

2 Historical development of windmills

2.1 Windmills with a vertical axis

According to historians, the first machines utilising wind energy were operated in the orient. As early as 1,700 B.C., it is mentioned that Hammurabi used windmills for irrigation in the plains of Mesopotamia [1]. There is written evidence of the quite early utilisation of wind power in Afghanistan: Documents from 700 AD confirm that the profession of a millwright was one of high social esteem there [1]. Even today, ruins of these windmills that were running for centuries can be found in Iran and Afghanistan (cf. [Fig. 2-1](#)).

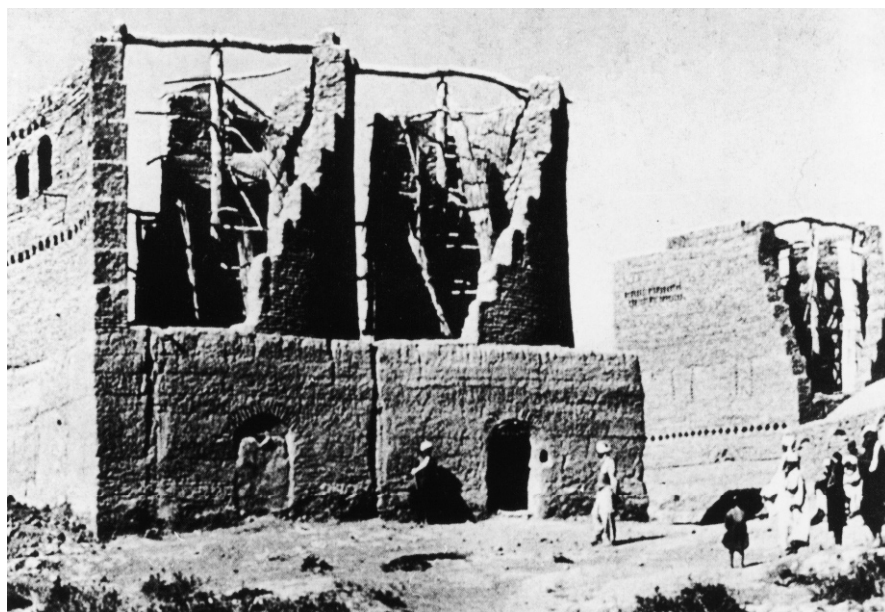


Fig. 2-1 Ruins of a vertical axis windmill in Afghanistan, 1977 [3]

The world's oldest windmills had a vertical axis of rotation. Braided mats were attached to the axis. The mats caused drag forces and, therefore, were "taken along with the wind". In Persian windmills, an asymmetry was created by screening half the rotor with a wall. This way the drag forces could be utilised for driving the rotor ([Fig. 2-2a](#)).

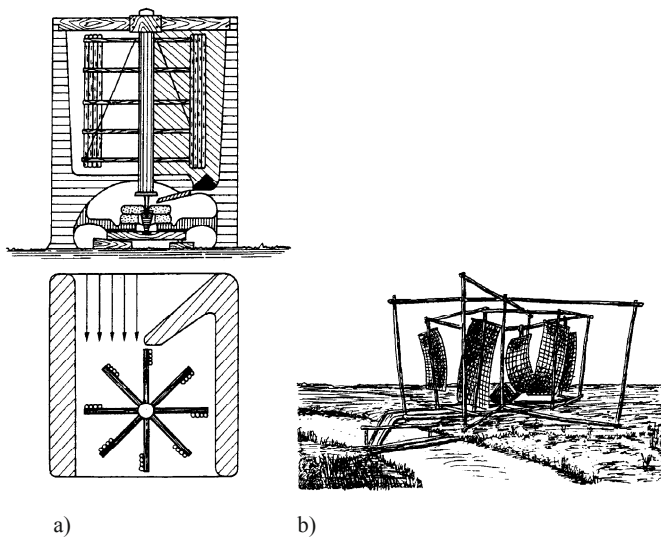


Fig. 2-2 a) Persian windmill [3] b) Chinese windmill with flapping sails [4]

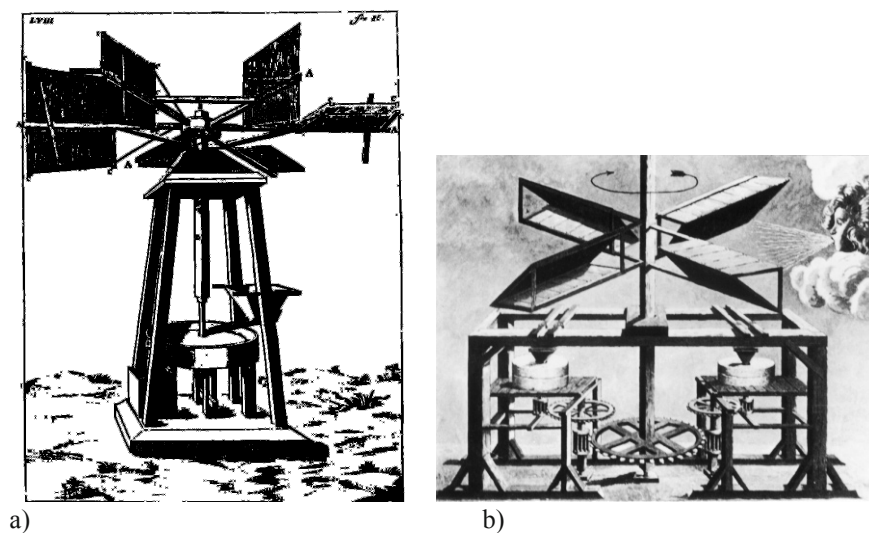


Fig. 2-3 Later design of vertical axis windmills: a) with flapping sails, France 1719 [2]; b) with bodies driven by drag forces, Italy, approx. 1600 [4]

In Chinese windmills - which also date back a long time -, a similar asymmetry is created by sails which rotate out of the wind on their way „back“, i.e. when they advance into the wind (Fig. 2-2b). These *Chinese drag wheels* date back to approx. 1000 AD. Similar to the Persian mills, they had a vertical axis and used

braided mats as „sails“. However, in contrast to the Persian mills, they had the typical advantage of vertical axis windmills to utilise the wind independent of its direction.

The simplicity of this construction can be appreciated in Fig. 2-3a which shows a later version of a vertical axis mill with flapping sails: The millstone is attached directly to the vertical drive shaft without redirecting the rotational movement and without an intermediate gear. The more recent horizontal axis windmills, such as the Dutch smock mills (Fig. 2-8), designed for a higher tip speed ratio, require a far more sophisticated construction not only for redirecting and back-gearing the rotational movement from the horizontal to the vertical axis, but also for the much more complicated bearing of the faster and heavier horizontal shaft. The windmill of Veranzio (Fig. 2.3b) like the cup anemometer (Fig. 2.19a), belongs to drag driven rotors with a relatively low tip speed ratio. For more details on the operation of these devices see section 2.3.2.

The simplicity of the vertical axis design is also central to the Savonius rotor (1924, Fig. 2.4a) and the Darrieus rotor (1929, Fig. 2.4b). But as late „occidental“ versions of the vertical axis principle they utilise - partially or exclusively - the lift force as their driving power, see 2.3.3 for a more detailed discussion.

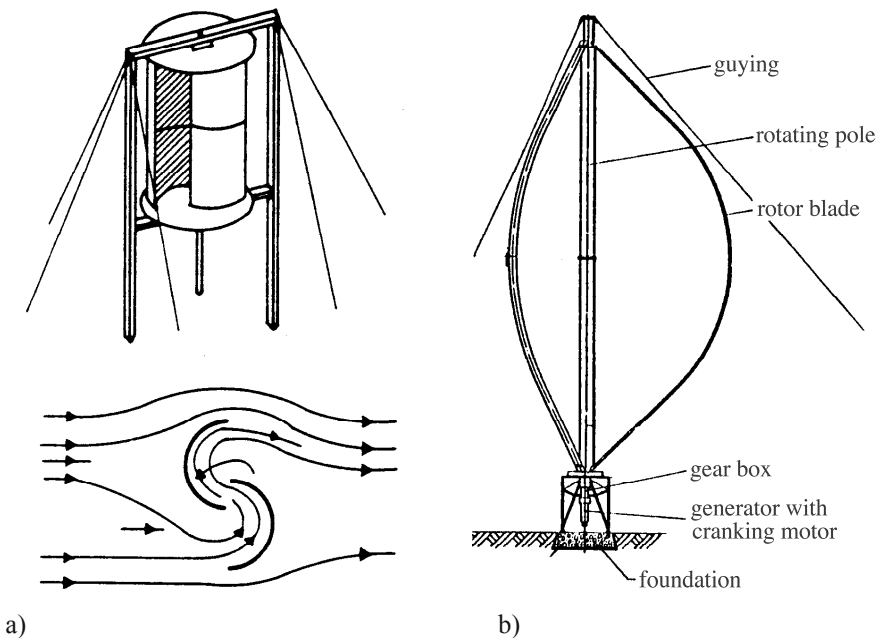


Fig. 2-4 a) Savonius rotor [5]; b) Darrieus rotor [6]

2.2 Horizontal axis windmills

2.2.1 From the post windmill to the Western mill

In the Occident, a windmill type different from the oriental vertical axis mills was developed, albeit very much later. The most prominent distinctive feature is the rotor with a horizontal axis whose sails rotate in a plane vertical to the wind, just like an aircraft propeller. In that case a driving principle different from the blade area obstruction of the drag driven rotors has to operate.

The first theoretical descriptions regarding lift forces of blades intercepting the wind, i.e. the driving power of horizontal axis devices, date back only to the beginning of the 20th century. Millwrights in earlier centuries may have used the idea that a wheel intercepts the airflow just like a screw („airscrew“).

