Optimization-Based Bidding in Day-Ahead Electricity Auction Markets: A Review of Models for Power Producers

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Abstract We review some mathematical programming models that capture the optimal bidding problem that power producers face in day-ahead electricity auction markets. The models consider both price-taking and non-price taking assumptions. The models include linear and non-linear integer programming models, mathematical programs with equilibrium constraints, and stochastic programming models with recourse. Models are emphasized where the producer must self-schedule units and therefore must integrate optimal bidding with unit commitment decisions. We classify models according to whether competition from competing producers is directly incorporated in the model.

Keywords Auctions • Bidding • Day-ahead electricity markets • Day-ahead markets • Mathematical programming • Unit commitment

1 Introduction

The transformation from regulation to competition in power industries around the world have led to the development of markets for power. Day-ahead electricity markets are emerging as an important medium through which power is allocated in many de-regulated environments. A day-ahead electricity market is a short term hedge market that operates a day in advance of the actual physical delivery of power. In these environments, the generation decisions for the next day are in most cases the result of a double (two-sided) auction where producing (selling) and consuming (buying) agents submit a set of price-quantity curves (bids). The bids must be submitted by a deadline on the day before actual delivery of power.

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A clearing price based on the submitted bids is determined by the ISO (Independent System Operator) or market making agent and all subsequent trades are settled at this price.

A significant amount of power is allocated through day-ahead markets. For example in the Nord Pool day-ahead market in Scandinavia, Elspot, the volume of power traded in 2007 was more than 290 TWh which amounts to more than 65% of all consumption in the Nordic countries for that year [1]. Irastorza and Fraser [2] find that in the United States most electricity is traded through day-ahead markets or through bilateral forward contracts. Day-ahead markets are beneficial since they act as a short term forward market for power that in conjunction with a real-time market offers significant benefits to both producers and consumers of power through price transparency, the reduction of price uncertainty, reduction of strategic gaming, unit commitment certainty, and facilitation of demand-side (consumer) participation.

In a day-ahead market, a producer of power must decide their offer curve and a consumer must decide their bid curve. In addition, the ISO or market maker must clear the market by finding the equilibrium clearing price of the auction based on the submitted bids. Given the importance of the role of day-ahead markets in the generation and allocation of power normative models for the agents have been emerging in the literature over the last decade on optimal bid construction, unit commitment, and payment/pricing and other decisions in the context of day-ahead markets. In this paper, we give a review of the literature on the various types of optimization modeling of the power producers i.e. those agents that generate and supply power into the market.

The purpose of this chapter is to classify and characterize the emerging literature of optimization models for producers participating in day-ahead markets. Producer models considered in this paper involve a diverse set of mathematical programming approaches including non-linear integer programs, stochastic programming with recourse, and mathematical programs with equilibrium constraints.

We focus on models where producers and not the ISO have the responsibility of unit commitment decisions and so must integrate these decisions with the offer or bidding strategy. There are advantages to not having an ISO perform a centralized unit commitment as cost information of competing producers must be revealed to the ISO. Also, a centralized unit commitment in a decentralized market setting can be problematic for individual units. In particular, small cost changes in the centralized unit commitment can result in large differences to individual units/generators [3]. Transmission and congestion issues when included are dealt within models and we do not consider separate for models for a congestion or transmission entity. We do not intend to give an exhaustive coverage of the literature, but select models that represent the current state of the art or represent the major issues in modeling of optimal producer bidding strategy in the context of day-ahead markets.

Modeling approaches for producers in the day-ahead market environment reflect the decentralized nature of the market. In this setting separate models for entities (agents) of the market are developed e.g. models for optimal producer bidding. This is in marked contrast to models for energy under the older regulated environments where for example a public utility would schedule generation and decide unit commitment without the presence of competitive bids. Most of these models are centralized cost minimization models. For a good review of this class models see Hobbs [4]. For an excellent review of general market modeling trends see Ventosa et al. [5]. In addition, see Wallace and Fleten [6] for stochastic programming models in regulated environments. Wallace and Fleten [6] discuss models in de-regulated environments e.g. day-ahead models as well but in the current paper we address and focus on day-ahead markets in a more comprehensive and detailed manner and consider other modeling approaches in addition to stochastic programming models.

