

Preface

Fractional Order Calculus has existed for over 300 years as an abstract mathematical concept. A renewed interest among the research community has come in the last few decades as fractional calculus has found applications in numerous science and engineering disciplines. Signal processing and control systems are two broad disciplines where fractional calculus has percolated to a substantial extent. Though there are a few books on fractional order calculus and fractional control systems, there are none on Fractional Order Signal Processing (FOSP). The idea of writing this book is to serve as a starting point for researchers interested in FOSP techniques.

Conventional Digital Signal Processing (DSP) deals with rational pole-zero models of FIR/IIR structures and looks at stability, causality, etc. of these models. Many physical signals and other phenomena have been shown to possess inherently fractional order dynamics and hence fractional calculus is naturally able to model these processes with greater accuracy. Hence we believe that fractional order signal and system theory will play a pivotal role in the coming eras of scientific development.

Theory of Statistical Signal Processing developed in parallel with DSP to analyze and handle signals with stochastic disturbances. However the major contributions in the field of Adaptive and Statistical Signal Processing have implicitly assumed the underlying probability distribution of the random variables to be Gaussian. Though this assumption works for some simplified cases, it does not deal with all possible real world scenarios where the distribution is inherently non-Gaussian. Among the many non-Gaussian random variables, a family of stable distributions which are capable of modeling signals with bursty nature have been found to effectively model many real world phenomena. These stable distributions can also be modeled from the perspective of fractional calculus and serve as the building blocks of fractional order Statistical Signal Processing.

Fourier transform and conventional filtering in Laplace (s) or z domain are widely used techniques in DSP. Fractional Fourier Transform (FrFT) extends this notion of frequency domain analysis to a more generalized time–frequency representation of the signal and can be used to separate the undesired component

from the desired signal, even in cases where the integer order Fourier transform techniques fail. Classical Fourier Transform finds application in analyzing the desired and undesired components of a corrupted signal and the frequency domain information of a signal is usually used to design analog or digital filters. The flexibility of FrFT of any arbitrary order and filters with fractional pole-zero model together have opened new horizons in filter design theory.

Most of the literatures on FOSP are scattered in different research articles, technical reports and monographs. This book tries to assimilate these diverse topics and briefly present these concepts to the uninitiated researcher. The focus is on trying to cover a broad range of topics in FOSP and hence detailed discussion regarding each individual topic is out of scope of this book. However, this book can serve as a starting point and the researcher can delve into his topic of interest by following up the extensive list of references that have been included at the end of each chapter. An outline of the contents of the individual chapters is described next.

Chapter 1 gives a brief introduction to Generalized Non-integer Order Calculus. The basic definitions and useful realizations are presented briefly along with short explanations as to the physical interpretations of the same. The necessity of the application of fractional calculus in signal processing domain is next highlighted and probable application areas and current research focus in this domain is presented.

Chapter 2 looks at some Linear Time Invariant (LTI) representations of fractional order systems. Since the LTI theory is well developed, casting fractional order systems in state space or transfer function framework helps in the intuitive understanding and also the use of existing tools of LTI systems with some modifications. The discrete versions of fractional order models are also introduced since many practical applications in signal processing work on discrete time system models and discrete signals. Some basic signal processing techniques like continuous and discrete fractional order realizations, convolution and norms applied to fractional order signals and systems are also elucidated.

Chapter 3 explains long range dependent models which are useful in modeling real world signals having outliers or long tailed statistical distributions. The concept of stable distribution is outlined and many standard distributions like symmetric α -stable, Gaussian, Cauchy, etc. are obtained from the generalized description of the family of stable distributions. Self similarity for random processes is covered next and few other concepts like fractional Brownian motion, fractional and multi-fractional Gaussian noise, etc. are discussed in this context. The Hurst parameter which is a measure of self similarity is delineated next and a few methods to estimate it, is subsequently outlined.

Chapter 4 presents different transforms commonly encountered in signal processing applications. The importance of the fractional Fourier transform (FrFT) is highlighted and its interpretation and relation to various other transforms are also illustrated. The concept of filtering in the fractional Fourier domain and various other applications of signal processing using FrFT are also described. Other transforms like the fractional Sine, Cosine and the Hartley transforms which

are obtained by suitable modification of the FrFT kernel is discussed. The fractional BSplines and fractional Wavelet transforms are also briefly introduced.

Chapter 5 explains various time domain and frequency domain system identification methods for fractional order systems from practical test data. System identification is important in cases where it is difficult to obtain the model from basic governing equations and first principles, or where there is only input–output data available and the underlying phenomenon is largely unknown. As is evident, fractional order models are better capable of modeling system dynamics than their integer order counterparts and hence identification using fractional order models is of practical interest from the system designer’s point of view.

The first part of **Chap. 6** throws light on the different forms of Kalman filter for fractional order systems. The latter part deals with signal processing with fractional lower order moments for α -stable processes. Some methods of parameter estimation of different α -stable processes are proposed. The concept of covariation, which is analogous to covariance for Gaussian signal processing, is unveiled and its properties and methods of estimation are reviewed.

Chapter 7 mentions several different MATLAB based toolboxes and codes which can be used to simulate fractional order signals and systems in general. Most of the toolboxes are in public domain and are freely accessible by any user. The reader can use these tools for simulation right away to get a feel of fractional order signal processing. These tools can also serve as building blocks for simulating more complex systems during research.

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