The oldest construction of a lift-driven horizontal axis device is the *post windmill*. A picture was found in an English prayer-book from the 12th century (Fig. 2-5a). It is also mentioned at this time in the statutes of the French city of Arles (Provence). As the most important driving engine apart from the waterwheel, it spread from England and France via Holland, Germany (13th century) and Poland to Russia (14th century).

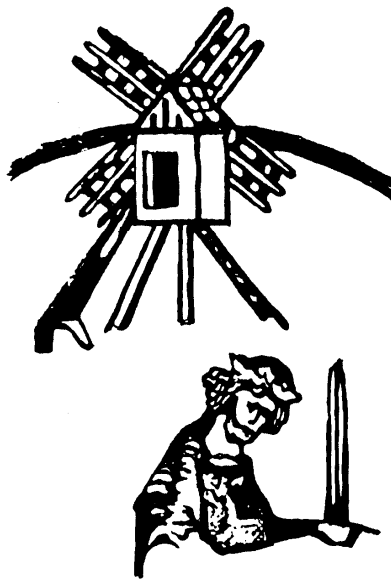


Fig. 2-5a Drawing of a post mill in an English prayer-book of 1270 [2]

It is disputed among historians who invented it and where it came from. However, there now seems to be a general agreement, „unlike previously believed, that the Crusaders did not come across windmills in Syria, but took them there themselves.“ [13] The post mill consists of a timber support holding the vertical central post around which the boxlike buck (i.e. the mill house) turns on a pivot, Fig. 2-5b. Using a tail pole, the buck together with the rotor was oriented into the wind.

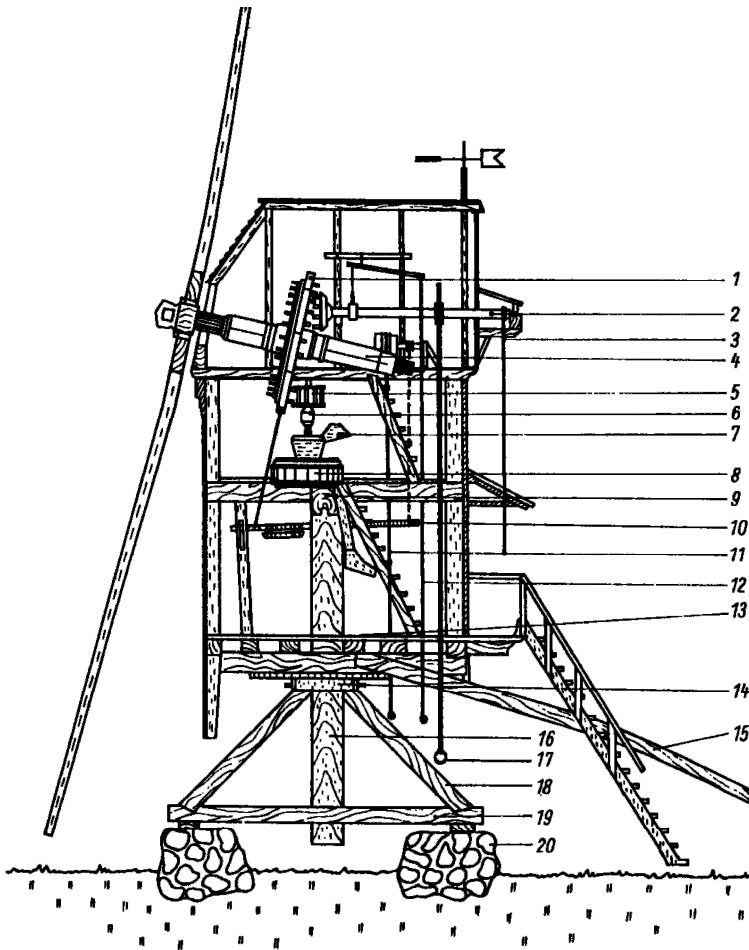


Fig. 2-5b Section of a post mill [3]

1 gear wheel with brake, 2 shaft for sack hoist, 3 hand-driven hoist, 4 rotor shaft, 5 lantern gear, 6 quant, 7 hopper, 8 millstones, 9 traverse beam, 10 brake lever, 11 brake rope, 12 hoist operating rope, 13 floor for flour, 14 saddle, 15 tailpole, 16 central post, 17 sack hoist, 18 quarter bars, 19 cross trees, 20 foundation

The main shaft with the rotor is almost horizontal. The brake wheel drives via the lantern gear the vertical shaft with the millstone. Only from the 19th Century onwards, post windmills were equipped with two lantern gears for parallel milling operations of two sets of millstones. The post windmill was exclusively applied for grinding grain.

In Holland there was an economic interest in the reclamation of land by draining the polders already in the 15th century. Therefore, first attempts were made to use the wind energy to drive pumps. The post mill had to be modified for that purpose. The driving power of the wind had to be transmitted to the pump that was situated under the mill. The result was the *wipmolen* which was first used about 300 years after the post mill was first documented, and these were especially designed for drainage purposes. The revolving mill house of a *wipmolen* contains only the gearbox (Fig. 2-6). The actual „machine“, e.g. a scoop wheel or Archimedean screw, is located below the pyramid-shaped support. The driving shaft had to be fitted through the hollow post - a masterpiece of carpenters' craftsmanship! Later on, also grain mills were built using this principle. There is the obvious advantage of having the set of stones on the ground because no longer heavy loads, like millstones and sacks with grains and flour, had to be carried up and down in the mill house.

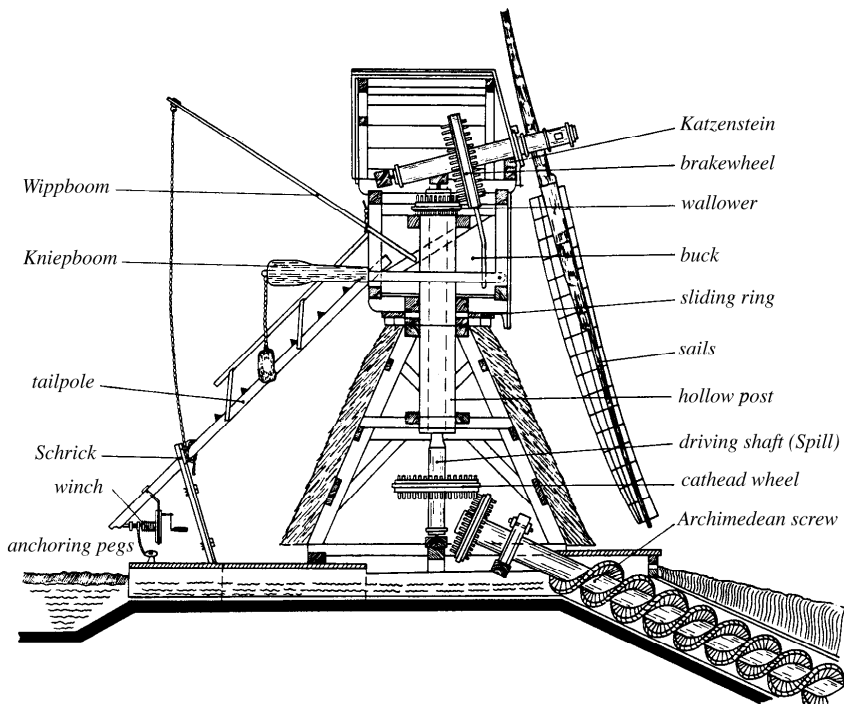


Fig. 2-6 Section of a wipmolen [7]

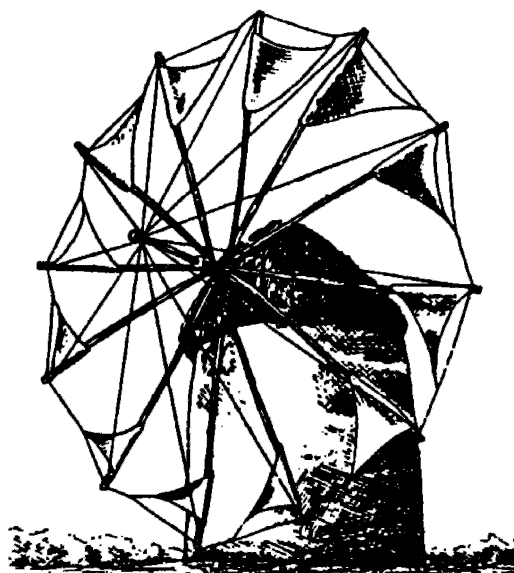


Fig. 2-7 Mediterranean tower mill with sails - an early version of the tower mill [8]

In Southern Europe, the post mill did not gain popularity. Another mill type was wide-spread there: the *tower mill*. Already very early on, the first wind mills of this kind were used for irrigation. The first documentation of these mills dates back to the 13th Century [1]. Main features of the older Mediterranean type are the cylindrical stone built mill house, a fixed thatched roof, and a guyed rotor with eight or more sails (Fig. 2-7). Later versions, mainly in Southern France, had a turnable wooden cap and a four-bladed wooden rotor like the post mills.

The turnable cap is the main characteristics of the *Dutch smock mill* which came into use in the 16th Century (Fig. 2-8). It is a further development of the tower mill as the lighter wooden construction of the octagonal tower could be easier erected on the wet Dutch marshland than the heavy stone construction of the tower mill. In Holland, the Dutch smock windmills were mainly used for the drainage of the polders, often arranged in a series to lift the water mill by mill over the embankments. In the rest of Europe, they were applied preferably for grinding grain.

With tens of thousands of Dutch smock mills being built, the use of wind power experienced its heyday in the Netherlands in the 18th and 19th Century. The large number of mills lead to a standardisation of its construction which was unusual for that time. Even in special versions such as the *gallery windmill* with its multi-storey socle (Fig. 2-9), the basic type of the Dutch smock mill can be easily recognised.