2 Producer Models

The main focus of models of producers in day-ahead markets is on constructing optimal offer curves to submit to the market coordinator e.g. ISO or market maker. The producers of power in the market have a multitude of important considerations in developing offers for power in the day-ahead market. A bidding model for a producer will depend on the particular market structure e.g. auction rules and protocols. In addition, producers face uncertainty in demand for power and in many cases uncertainty in the behavior of competing power producers. In addition, in some environments the unit commitment decisions may be the responsibility of the producer and thus any bids for power must consider the cost of operating generation units as well as inter-temporal operating constraints. The integration will depend on whether an ISO or other agent decides unit commitment.

It is typical in day-ahead markets to require producers to submit supply functions (offer curves) for each of the 24 h of the day-ahead schedule. The supply functions give for each hour the price per unit of power associated with a volume of power that the producer is willing to sell at. Offer curves are typically non-decreasing. We classify the models we consider in this chapter broadly into two classes. The first class of models directly incorporates bidding behavior of competing producers. The second class does not incorporate competition directly into the model. We call the first class “Producer models with strategy” and the second class “Producer models without strategy”.

3 Producer Models with Strategy

This class of models considers a producer’s optimal bidding problem where important model attributes include self-scheduling, integration with unit-commitment, and incorporation of demand load and competing producer bidding behavior. We emphasize the structural aspects of all models in general but note that the incorporation of competing producers will involve important estimation techniques (these techniques are complementary to the structural development/description of models and will not be pursued in detail in this chapter.) We give detailed examples for two representative models. They differ dramatically in how competition is incorporated. The first example is a non-equilibrium approach (i.e. does not use mathematical programming formulations with equilibrium constraints, MPEC) and the second example is an MPEC model i.e. an explicit equilibrium programming approach.

3.1 Non-equilibrium Example

The first model we present is by Wen and David [7]. The modeling framework was developed for day-ahead markets in California that pre-dates the California energy crisis of 2000–2001. In the pre-crisis environment, there was an ISO that managed the grid, and a separate market maker called PX (Power Exchange), that coordinated the day-ahead market. The structure of the newly emerging post-crisis energy markets in California will include a day-ahead market, however, the ISO now will act as the market coordinator. The framework of Wen and David [7] is nevertheless instructive for environments where ISOs are not the market coordinators and producers consider the strategies of other producers and self-unit commitment in the construction of offer curves. It is assumed that the producers are thermal producers of electricity.

In this framework, producer i submits linear non-decreasing offer curve (bid price) of the form $B^{(t)}{}_{(i)}(P_{(i)}{}^{(t)}) = \alpha_i{}^{(t)} + \beta_i{}^{(t)} \times P_{(i)}{}^{(t)}$ for each hour of in the day ahead market (there are 24 h) where $\alpha_i{}^{(t)}$ and $\beta_i{}^{(t)}$ are bidding coefficients for producer i and $P_{(i)}{}^{(t)}$ is the generation output.

3.1.1 Market Maker Model (i.e. PX Model)

The market maker, PX, after receiving bids from the producers computes the market clearing price R^t and solves the following problem in a manner similar to solving a classical economic dispatch problem to compute the generation output for each producer.

$$R^t = \alpha_i{}^{(t)} + \beta_i{}^{(t)} \times P_{(i)}{}^{(t)} \quad t = 1, 2, 3, \ldots, 24 \tag{1}$$

$$\sum_{j=1}^{n} P_j{}^{(t)} = Q_t \quad t = 1, 2, 3, \ldots, 24 \tag{2}$$

$$P_{j\min}{}^{(t)} \leq P_j{}^{(t)} \leq P_{j\max}{}^{(t)} \quad j = 1, 2, , \ldots, n \quad t = 1, 2, 3, \ldots, 24 \tag{3}$$

In (1), generation is assigned to each producer so that bid prices coincide with the market clearing price for each hour. Equation 2 ensures that generation is assigned so that the load for each hour Q_t is met by total of generation from all producers. Equation 3 ensure that all generation is assigned this is between the lower and upper bounds for each producer where ${P_{j\,\min}}^{(t)}$ and ${P_{j\,\max}}^{(t)}$ are the respective bounds for each producer.

3.1.2 Producer Strategies

The strategy of a producer in this framework is to determine offer curves for each hour with the aim of "maximizing hourly benefit" or providing "minimum stable output". The two strategies ensure that enough generation is dispatched so that offer curves are profitable of at least enough generation output dispatched for the generator to remain in continuous operation. The producer considers an optimization model for each of these problems and then formulates a unit commitment model that that incorporates the strategies from the previous two models.

