential geometry of the surfaces with applications to the action of differential operators on surfaces. In Chap. 7 we derive the theory of motion of curves, both in two-dimensions, and in the general case. We devoted a section on the axiomatic deduction of the theory of motions based on differentiable forms and Cartan connection theory. We relate these motions with soliton solutions and find the nonlinear integrable systems that can be represented by such motions of curves. In Chap. 8 we discuss the theory of motion of surfaces, and we also relate it to integrable systems.

The second part of the monograph contains an exposition of the basic branches of nonlinear hydrodynamics. The working frame of hydrodynamics is the main content of the first part of the monograph, namely Chap. 9. In Chap. 10 we discuss problems on surface tension effects and representation theorems for fluid dynamics models. Chapter 11 concentrates with one-dimensional integrable systems on compact intervals, and their periodic solutions. Chapters 12 and 13 deal with nonlinear shape excitations of two-dimensional and three-dimensional liquid drops and bubbles. Chapter 14 is devoted to various applications of three-dimensional nonlinear drops, and also to compact supported solitons.

In the third part of the book, as a final goal for the first two parts, we present additional physical applications of nonlinear systems and their soliton solutions on various systems of different scales. In Chap. 15 we study the vortex filaments and other one-dimensional flows. In Chap. 16 we describe microscopic applications like elementary particles as solitons, instantons, exotic shapes in heavy nuclei, exotic radioactivity and quantum Hall drops. Chapter 17 deals with macroscopic applications like magnetohydrodynamic plasma systems, elastic spheres, nonlinear surface diffusion and neutron stars.

The book is closed by a mathematical annex including a section on nonlinear dispersion relations and their use for nonlinear systems of partial differential equations.

A legitimate question of the potential reader would be: "Why one more book on solitons?" First of all we have to acknowledge the importance of the interactions between compact boundary manifolds and the dynamics of particles and fields in mathematical in physical models. Historically the solitons are observed in sort of "infinite" systems like infinite long lines or curves, planes or open surfaces, or unbounded space. However, there is more and more evidence of the existence solitons or of localized patterns (like vortices) in compact lower dimensional spaces, like closed curves and/or surfaces. As examples, we can mention the unprecedent information technology advances in optical communication (light bullets and ultra-short optical pulses), solid-state spectroscopy, ultra-cold atom studies, soliton molecules, spinning solitons, quantum computers, spintronics and mass memory systems, femtosecond laser pulses, mesoscopic superconductivity, etc. Consequently, the reasons for writing this book are generated by a constantly increasing number of new challenges, vivid topics and hundreds of published articles.

If a substantial percentage of users of this book feel that it helped them to enlarge their outlook in the intersection between the fascinating worlds of nonlinear waves and compact surfaces and closed curves, its purpose has been fulfilled.

While writing the second edition of this book I have greatly benefited from discussions with my colleagues. I am particularly grateful to Ivailo Mladenov, Thiab Taha, Annalisa Calini who provided an inspirational and valuable help in the elaboration of this second edition. I am glad to mention the useful help from two of my students, Harry Wheeler and Tamika Thomas. For the best advices and uninterrupted encouragement I am indebted to my family.

Andrei Ludu
Daytona-Beach, Florida
December 2011

http://www.springer.com/978-3-642-22894-0

Nonlinear Waves and Solitons on Contours and Closed Surfaces
Ludu, A.
2012, XVIII, 490 p., Hardcover
ISBN: 978-3-642-22894-0