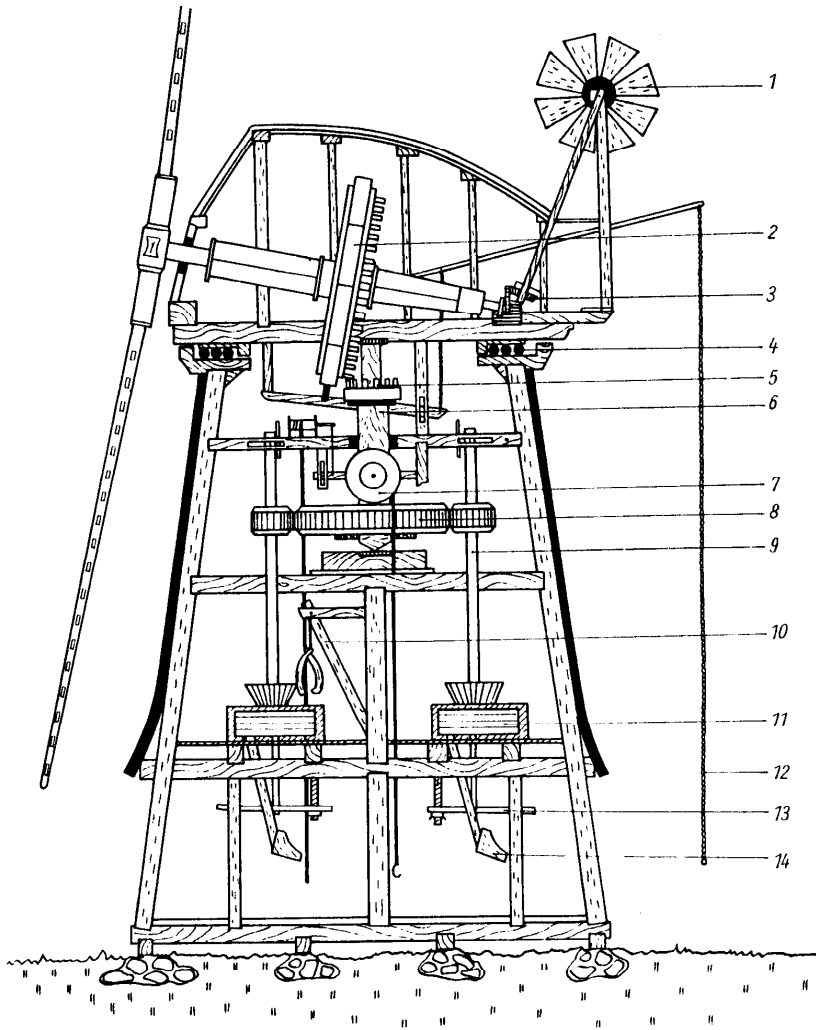


Fig. 2-8 Section of a Dutch smock mill [3]

1 fantail; 2 gear wheel with brake; 3 gear for cap rotation; 4 rollers; 5 wallower; 6 main shaft; 7 sack hoist; 8 great spur wheel; 9 spindle drive; 10 millstone crane; 11 millworks with chute; 12 brake chain; 13 stone adjustment; 14 flour chute

A somewhat exotic development is the 17th century *Paltrock mill* which shows that wind energy can be utilised universally as a driving force. The whole mill (as the cap of the Dutch smock mill) rested on a live ring. This way an entire sawmill can be driven by a windwheel (Fig. 2-11).

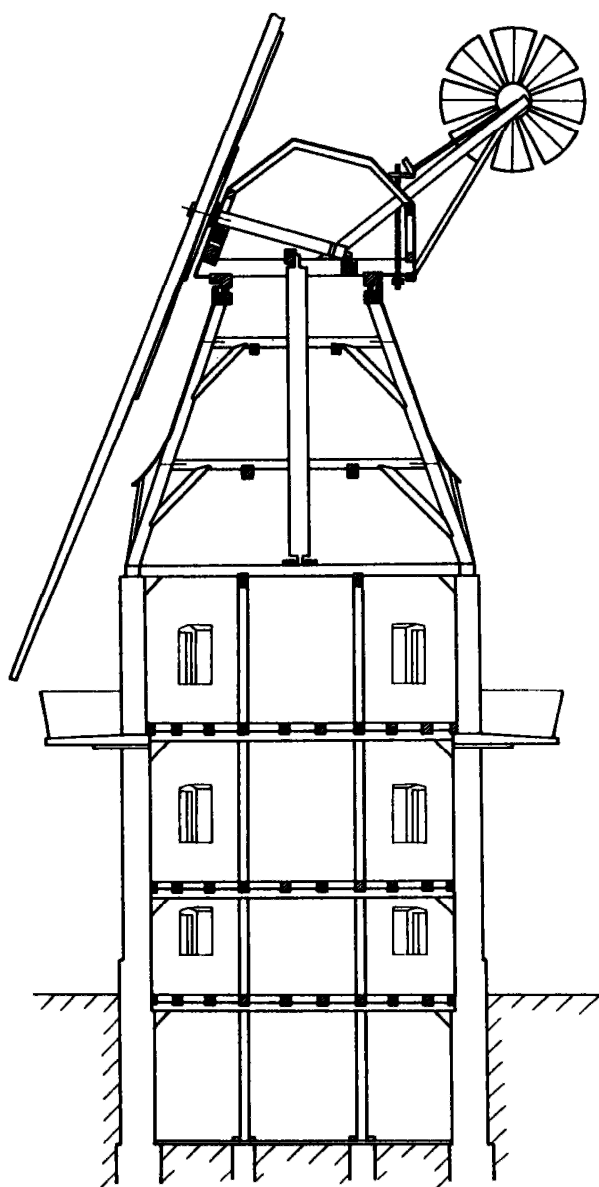


Fig. 2-9 Sketch of a gallery windmill [9]



Fig. 2-10 Western mills as wind pumping systems [10]

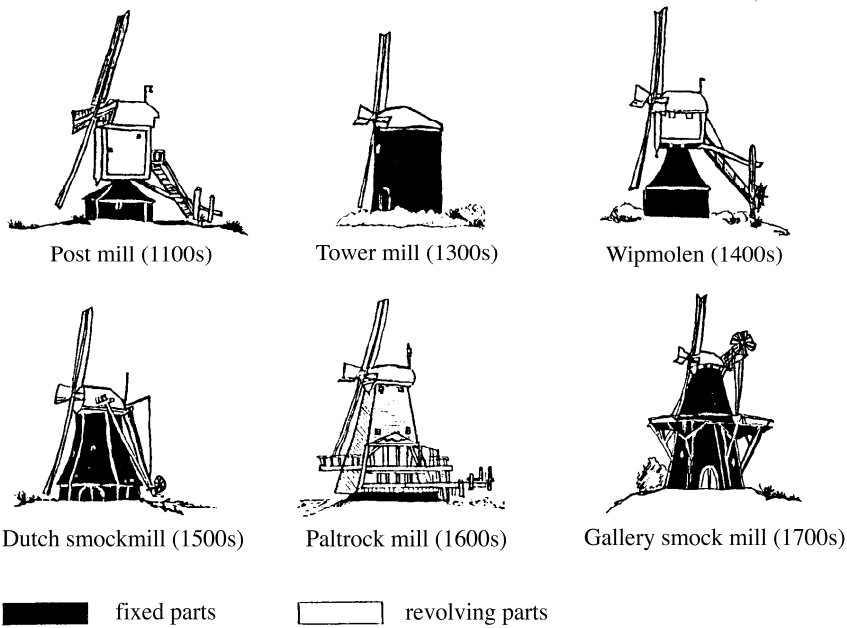


Fig. 2-11 Overview of historical horizontal axis windmill types [11]

The last type of historical windmills is the American farm windmill ('Western mill'), which was developed in the mid-19th century. The Western mill was mainly used for providing drinking water for both people and cattle in North America. Moreover, it assured the water supply for the steam locomotives of the new railways expanding into the West. The main characteristics of this windmill is the is the "rotor rosette" of a diameter between 3 and 5 m, with more than 20 metal sheet blades, situated on top of a metal lattice tower. It uses a crank shaft to drive a piston pump (Fig. 2-10).

The Western mill was the first windmill type with a fully automatically controlled yaw system including a storm control (see chapter 12). So the Western mill is still nowadays a "modern" machine of which tens of thousands are installed with a nearly unchanged design in Australia, Argentina and the USA.

2.2.2 *Technical innovations*

In contrast to the modern wind turbines, the old windmills needed the permanent attendance of a miller who was not only responsible for milling but also for the safe operation of the windmill. The main tasks in windmill operation were: adjusting the wind mill rotor to the wind direction, hoisting or reefing the sails according to the wind speed for power control and braking the rotor early enough when a storm was brewing. Only the Western mill was the first type without need for an operator.

The miller or his donkey pulled at the so-called tailpole and turned the mill into the wind. Later, winches were attached to the tailpole. This way, the tailpole could be pulled towards pegs which were situated in a circle around the mill (Fig. 2-6). Even later still, a small fantail, that was situated at right angles to the large rotor, drove the winch. Therefore, it always intercepted the wind, when there were skew winds at the rotor. This mechanism was much easier to use with Dutch smock mills as the fantail could be mounted directly to the cap (from approx. 1750, Fig. 2-12).