3.1.3 The Producer Hourly Benefit Model

This model seeks to find generation offers that would maximize hourly benefit given data about estimated loads and estimated bidding behavior of other bidders (power producers). The model is as follows:

$$\begin{aligned}&\text{maximize}\ \psi^{(t)}({\alpha_i}^{(t)}, {\beta_i}^{(t)}) = R_t P_i^{(t)} - C_i({P_i}^{(t)}) \qquad (4)\\ &\text{subject to (1) to (3)}\end{aligned}$$

where $C_i({P_i}^{(t)})$ is the cost of generation for producer i which is a function of the generation. $\psi^{(t)}({\alpha_i}^{(t)}, {\beta_i}^{(t)})$ is the hourly benefit objective function which is a measure of hourly benefit or profit. It should be noted that constraint (1) in the context of the producer model requires that the bidding coefficients of other producers to be estimated since a producer would not have access to this information directly see Wen and David [7] for details. In this approach, they use a joint probability distribution to estimate all other producers and then the hourly benefit model becomes a stochastic optimization problem.

3.1.4 Minimum Stable Output Bidding Strategy

This model aims to ensure that average output from a producer achieves near the minimum generation level. The model is as follows:

$$\text{Minimize}\ \varsigma^{(t)}(\alpha, \beta) = |\bar{P}_i^{(t)} - {P_{\min}}^{(t)}| + \gamma(\bar{P}_i^{(t)} - P_{\min}^{(t)})^2 \qquad (5)$$

$$\text{subject to}\, {P_{i\,\min}}^{(t)} \le \bar{P}_i^{(t)} \le {P_{i\,\max}}^{(t)} \tag{6}$$

where γ is a positive penalty parameter.

3.1.5 Overall Producer Bidding Model

This model integrates the strategies from the hourly benefit and the minimum stable output models to determine which generators are to be on for each time period.

$$\text{Maximize}: \; \Omega(\mu_t) = M + \sum_{t=1}^{24} [\mu_t \psi^{(t)}(\alpha_i^{(t)}, \beta_i^{(t)}) - S(\tau)\mu_t(1-\mu_{t-1})] \tag{7}$$

$$\text{subject to} \sum_{t=1}^{24} (\mu_t - \mu_{t-1})^2 \le N \tag{8}$$

where the objective function in (7) is constructed to be positive (M is a sufficiently large number) in all cases since the formulation will be solved be a genetic algorithm representing the fitness of bidding strategies. μ_t is a binary variable that is 1 if a unit is up for hour t, 0 otherwise and $S(\tau)$ is the start up cost of the generator. Constraint (8) ensures that a unit has a maximum number of start ups and shutdowns in a day.

Recent papers similar to Wen and David [7] that make estimates of load forecast and competitor behavior have emerged that also incorporate reserve markets as well as the spot (day-ahead) markets. Attaviriyanupap et al. [8] consider thermal producer optimization models that incorporate self-scheduling and unit commitment based on estimates of competing producers in both spot and reserve markets. The resulting producer models are non-convex and non-differentiable and evolutionary programming heuristics are used to solve the models. An alternative approach by Swinder [9] considers the spot market to be a price-taking market but assumes that bidders behave strategically in the reserve market and that the behavior is captured in a joint probability distribution. A simultaneous bidding model is developed that is a stochastic non-linear profit maximization model.

Other models that incorporate competitor behavior and load estimates include Zhang et al. [10] in which they consider a Lagrangian relaxation-based approach after obtaining a closed form solution for the ISO problem. A Lagrangian relaxation approach is also adopted in Gross and Finlay [11] where the load forecast and competitor behavior is incorporated. An ordinal optimization approach is presented in Guan et al. [12] where the idea is to generate good enough solutions. Anderson and Philpott [13] consider the more general problem of a producer that makes offers into a wholesale power market for which the prices are determined by a sealed-bid auction (this case would be applicable to day-ahead markets). Demand and behavior of competing producers are represented in a probability distribution for which models are then defined and necessary optimality conditions are derived for these models.

3.2 Equilibrium Approaches for Producer Models

Next, we consider an MPEC (mathematical program with equilibrium constraints) model by Bakirtzis et al. [14] for a generator's offering strategy with step-wise offers. An MPEC model is a non-linear programming problem in which there are constraints defined by a parametric variational inequality or complementarity system [15]. The MPEC framework is useful for modeling strategic interaction that follows a Stackleberg game [16]. In this model, a "leader" producer can affect the market price and estimates the demand declarations as well as the supply functions of competing producers to be used in the leader's optimal offering strategy problem. The competing producers are the "followers" in the sense that the "leader" producer faces a residual demand function of the aggregate of the competing producers. An MPEC generator model will have an outer problem of maximizing profit given that the ISO will solve an economic dispatch problem (this latter problem is the inner problem). The ISO problem (given supply functions from all generators) is to minimize revealed cost while determining the dispatched quantity for each generator. In the model of Bakirtzis et al. [14], a producer j constructs optimal energy offers in the form of non-decreasing step-wise bids i.e. selects a numberB^j of steps and a set of quantity-price pairs (Q_{jb}, π_{jb}) for each step $b \in \{1, ..., B^j\}$ where Q_{jb} is the quantity offered by producer j for block b and π_{jb} is the corresponding offer price.

The deterministic ISO problem is of the following form:

$$Minimize \sum_j \sum_b \pi_{jb} \cdot q_{jb} \tag{9}$$

subject to (10) to (12)

The objective (9) is to minimize the cost of dispatched energy with respect to the revealed price of energy where q_{jb} is the dispatched power for unit j step b. The constraints are defined as follows:

$$\sum_j \sum_b q_{jb} = d \tag{10}$$

$$q_{jb} \leq Q_{jb} \text{ for all } j \in A, b \in B^j \tag{11}$$

$$q_{jb} \geq 0 \text{ for all } j \in A, b \in B^j \tag{12}$$

Constraint (10) ensures that demand d is met through all dispatched power and (11) ensures that accepted power does not exceed the offered volume (i.e. a producer is never dispatched more power then bid for).

The producer model assumes a 1 h horizon and pertains to a producer with a set of thermal units$A$ and is defined as follows:

$$Maximize \sum_{s \in S} \left\{ p^s \sum_{j \in A} \left[\sum_{b \in B^j} \lambda^s q^s_{jb} - c_j \left(\sum_{b \in B} q^s_{jb} \right) \right] \right\} \tag{13}$$

subject to (14) to (28)

The objective (13) is to maximize the expected profit where the system marginal price for power in scenario s is λ^s and q^s_{jb} is the quantity of step b of unit j offer accepted by the ISO in scenario s. $c_j(\bullet)$ is the hourly non-linear cost of unit j as a function of generation level.

3.2.1 Producer Constraints

$$0 \leq Q_{jb} \leq Q^{\max}_j \text{ for all } j \in A, b \in B^j \tag{14}$$

$$\sum_b Q_{jb} = Q^{\max}_j \text{ for all } j \in A \tag{15}$$

$$Q_{jq} \leq \delta \cdot Q^{\max}_j \text{ for all } j \in A \tag{16}$$

Constraints (14) ensure that any offer is not greater than the capacity of a unit j and (15) ensures that all of the capacity of a unit j is offered. Constraint (16) ensures that the first step of an offer is a fraction of the available capacity of a unit j (as required by Greek power markets).

$$0 \leq \pi_{jb} \leq \pi^{\max} \text{ for all } j \in A, b \in B^j \tag{17}$$

$$\pi_{jb} \leq \pi_{j(b+1)} \text{ for all } j \in A, b = 1, \ldots, B^j - 1 \tag{18}$$

$$\pi_{jb} \geq MVC_j \text{ for all } j \in A, b = 1, \ldots, B^j \tag{19}$$

$$\sum_{b \in B^j} Q_{jb} \cdot \pi_{jb} \geq MVC_j \cdot \sum_{b \in B^j} Q_{jb} \text{ for all } j \in A \tag{20}$$

Constraint (17) places a price cap of all price offers and (18) ensures the offer prices are non-decreasing. Constraints (19) and (20) ensure that offer prices are at least the minimum variable cost, MVC_j, for a unit j.

3.2.2 ISO Market Clearing Problem

$$\sum_{j,b} q_{jb}^s = d^s \text{ for all } s \in S \tag{21}$$

$$q_{jb}^s \leq Q_{jb} \text{ for all } j \in A, b \in B^j, s \in S \tag{22}$$

$$q_{jb}^s \leq \bar{Q}_{jb}^s \text{ for all } j \in \bar{A}, b \in B^j, s \in S \tag{23}$$

$$q_{jb}^s \geq 0 \text{ for all } j \in J, b \in B^j, s \in S \tag{24}$$

$$\mu_{jb}^s \geq 0 \text{ for all } j \in J, b \in B^j, s \in S \tag{25}$$

$$\pi_{jb} + \mu_{jb}^s \geq \lambda^s \text{ for all } j \in A, b \in B^j, s \in S \tag{26}$$