The adaption of the power consumption by the rotor to the prevailing wind conditions was far more crucial. For this purpose, more or less sail was on the sail frames of the windmill blade (variation of the area a , in eq. 2-13). This was a precarious issue especially when the miller under-estimated the wind, and suddenly a stronger breeze or even a storm came up and there was the danger of an over speeding of the rotor. Then, to reef the sails completely the miller had to brake the rotor as fast as possible using a wooden brake acting on the wooden gear wheel. Due to excessive frictional heat many windmills burnt down because the miller had started braking the rotor too late.

Therefore, the invention of the spring sails (in the 17th century, Fig. 2-12), which were easily controlled by a lever, brought a significant relief for the millers. They allowed braking the rotor even during a storm because even under full

operation, the shutters of the spring sails could be opened completely from inside the mill house so that the wind could blow through the blade surface.

The discovery of the blade twist was most relevant for an increase in the rotor efficiency. This innovation was studied by John Smeaton who presented the results of his wind rotor experiments to the Royal Society of England in 1759 [1]. With a sophisticated test rig (Fig. 2-13), which nowadays is replaced by the wind tunnels, he verified and improved the existing rules of wind mill construction. He recommended a blade pitch angle between the blade and the rotor plane of 18° at the rotor hub and 7° at the rotor tip. He also found out that for a given rotor diameter an increase of sail surface beyond a certain surface size no longer increases the rotor power output.

Moreover, Smeaton determined the power and the tip speed ratio - the ratio of circumferential speed at the blade tip to the wind speed - whose values were in the range between 2.2 and 4.3.

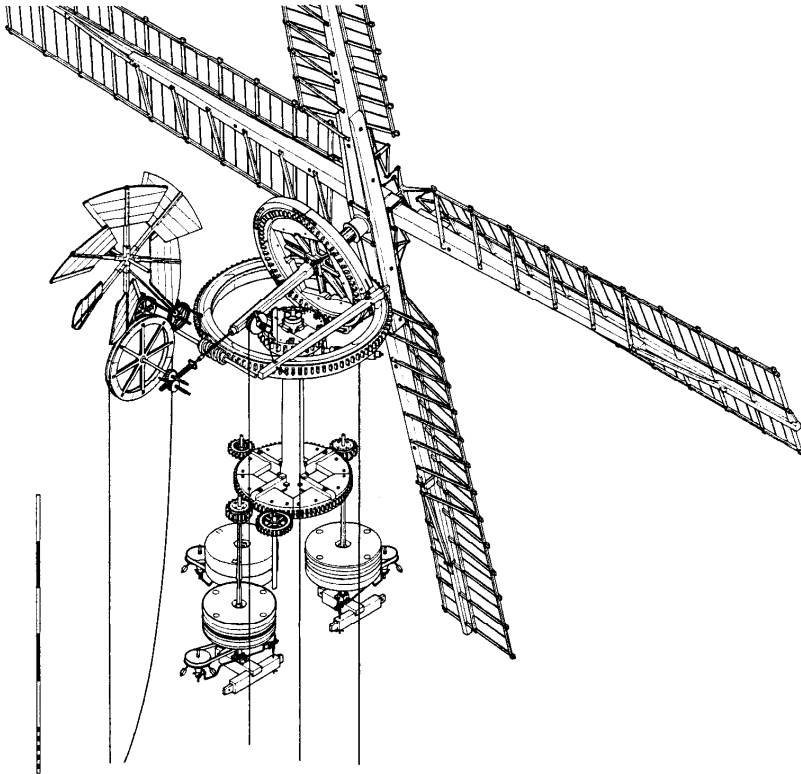


Fig. 2-12 Drive train of a large corn mill with fantail yawing, spring sails, and three sets of millstones [10]

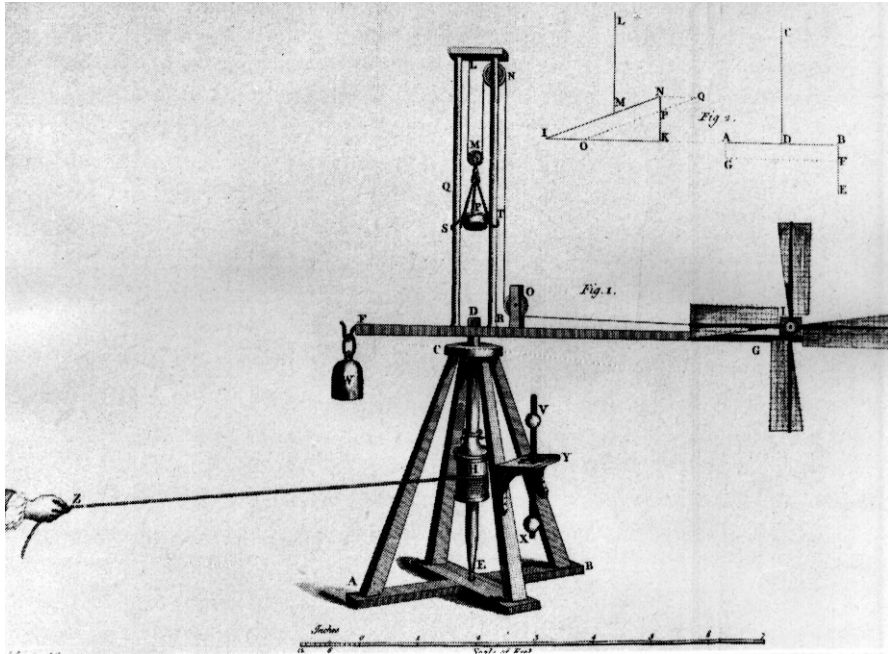


Fig. 2-13 Test rig by Smeaton for measuring the power characteristics of windmill rotors [12]

With the development of the Western mill in the 19th century, a completely new era began for the application of wind energy: it reflects the industrialisation of the wind energy application.

The Western mill was not only the first wind turbine in industrial series production and made from metal, it was the first wind turbine with a completely automatic control requiring no supervision by a human operator: A sophisticated system of wind vanes served for yawing and protecting against overspeed during stormy weather, see chapter 12. Therefore, the turbines operated self-governed on vast pasture land. Naturally, it became obvious to apply this “stand-alone” Western mill also for generating electricity, first experiments started in 1890 in the USA [20].

Paul LaCour, professor at the Askov School of engineering in Denmark, investigated since 1891 systematically the application of wind energy for electricity generation – and noticed at once that due to its low tip speed ratio the Western mill was not very suitable for this purpose. He developed a very perfect self-governing four-bladed wind turbine with a DC generator for remote dwellings. During the First World War (1914-18) more than 250 turbines of this type operated in Denmark. For more than 50 years, these “LaCour machines” were produced [21].

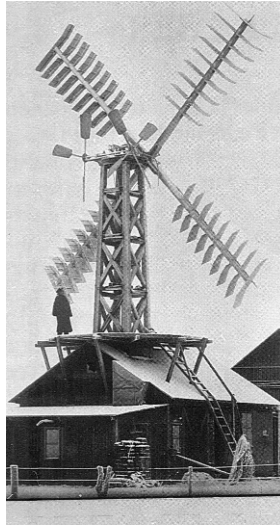


Fig. 2-14 First wind turbine for the production of direct current by Paul LaCour, Askov, Jütland, 1891 [21]

2.2.3 Begin and end of the wind power era in the Occident

From the 12th century to the early 20th century, water and wind power were the only relevant sources of mechanical energy. Braudel remarks the following [13]:

„In the 11th, 12th and 13th centurys, the Occident experienced its first mechanical revolution. We refer to ‘revolution’ as the total of all changes that were caused by the increased number of water and windmills. Though the output of these ‘primary drives’ was very limited (between 2 to 5 HP for a watermill, 5 to max. 10 HP for a windmill), in a world with a very poor power supply, this constituted a considerable increase in power and was decisive for the first growth phase of Europe.“

In the 19th century, the steam engines and combustion engines began to replace the wind and watermills. However, the low speed of the second mechanical revolution in the area of driving engines is shown by the 1895 industrial census of the German Reich [7]:

18,362 wind turbines
 54,529 water turbines
 58,530 steam engines
 21,350 combustion engines and others.

130 years after the invention of the steam engine, half of the drive units were still of traditional origin!

2.2.4 The period after the First World War until the end of the 1960s

After the First World War (1914-18) the scientific understanding of wind turbine design made a great leap forward, partially based on the experience of propeller design for military and civil aeroplanes.

In 1920 Betz applied the actuator disc theory to the wind turbine and found that a maximum of 59% of the kinetic wind energy can be converted into mechanical energy by a free-stream turbine [18]. This was previously discovered by Lanchester in England [19]. But Betz continued, linking these considerations, rooted in the theory of linear momentum and the energy conservation law, with the airfoil theory (Blade element momentum theory). This resulted in simple design rules for the blade geometry of optimised wind turbine rotors. With small modifications these basics developed by Betz are still used in wind turbine design.

With this new theoretical background for wind turbines, many promising approaches for the modern wind turbine design emerged, e.g. in France, Germany (Bilau, Kleinhenz-MAN, Honeff and others) and Russia (Sabinin, Yurieff and others). In Crimea close to Yalta, the wind turbine WIME D-30 with a diameter of 30 m and a power of 300 kW was operated from 1931 to 1942 [23], feeding into a small 20 MW grid.

But the start of the Second World War by the German National Socialists quickly disrupted these beginnings. Only in the USA the development of wind turbines was continued during the war. The engineer Palmer C. Putnam designed together with the water turbine firm “Smith” the first grid-connected Megawatt wind turbine ($D = 53$ m; 1,250 kW). Well-known scientists participated in the development of its concept (Fig. 2-15 d). It was commissioned in 1941 and operated until 1945. Unfortunately the economic balance showed that the power production costs were 50% higher than that for the conventional power generation. Therefore, the improvements of the technical concept proposed by Putnam were not put into practice.