$$\bar{\pi}_{jb} + \mu_{jb}^s \geq \lambda^s \text{ for all } j \in \bar{A}, b \in B^j, s \in S \tag{27}$$

$$\sum_{j\in A, b\in B^j} \pi_{jb} q_{jb}^s + \sum_{j\in A, b\in B^j} \mu_{jb}^s Q_{jb} + \sum_{j\in \bar{A}, b\in B^j} \bar{\pi}_{jb}^s q_{jb}^s + \sum_{j\in \bar{A}, b\in B^j} \mu_{jb}^s \bar{Q}_{jb}^s = \lambda^s d^s \text{ for all } s \in S \tag{28}$$

Constraints (21) to (28) represent the expected value (scenario-based) ISO problem via the Karush-Kuhn-Tucker conditions whereλ^s is the dual multiplier associated with demand balance constraint (10) for scenario s and μ_{jb}^s is the dual multiplier for the constraint (11) for scenarios s.

It should be noted that the producer MPEC formulation is converted to a mixed integer linear program by a binary expansion of the offer prices and quantities using the techniques in Pereira et al. [17]. Other bi-level level mathematical programming-based approaches which involve an outer and an inner problem corresponding to producer and ISO, respectively, include Gountis et al. (2004) [29] where each producer submits a linear supply curve and estimates the competing producers and demand behavior using Monte Carlo approaches. In addition, risk aversion is modeled and is seen to have an impact on optimal offer strategies. Earlier bi-level model is given in Weber and Overbye [18].

4 Producer Models Without Strategy

In this section, we consider producer models where there is no explicit incorporation of competing producers in the models. We detail two models. The first is a deterministic mixed integer linear programming modeling framework and the second a stochastic integer programming model. Both models are price-taking in the sense that the model assumes that the producer can not impact the market clearing price.

4.1 Mixed Integer Linear Programming Model

The first modeling framework is by Conejo et al. [19] for constructing offer curves for price-taking thermal producers (producers). In this approach, hourly prices are forecast and a self-schedule is obtained by an optimization model. The framework is for general pool type electricity markets. Generators submit a bid for each hour in the day-ahead time frame and consists a set of blocks of power along with corresponding unit prices. Then, hourly offer curves are constructed based on the optimal self-schedule by the use of a simple bidding strategy.

The producer (price-taking) optimization model is

$$\text{Maximize}\, E_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T}\lambda_t p_t\right\} - \sum_{t=1}^{T} c_t \tag{29}$$

$$\text{subject to}\, p_t \in \Pi \tag{30}$$

where λ_t are the random prices of power for hour t (the distributions are approximately lognormal), c_t is the (non-linear) cost of generating for hour t (see Conejo et al. [19] for details), p_t is the power generation for hour t, and Π is the set of feasible generation outputs that satisfy operational constraints with respect to minimum up and down times, ramping, and power output limits. Thus, the model seeks to maximize optimal expected profit subject to operating constraints. This model is seen to be equivalent to a mixed integer program (MIP), where Π is defined by the following constraints:

$$p_t \geq \underline{P}\, v(t) \text{ for all } t = 1, \ldots, T \tag{31}$$

$$p_t \leq \bar{P}\,[v(t) - z(t+1)] + z(t+1)SD \text{ for all } t = 1, \ldots, T \tag{32}$$

these constraints ensure the lower and upper limits on power production $(z(t)(v(t))$is a binary variable which is equal to 1 is the generator is shut down (on-line) at the start of hour t, SD is the shut down ramp rate limit in MW/h.)

$$p_t \leq p_{t-1} + RUv(t-1) + SUy(t) \text{ for all } t = 1, \ldots, T \tag{33}$$

$$p_{t-1} - p_t + RDv(t) + SDz(t) \text{ for all } t = 1, \ldots, T \tag{34}$$

these constraints ensure that ramp rates are obeyed (where RU (RD) is the ramp-up (ramp-down) rate limit and SD is the shut down ramp rate limit)

$$\sum_{t=1}^{G} [1 - v(t)] = 0 \tag{35}$$

$$\sum_{j=t}^{t+UT-1} v(j) \geq UTy(t) \quad t = G+1, ..., T-UT+1 \tag{36}$$

$$\sum_{j=t}^{T} [v(j) - y(t)] \geq 0 \quad t = T-UT+2, ..., T \tag{37}$$

$$\sum_{t=1}^{F} v(t) = 0 \tag{38}$$

$$\sum_{j=t}^{t+DT-1} [1 - v(j)] \geq DTz(t) \quad t = F+1, ..., T-DT+1 \tag{39}$$

$$\sum_{j=t}^{T} [1 - v(j) - z(t)] \geq 0 \quad t = T-DT+2, ..., T \tag{40}$$

Constraints (35) to (40) ensure that the minimum up and down times of a generator are satisfied where DT (UT) is the minimum downtime (uptime) and F (G) is the required number of time intervals that a generating unit must be off-line (on-line) because of downtime (uptime) constraints.

$$y(t) - z(t) = v(t) - v(t-1) \quad t = 1, ..., T \tag{41}$$

$$y(t) + z(t) \leq 1 \quad t = 1, ..., T \tag{42}$$

$$z(t) \in \{0, 1\} \quad t = 1, ..., T \tag{43}$$

Constraints (41) to (43) ensure the correct logical relationship between a generator' shut-down, start-up, and running states.

The offer curve for a generator is obtained by solving the MIP model to obtain the optimal production schedulep_t^* for each of the 24 h in the day-ahead time frame. Then the bidding strategy to realizep_t^* for hour t is summarized as follows:

1. If $p_t^* = 0$ then offer curve will consist of a single block of power equivalent to maximum capacity of a thermal generator, $\bar{P}$, at unit price of $\lambda_t^{est} + a_t\sigma_t^{est}$
2. If $0 < p_t^* < \bar{P}$, then the offer curve will consist of two blocks of power p_t^* and $\bar{P} - p_t^*$ at prices $\lambda_t^{est} - b_t\sigma_t^{est}$ and $\lambda_t^{est} + a_t\sigma_t^{est}$, respectively.
3. If $p_t^* = \bar{P}$, then the offer curve will be a block of power $p_t^* = \bar{P}$ at a price per unit of $\lambda_t^{est} - b_t\sigma_t^{est}$.

where λ_t^{est} is the expected value of λ_t and σ_t^{est} the estimate of the standard deviation of λ_t. The prices (i.e. coefficients a_t and b_t) are chosen so that the offer curves guarantee with a level of 99% confidence that the power accepted by the market maker is the specified power amount in the offer curve.

Other price–taking producer models without strategy include Ladurantaye et al. [20] where a bidding model for a price-taking hydro-producer is formulated. Gonzalez et al. [21] consider a profit-based hydro-producer model for day-ahead markets. Risk aversion is incorporated into the model via the conditional Value at Risk (CVaR) measure [22]. Scenarios are generated for market prices via hidden markov models. Also considered are minimum profit-based models. The resulting formulations are mixed integer linear programs. Conejo et al. [23], Yamin et al. (2004) [31] and Dicorato et al. [24] also incorporate risk aversion into producer bidding models where the first two consider as a measure of risk the variance of market clearing price and the latter the CVaR measure.

4.2 Stochastic Programming Model

Stochastic programming has been emerging as an important modeling framework for problems arising in the energy sector such as hydro and thermal scheduling [6], unit commitment (Takriti et al. 1996) [30] and structuring energy forward contracts [25] among many other classes of problems. Uncertainty in deregulated energy markets often takes the form of uncertainty in spot prices and uncertainty in weather which relates to uncertainty in demand (load). For an excellent survey of the use of stochastic programming in both regulated and deregulated energy markets see Wallace and Fleten [6].

We detail the model of Fleten and Kristoffersen [26] where a two-stage stochastic programming model is developed for optimizing the offer strategy for a Nordic hydropower producer for the day-ahead market in the Nord Pool, the Elspot. The hydropower producer is assumed to be a price-taker and the model incorporates price uncertainty. Another stochastic programming model in the same spirit of Fleten and Kristoffersen [26] is given in Ladurantaye et al. [20]. The rationale for stochastic programming with recourse stems from the fact that the clearing price cannot be known before dispatch making it a challenge to commit generators before the actual trading price is known. In particular, a decision (offer curve) must be made now before uncertainty of price is resolved. Ideally, the “here and now” decision should reflect the possible *recourse* (corrective actions) required after price uncertainty has resolved e.g. the actual production decisions occurring in the future based on realized prices at that time. Furthermore, the “here and now” decision should be constructed as to minimize the costs of recourse actions over all possible random outcomes. Stochastic programming with recourse is a natural framework for this situation. The stochastic programming model incorporates a finite set of scenarios that capture different price possibilities to generate offers that are robust to price uncertainty. There are two major decision stages where the first stage involves decisions related to day-ahead bid construction i.e. offer volume and the second stage "*recourse*" decisions reflect production of power e.g. hydro power production and dispatch subject to operating and balance constraints. The model assumes a simple hydro-plant with two reservoirs in a cascade, one larger upper reservoir and one smaller reservoir downstream.