With the reconstruction of Europe after the Second World War and the growing realisation that coal reserves were continuously decreasing, interest in wind energy application arose again in the 1950s. Through the „Organisation for European Economic Cooperation (OEEC), Working Group 2”, experts from England (Golding), Denmark (Juul), Germany (Hütter), France (Vadot) and others met to discuss their experiences in the wind turbine design.

In Germany, Hütter followed with the “Research Association Wind Power” a very innovative turbine concept, leading in 1958 to the prototype W34 ($D = 34$ m, 100 kW). It had fibre glass blades, an electro-hydraulic pitch control and produced 50 Hz AC power with a synchronous generator, Fig. 2-15c. A teetering hub dampened the dynamics of this two-bladed rotor. With many interruptions this wind turbine operated until 1968.



a) Gedser wind turbine
(200 kW, $D = 24$ m, DK 1957)



b) TVIND wind turbine
(2,000 kW, $D = 54$ m, DK 1977)



c) Hütter wind turbine
(100 kW, $D = 34$ m, D 1958)



d) Smith-Putnam wind turbine
(1,250 kW, $D = 53$ m, USA 1941)

Fig. 2-15 Historical wind turbine prototypes

Johannes Juul in Denmark followed a completely different course. His aim was a simple and robust turbine concept for the grid connection. He was a “wind electrician” trained by Paul LaCour in Askov. Later, he became the leader (Linjemester) of the grid expansion division of the Sjaelandic electricity supplier (SEAS). With SEAS he erected the famous Gedser wind turbine (Fig. 2-15a, $D = 24$ m, 200 kW). This wind turbine provided electricity to the grid for thousands of working hours from 1957 to 1962.

His electrical concept was ingenious: an asynchronous motor was pushed by the rotor into the super synchronous rotational speed range and thus became a generator without any effort of synchronisation. The rotor, although of simple design (plywood profiles on a guyed steel spar), had a skilful aerodynamic design which caused flow separation in the strong wind range acting as a completely passive power limitation. The turnable blade tips served as braking flaps and were activated by centrifugal forces at a grid failure.

However, at the beginning of the 1960s, cheap oil from the Near East came to Europe. The calculations by Juul himself showed that the wind generated electricity was too expensive to compete with the conventional fossil generation. This caused the breakdown of the second break-up.

2.2.5 The Renaissance of the wind energy after 1980

The oil price shocks in 1973 and 1978 initiated again a reflection on the future energy supply. In 1977, even the Gedser wind turbine of J. Juul was re-activated for research purposes and again coupled to the grid.

But the Renaissance of wind energy began with a tremendous crash. In the USA, Germany, Sweden and some other countries, supported by the governments, giant wind turbines were designed by the aerospace industry, Fig. 2-16. Nearly all of them failed after some hundred hours of operation due to technical problems: too early, too big and too expensive.

An exception was the Maglarp wind turbine WTS-3 ($D = 78$ m, 3.000 kW) which operated grid-connected more than 20,000 hours, and also the Tvind wind turbine, developed by “amateurs”, Fig. 2-15b, which is still operating today (but with two thirds of its original rated power).

In contrast to this, the small Danish manufacturers of agricultural machines (Vestas, Bonus, Nordtank, Windworld, etc.) were very successful in the beginning of the 1980s with wind turbines consisting of a rotor diameter between 12 and 15 m produced in series and equipped with an asynchronous machine, according to the concept of Juul. However, the blades of these turbines were manufactured with fibre glass, following Hütter’s blade design. With a rated power of 30, 55 or 75 kW they were technically and economically successful because an appropriate feed-in tariff was set and granted by the Danish government. This first small market grew continuously.

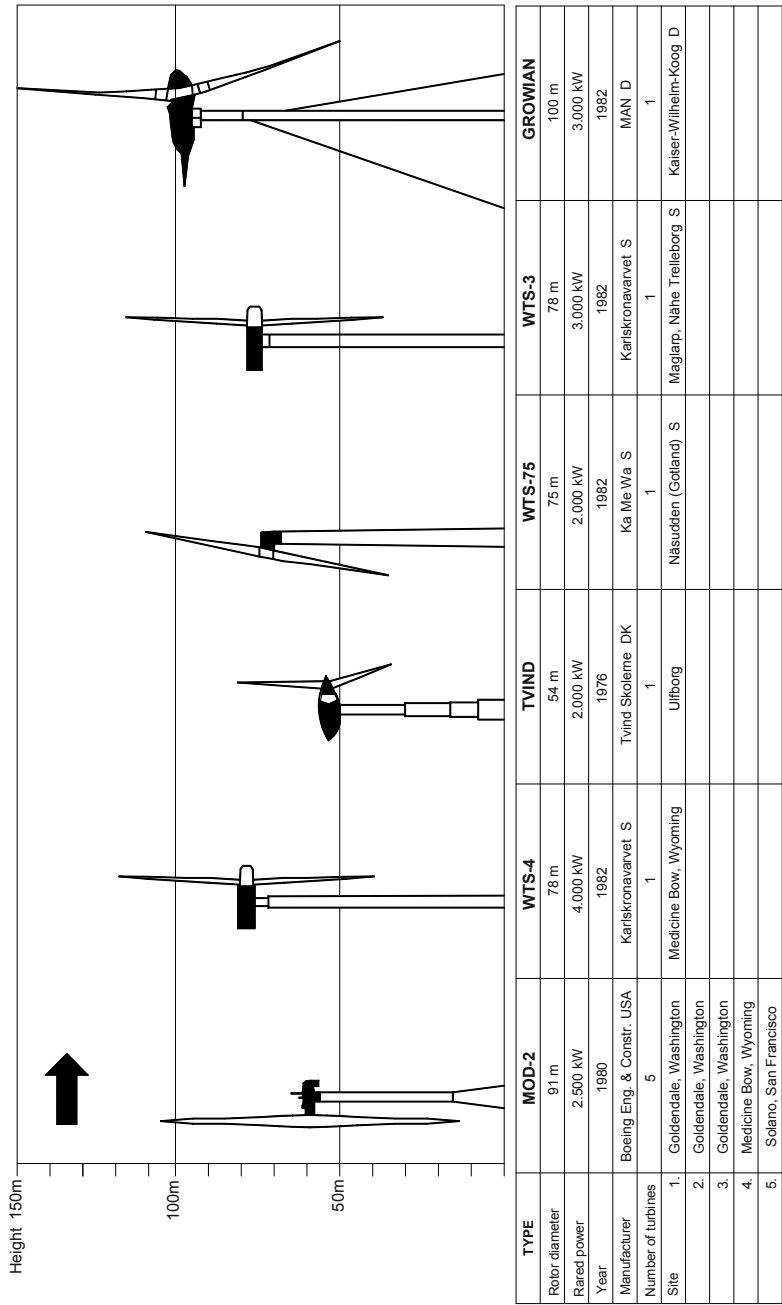


Fig. 2-16 The Multi-Megawatt class in the beginning of the 1980s [24]

Today, after more than 30 years of continuous research and development on the wind turbines these formerly “small-scale manufacturers” produce successfully in the rotor diameter range from 80 to 126 m, where formerly the aerospace industries failed.

2.3 The physics of the use of wind energy

2.3.1 Wind power

The power of the wind that flows at a velocity v through an area A is

$$P_{\text{wind}} = \frac{1}{2} \rho A v^3. \quad (2.1)$$

It is proportional to the air density ρ , the cross sectional area A (perpendicular to v) and the third power of the wind velocity v . The third power of the wind velocity can be explained as follows: the power P_{wind} in the wind is the kinetic energy

$$E = \frac{1}{2} m v^2 \quad (2.2)$$

of the air mass m , passing through the area A in a given time. Since the resulting mass flow

$$\dot{m} = A \rho \frac{dx}{dt} = \rho A v \quad (2.3)$$

itself is proportional to the wind velocity (Fig. 2-17), the power (energy per unit of time) is expressed by

$$P_{\text{wind}} = \dot{E} = \frac{1}{2} \dot{m} v^2 = \frac{1}{2} \rho A v^3. \quad (2.4)$$

The power of the wind is converted into mechanical power of the rotor by deceleration of the flowing air mass. On one hand, it cannot be converted completely, since this would decelerate the mass flow to zero, it would block the cross sectional (rotor) area A for the following air masses.

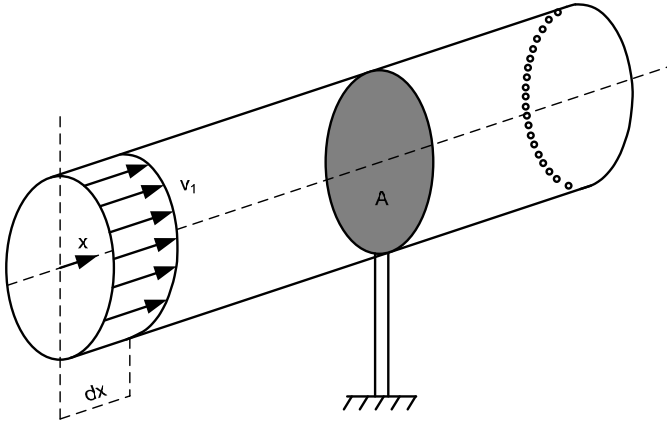


Fig. 2-17 Scheme of a stream tube with mass flow through the cross sectional area A

On the other hand, if the air flows through the area without any deceleration of the wind velocity, there would be no power conversion as well. So there must be an optimum of wind energy conversion through flow deceleration between these two extremes.

Betz [14] and Lanchester [19] discovered that the maximum power is extracted by a free (i.e. unshrouded) wind turbine if the original upstream wind velocity v_1 is reduced to a velocity $v_3 = v_1/3$ far downstream the rotor.