4.2.1 Bid Structure

A hydro-producer can submit hourly bids, block bids, of flexible hourly bids. An hourly bid has the form (x_{it}, p_i) where the first component is the amount of power offered (i.e. bid volume) by a producer for hour t and the second component is the associated unit price of power (the i indexes a finite set of predetermined prices). These bids are seen to be points on a bidding curve constructed by making an linear interpolation between the points. For example, the unit price ρ_t for hour t associated with a volume y_t is given by

$$\rho_t = p_{i-1} + \frac{p_2 - p_1}{x_{2t} - x_{1t}}(y_t - x_{1t}) \text{ for } x_{i-1,t} \leq y_t \leq x_{it}. \tag{44}$$

4.2.2 Two-stage Stochastic Programming Model

First Stage decisions: x_{it} (x_{ib}) represents offer volume for hour t (block b)

Second Stage decisions: (Scenario dependent decisions) $y_t^s(y_b^s)$ volume dispatched for hourly (block) bids, $z_t^{+,s}(z_t^{-,s})$ is the positive (negative) imbalance between volume dispatched and volume produced, l_{jt}^s is storage level for reservoir j for time period t, μ_{jt}^s is the on or off state of a generator, and v_{jt}^s is the volume of discharge from generator j for time period t. (Scenario independent decisions) w_{jt} is the generation level for generator j for time period t, and l_{jt} is the storage level for reservoir j for time period t.

Parameters: π^s is the probability of occurrence of scenario s, ρ_t^s is the unit price of power for hour t in scenario s, $\overline{\rho_b^s}$ is the average unit price of power in block b in scenario s, $\mu_t^+(\mu_t^-)$ are the penalty (reward) for power imbalances.

$$\begin{aligned} Maximize \sum_{s \in S} \pi^s \Bigg(&\sum_{t \in S} \rho_t^s y_t^s + \sum_{b \in B} \overline{\rho_b^s} y_b^s - \sum_{t \in T} (\mu_t^+ z_t^{+,s} - \mu_t^- z_t^{-,s}) \\ &- \sum_{j \in T} (V_j(l_{j0}^s) - V_j(l_{jT}^s)) - \sum_{t \in T} \sum_{j \in J} S_j(\mu_{jt-1}^s, \mu_{jt}^s) \Bigg) \end{aligned} \tag{45}$$

subject to (46) to (59)

The objective (45) is to maximize the expected profit from offers and power production where the function $V_j(\bullet)$ is such that $\sum_{j \in T} (V_j(l_{j0}^s) - V_j(l_{jT}^s))$ gives the opportunity costs of storing water for generator j and $S_j(\mu^s_{jt-1}, \mu^s_{jt})$ is the direct cost function of starting up a generator (the functions are defined to so that the model will correspond to a linear MIP).

$$y^s_t = \frac{\rho^s_t - \rho_{i(t,s)}}{\rho_{i(t,s)+1} - \rho_{i(t,s)}} x_{i(t,s)+1t} + \frac{\rho_{i(t,s)+1} - \rho^s_t}{\rho_{i(t,s)+1} - \rho_{i(t,s)}} x_{i(t,s)t} \text{ for all } t \in T, s \in S \tag{46}$$

Constraint (46) gives the representation of the actual dispatch in hour t under scenario s implied by the piecewise linear offer curve (44) where $i(t,s) = \max\{i \in I : p_i \leq \rho^s_t\}$.

$$x_{it} \leq x_{i+1t} \text{ for } i \in \mathrm{I} \backslash I,\ b \in B \tag{47}$$

$$y^s_b = \sum_{j \leq i(b,s)} x_{jb} \text{ for } b \in B,\ s \in S \tag{48}$$

Constraint (47) ensures that bid (offer) curve is non-decreasing, and (48) relates the actual power dispatched for block bids to offer volumes under all scenarios.