Then, the resulting velocity in the rotor plane is $v_2 = 2v_1/3$ (Fig. 2-18). In that case of a theoretically maximum power extraction, the result is

$$P_{\text{Betz}} = \frac{1}{2} \rho A v^3 c_{P,\text{Betz}} \quad (2.5)$$

with the maximum power coefficient $c_{P,\text{Betz}} = 16/27 = 0.59$. Even in this best case of power extraction without any losses, only 59 % of the wind power is extractable.

Real power coefficients c_p are lower. For the drag driven rotors they are below $c_p = 0.2$, and for lift driven rotors with good airfoil profiles they reach up to $c_p = 0.5$. Betz' theory is discussed in detail in chapter 5.

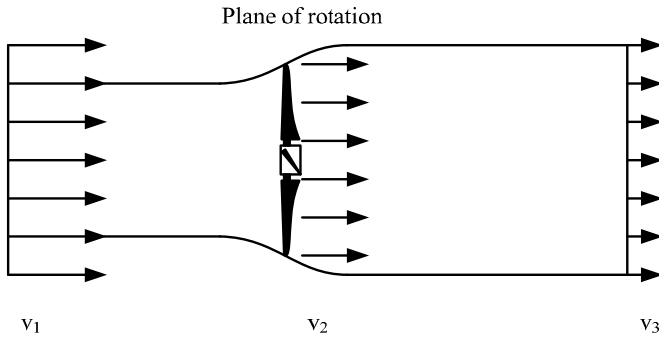


Fig. 2-18 Air flow through the rotor of a wind turbine, divergence of the stream tube resulting from the flow deceleration

2.3.2 Drag driven rotors

The drag devices utilise the force that acts on an area a perpendicular to the wind direction, Fig. 2-19, This force, referred to as

$$D = c_D \frac{\rho}{2} a v^2 \quad (2.6)$$

is proportional to the area a , to the air density ρ and to the square of the wind velocity v .

This formula is also used to describe the drag of other bodies placed in a flow, where a is their projected area perpendicular to the flow velocity.

The drag coefficient c_D is the proportional constant and describes the „aerodynamic quality“ of the body: the higher the aerodynamic quality of a body, the lower is c_D and thus the corresponding drag force (Fig. 2-19).

The torque, rotational speed and power of the early Persian (or Chinese) vertical axis windmill, using the drag principle, can be easily estimated, based on the assumption that the torque of the simplified model shown in Fig. 2-20b is equivalent to that of the real windmill in Fig. 2-20a. The simplified model neglects the coming and going of the blades and the effect of the preceding and the following blades, respectively.

In that case, the actual relative velocity $w = v - u$ at the rotating plate is the difference between wind velocity v and the circumferential velocity $u = R_M \Omega$ at the mean radius R_M of the area a . The angular velocity due to the rotational speed n is $\Omega = 2 \pi n$.

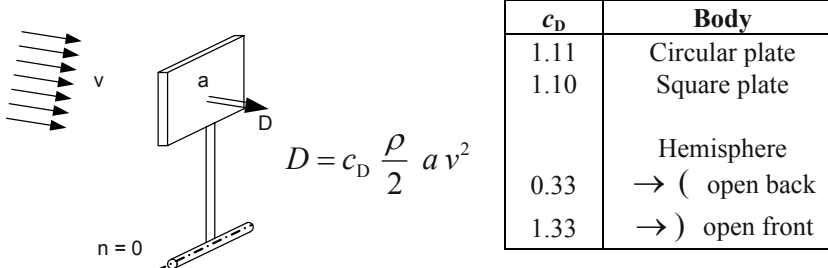


Fig. 2-19 Drag force on a plate and drag coefficients c_D for some typical bodies

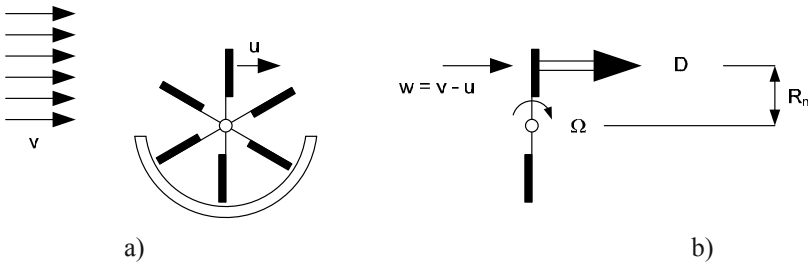


Fig. 2-20 a) Principle of a Persian windmill, b) simplified model

Thus, the resulting drag force on the rotating plate is

$$D = c_D \frac{\rho}{2} a w^2 = c_D \frac{\rho}{2} a (v - u)^2. \quad (2.7)$$

Hence, the mean driving mechanical power – which in reality is slightly pulsating – amounts to

$$P = D u = \frac{\rho}{2} a v^3 \left\{ c_D \cdot \left(1 - \frac{u}{v} \right)^2 \cdot \frac{u}{v} \right\} = \frac{\rho}{2} a v^3 c_P. \quad (2.8)$$

This driving power in eq. (2.8) is - like the power in the wind - proportional to the projected area a and to v^3 , the cube of the wind velocity.¹ The expression in the braces is equal to the power coefficient c_P , the aerodynamic efficiency of the rotor. It gives the portion of the wind power which is converted into mechanical power.

¹ Usually, the reference area is the swept rotor area A , not the projected „drag“ area a of this simplified consideration. The area A would be in this case the rotor height multiplied by half the rotor diameter (unshaded side). The resulting $c_P(\lambda)$ would even be lower by the factor a/A .

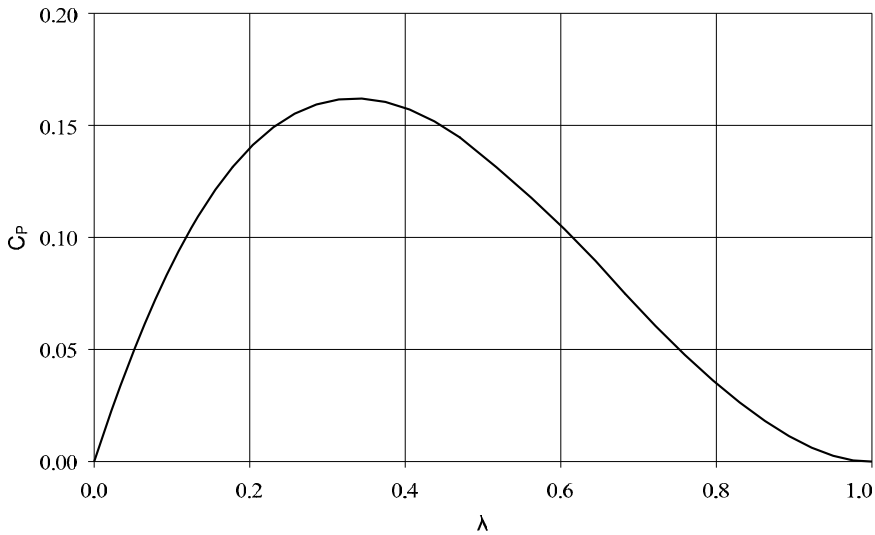


Fig. 2-21 Power coefficient versus tip speed ratio $\lambda = \Omega R_M / v$ of the Persian windmill (approximation for the simplified model)

This coefficient must be lower than the theoretical maximum value $c_{p,\text{Betz}} = 0.59$ determined by Betz. It depends on the ratio tip speed ratio $\lambda = u / v$, which was introduced in chapter 1, of the circumferential velocity $u = \Omega \cdot R_M$ to the wind velocity v .²

For a given wind velocity v , the diagram of $c_p(\lambda) = c_p(\Omega \cdot R_M / v)$ shows which portion of the wind power ($\rho/2$) $a v^3$ can be extracted. It depends on the circumferential speed u , respectively the angular velocity Ω (i.e. the rotational speed n).

Fig. 2-21 shows such a diagram for the simplified model of the *Persian windmill* (Fig. 2-20) using the drag coefficient $c_D = 1.1$ of the square plate. At complete standstill ($\lambda = 0$) no mechanical power is extracted from the wind. Neither it is at idling with maximum rotational speed ($\lambda = \lambda_{\text{idle}} = 1$), where the circumferential velocity is equal to the wind velocity. In between these extreme cases, the maximum power coefficient $c_{p,\text{max}} \approx 0.16$ is reached at a tip speed ratio of about $\lambda_{\text{opt}} \approx 0.33$. Merely 16% of the wind energy can be converted to mechanical energy.

Even worse is the power output of the *cup anemometer* (Fig. 2-22): On the “way back into the wind”, the cup has to be pushed against the drag resulting from the relative velocity $w = v + u$, causing additional losses.

² For the horizontal axis machines which are the main topic of this book, the tip speed ratio is defined as ratio of the circumferential velocity at the blade tip to the undisturbed upstream wind velocity.

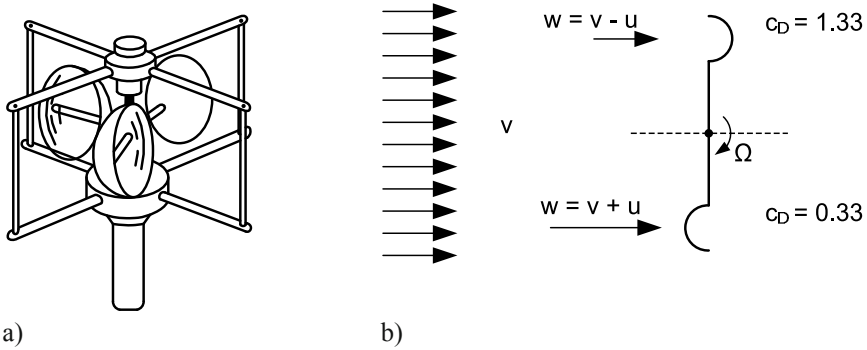


Fig. 2-22 a) Cup anemometer, b) simplified model

The aerodynamic efficiency of this “wind turbine” is roughly calculated with a similar simplified model (for the drag coefficients of the hemispheres see Fig. 2-22). From the difference of the driving (dr) drag force

$$D_{\text{dr}} = c_{D,\text{dr}} \frac{\rho}{2} a w^2 = 1.33 \frac{\rho}{2} a (v - u)^2 \quad (2.9)$$

and the slowing (sl) drag force

$$D_{\text{sl}} = 0.33 \frac{\rho}{2} a (v + u)^2 \quad (2.10)$$

results the mechanical power

$$P = (D_{\text{dr}} - D_{\text{sl}}) u = \frac{\rho}{2} a v^3 \left\{ \lambda \left(1 - 3.32 \lambda + \lambda^2 \right) \right\}. \quad (2.11)$$

Again, the expression in the braces corresponds to the power coefficient $c_P(\lambda)$. Its maximum of $c_{P,\text{max}} \approx 0.08$ (at $\lambda_{\text{opt}} = 0.16$, Fig. 2-23) is even lower than the one of the Persian windmill (Fig. 2-21). Therefore, this type of “wind turbine” is not used for extracting power from the wind. But running idle, it is suitable for measuring the wind velocity (see chapter 4). The tip speed ratio $\lambda_{\text{idle}} \approx 0.34$ gives the “calibration factor” between rotational speed n and wind velocity v , since from $\lambda = \Omega R_M / v = 2\pi R_M n / v$ results

$$v = \Omega \left(\frac{R_M}{\lambda_{\text{idle}}} \right) = 2\pi \left(\frac{R_M}{\lambda_{\text{idle}}} \right) n. \quad (2.12)$$

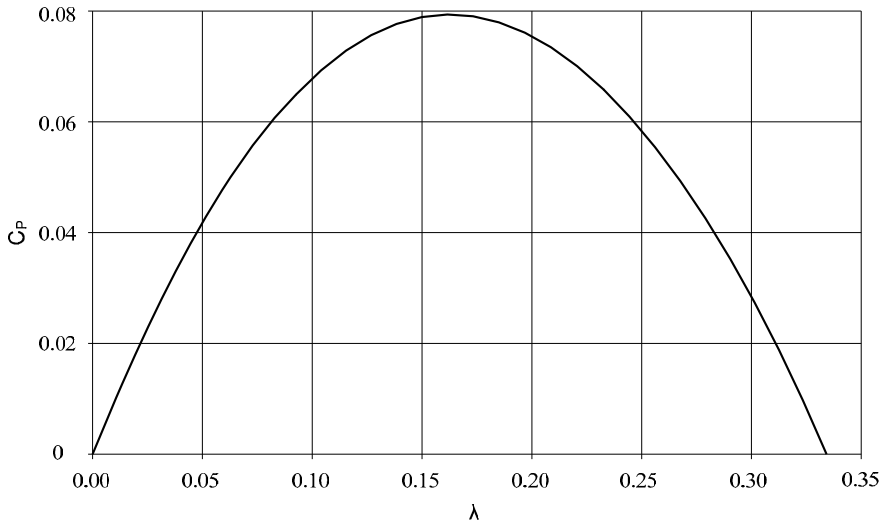


Fig. 2-23 Power coefficient versus tip speed ratio of a cup anemometer (approximation for the simplified model)

This roughly calculated value of $\lambda_{idle} \approx 0.34$ corresponds quite well to the values found in measurements [16]. But for wind measurements, each anemometer should be calibrated correctly in a wind tunnel.

2.3.3 Lift driven rotors

For many bodies, like airfoils or also the flat inclined plate, the force resulting from the attacking flow has not only a drag component D parallel to the direction of the flow velocity w , but also a component L perpendicular to it (Fig. 2-24), the lift force

$$L = c_L \frac{\rho}{2} a w^2. \quad (2.13)$$

Similar to the drag force, it is proportional to the projected area $a = c b$ and the dynamic pressure $(\rho / 2) w^2$. For small angles of attack α_A the lift force L acts at approx. a quarter of the cord length c behind the leading edge.

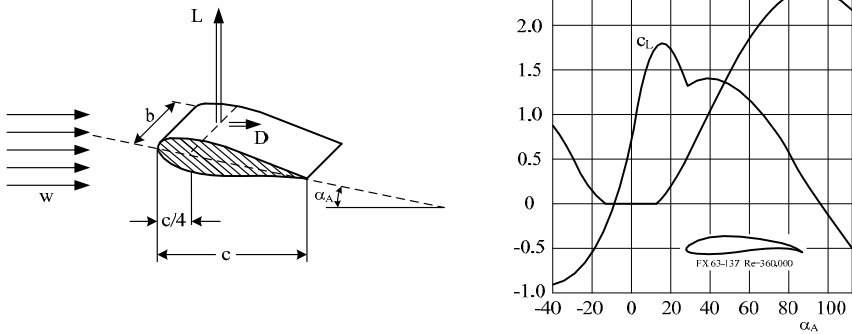


Fig. 2-24 Lift force L and drag force D of an airfoil and the corresponding coefficients c_L and c_D versus angle of attack α_A

Fig. 2-24, right, shows that in the range of small angles of attack (approx. $\alpha_A \leq 10^\circ$), the lift coefficient c_L - and hence the lift force - is directly proportional to the angle of attack:

$$L = c_L(\alpha_A) \frac{\rho}{2} a w^2, \quad (2.14)$$

$$\text{with } c_L(\alpha_A) \approx c_A' \alpha_A \text{ for } \alpha_A \leq 0.1745 (\approx 10^\circ).$$

In the case of an ideal thin rectangular plate of infinite width b , c_A' is 2π . Real values of actual profiles are slightly lower, approx. $c_A' \approx 5.5$.

Of course, there is also a drag force D existing, but it is very small for good quality aerodynamic profiles in the range of a small angle of attack:

$c_D = (0.01 \dots 0.20) c_A$. Only beyond approx. $\alpha_A = 15^\circ$ the drag coefficient begins to grow drastically, Fig. 2-24, right.

The lift L_V force is the driving force of the wind turbines based on the lift principle. In order to distinguish them clearly from drag driven rotors discussed in the previous section 2.3.2, the basic principle is explained using the example of the Darrieus rotor (Fig. 2-25; cf. Fig 2-4b). Though being a turbine working with the lift principle, the Darrieus rotor has a vertical axis, which is typical for the drag driven rotors; and it has a tip speed ratio (i.e. the ratio of the circumferential blade velocity to the wind velocity) significantly higher than the discussed drag driven rotors reaching at maximum $\lambda_{\max} = 1$.

At the Darrieus rotor, due to its high tip speed ratio of 4 or 5, the two blades considered in Fig. 2-25 are attacked by the relative velocity nearly tangentially at their leading edge. The lift force is very much higher than the drag force, and therefore also the relevant force (L and L') driving the rotor. By definition, the lift force is perpendicular to the relative velocity and causes with the levers (h and h') the necessary driving torque.

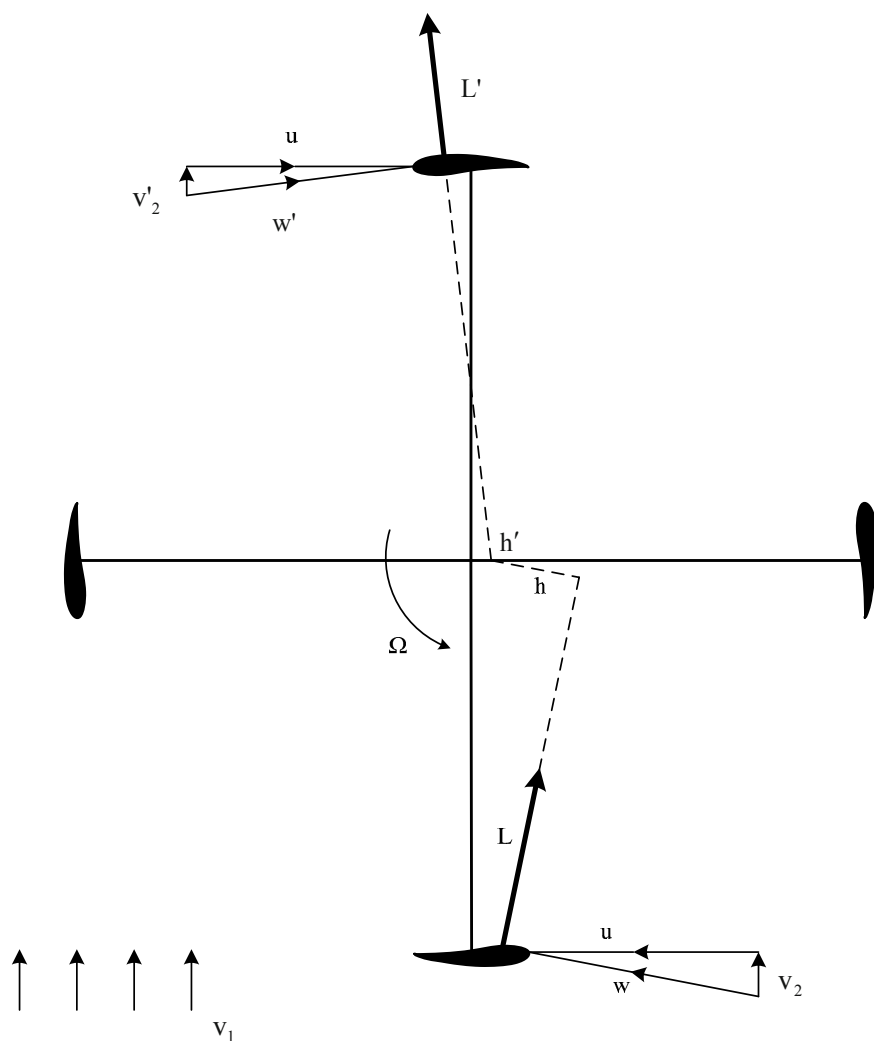


Fig. 2-25 Lift forces L and L' at the Darrieus rotor producing the driving torque

All horizontal axis wind turbines, like the post windmill, the Dutch smock mill or the Mediterranean sail windmill as well, are driven by the lift principle (Fig. 2-26). Their power coefficients are in the range of $c_{p,\max} \approx 0.25$ and therefore significantly higher than the maximum values of the drag driven rotors. Modern horizontal axis wind turbines with good aerodynamic profiles (which show small drag coefficients) reach power coefficients up to $c_{p,\max} = 0.5$. So they are already very close to the limit value of $c_{p,\text{Betz}} = 16 / 27 = 0.59$ found by Betz and Lanchester.

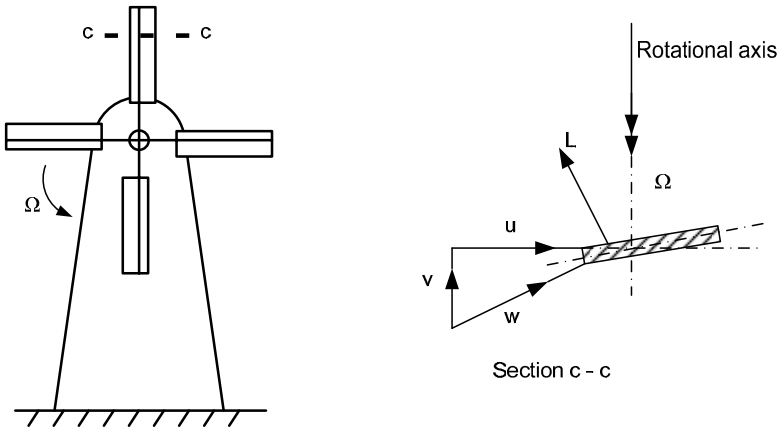


Fig. 2-26 Horizontal axis windmill driven by the lift force L

2.3.4 Comparison of rotors using drag principle and lift principle

The theoretical consideration of Betz and Lanchester that the maximum 59% of wind power is extractable leaves it completely open how the power is extracted in the rotor plane (Fig. 2-18). From this point of view, the oriental drag driven rotor and the occidental lift driven rotor are equally suitable for the wind energy utilisation.

Only a closer analysis reveals, why already Smeaton in 1759 measured for Dutch smock mills a $c_{P,\max} = 0.28$, and by the application of modern high-lift profiles a power coefficient of $c_{P,\max} = 0.50$ is attainable. Whereas the approximation for the drag driven rotors yields only $c_{P,\max} = 0.16$, as calculated in section 2.3.2. What is the cause for the better performance of the lift driven turbines?

The reason is the different magnitude of the aerodynamic forces attainable with the same blade area a . The maximum aerodynamic coefficients $c_{W,\max}$ and $c_{A,\max}$ are in the same order (Fig. 2-24 and 2-27), but there is a fundamental difference in the magnitudes of the attacking relative velocity w . For the drag driven rotor, the relative velocity $c = v - u = v \cdot (1 - \lambda)$ is always lower than the wind velocity since it is reduced by the circumferential velocity u .

The lift driven rotor has a relative velocity w that results from the geometrical addition of wind velocity v and circumferential velocity u which means it is always higher than the wind velocity: $w = (v^2 + u^2)^{1/2} = v^2(1 + \lambda)^{1/2}$. Depending on the tip speed ratio it achieves the multiple of the wind velocity.

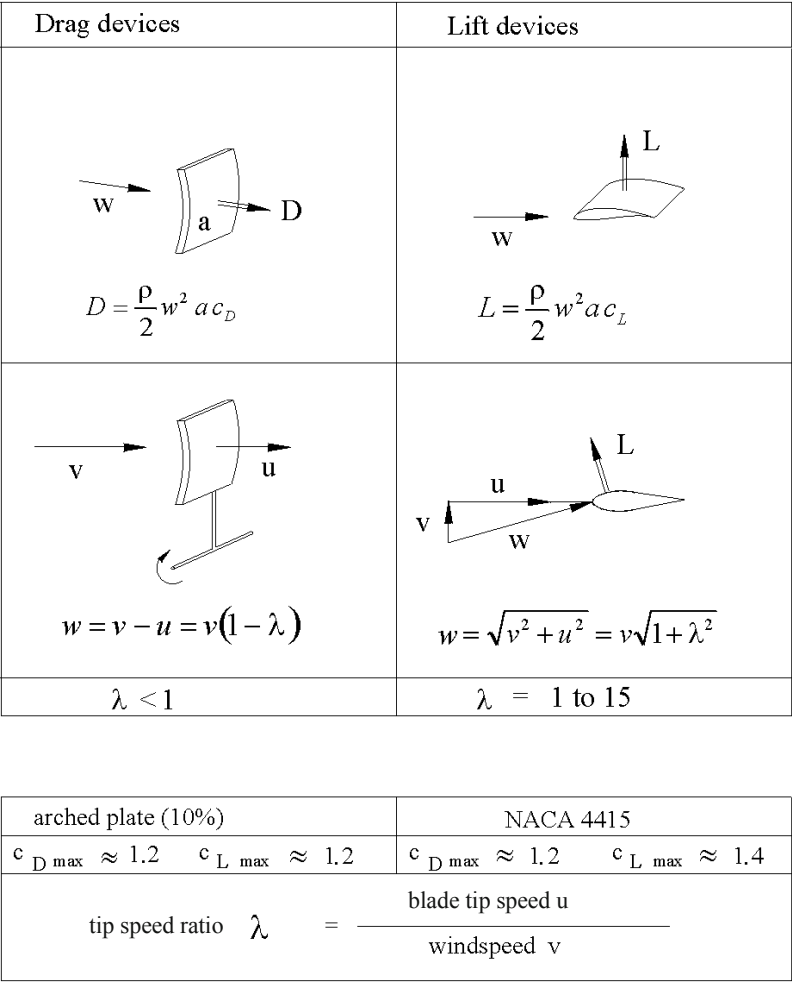


Fig. 2-27 Comparison of rotors driven by drag and lift

Therefore, with the same area *a*, the magnitude of the aerodynamic force - being proportional to the square of the attacking relative velocity - of the lift force driven rotor is a multiple of the force attainable by the drag force driven rotors. The aerodynamic forces obtained by the drag principle in the “active rotor plane” (see Fig. 2-18) are too small to come even a little close to the optimum extractable power of 59%. The fact that also the lift driven rotors do not completely attain the ideal power coefficient is due to some losses under real flow conditions, neglected in the considerations of Betz and Lanchester (see chapter 5).

It is worthwhile noting that the lift principle which is the basic principle of all wind turbines with a horizontal axis of rotation – from the post windmill to the Western mill - was used intelligently and efficiently for more than 700 years without

being explained by a technical and physical theory. Still in 1889, Otto Lilienthal correctly notes: “Technical handbooks give for this kind of aerodynamic drag force (the technical term Lilienthal uses for lift and aerodynamic forces in general - the author) such formulae which are mostly the result of theoretical considerations and are based on assumptions which cannot be found in real-life conditions.”

The ideas of physicists regarding the fluid mechanics of the lift force were wrong (e.g. Newton 1726 and Rayleigh 1876, Fig. 2-28). The estimations of Lilienthal based on bird flight and his subsequent experiments showed how powerful the lift forces of plane and cambered plates with a small angle of attack really were.

Only in 1907, long after the glides of Lilienthal and four years after the first successful motorized flights of the Wright brothers, Joukowski using the potential theory, found a sufficient theoretical explanation for the success of the practitioners, e.g. the millwrights and aircraft constructors.

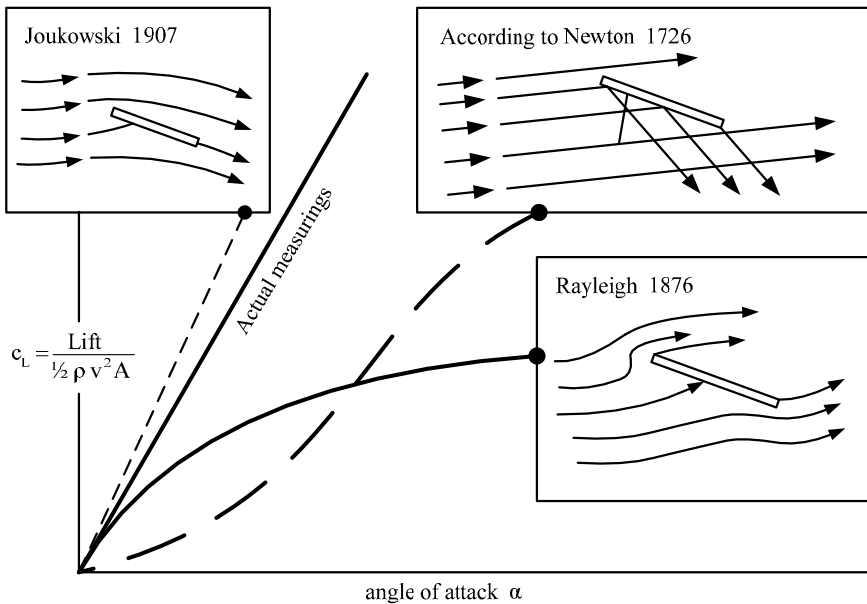


Fig. 2-28 Historical theories on the aerodynamic lift force, [17]

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