$$w^s_{1t} = \mu^s_{1t} w^{\max}_1 \text{ for } t \in \mathrm{T} \text{ and } s \in S \tag{49}$$

$$\mu^s_{2t} w^{\min}_2 \leq w^s_{2t} \leq \mu^s_{2t} w^{\max}_2 \text{ for } t \in \mathrm{T}, s \in S \tag{50}$$

$$v^{\min}_j \leq v^s_{jt} \leq v^{\max}_j \text{ for } j \in \mathrm{J}, t \in \mathrm{T}, s \in S \tag{51}$$

$$l^{\min}_j \leq l^s_{jt} \leq l^{\max}_j \text{ for } j \in \mathrm{J}, t \in \mathrm{T}, s \in S \tag{52}$$

Constraints (49) to (52) impose water discharge bounds e.g. (49) enforces that the maximum amount of water is discharged or no water is discharged whereas (50) allows the second reservoir to discharge any amount between specified upper and lower bounds.

$$l^s_{1t} - l^s_{1t-1} + v^s_{1t} + r^s_{1t} = v^s_{1t} \text{ for } t \in \mathrm{T}, s \in S \tag{53}$$

$$l^s_{2t} - l^s_{2t-1} + v^s_{2t} + r^s_{2t} = v^s_{1t-\tau} \text{ for } t \in \mathrm{T}, s \in S \tag{54}$$

Constraints (53) to (54) are the reservoir balance equations.

$$w^s_{jt} = \gamma_j v^s_{jt} \text{ for } j \in \mathrm{J}, t \in \mathrm{T}, s \in S \tag{55}$$

Constraint (55) gives the generation efficiency of reservoirs.

$$y^s_t + \sum_{b \in B: t \in b} y^s_b - \sum_{j \in J} w^s_{jt} = z^{+,s}_t - z^{-,s}_t \text{ for } t \in \mathrm{T}, s \in S \tag{56}$$

Constraint (56) measures the imbalances between volumes produced and volumes dispatched.

$$x_{it}, x_{ib} \in R_{+} \text{ for } i \in \mathrm{I}, t \in \mathrm{T}, b \in \mathrm{B} \tag{57}$$

$$\mu_{jt}^{s} = 0 \text{ or } 1 \tag{58}$$

$$y_t^s, y_b^s, z_t^{+,s}, z_t^{-,s}, v_{jt}^s, w_{jt}^s, l_{jt}^s \in R_{+} \text{ for } j \in \mathrm{J}, t \in \mathrm{T}, b \in \mathrm{B}, s \in S \tag{59}$$

The stochastic programming model above has been extended by Faria and Fleten [1] to incorporate the adjustment market called the Elbas in NordPool. The Elbas market allows adjustment in accepted bids up to 1 h before scheduled dispatch. The rationale is that accepted offers are made before prices, loads, and inflow are known so after realization of these uncertainties a rebalancing or recourse should occur. It is found, however, that the incorporation of the Elbas does not significantly change bidding. Nowak et al. [27] consider a stochastic integer program for incorporating day-ahead trading in hydro-thermal unit commitment decisions made for a week ahead for a German power utility. The main sources of uncertainty are the bids made by competitors. The stochastic model is fully linear which allows a Lagrangian-based branch and bound procedure to be applied.

5 Discussion of Model Features

Common to most models that incorporate strategy is the need for estimation of demand load and competitor bidding behavior while models without competitive behavior need estimation of market clearing prices. In addition, Table 1 lists additional features (as done in a manner analogous to Ventosa et al. [5]) for models that incorporate strategy and Table 2 lists features for those models without strategy. Besides the classification of a model as one with or without strategy and author names, the features in Tables 1 and 2 include the type of optimization model e.g. integer program, solution methodology, features particular to a specific model, and intended market.

Almost all day-ahead producer models are seen to be non-convex and non-differentiable e.g. mixed-integer programming models with the exception of Dicorato et al. [24], where the unit commitment decisions are assumed to have been made *ex ante*. Thus, the computational tractability is an issue for most day-ahead producer models as bidding and unit commitment (which alone is a difficult problem to solve) are combined and is dealt with using a variety of methods as seen in Tables 1 and 2. The most common techniques involve commercial branch and bound solvers, Lagrangian relaxation, and evolutionary heuristics such as genetic algorithms. Some models deal with the complexity of the producer optimization by decoupling and solving separately the bidding strategy and scheduling of power

Table 1 Models with strategy

Strategic models	Year	Model-type/solution technique	Features	Intended market
Wen and David	2001	Mixed-integer program/ genetic algorithms	Integrates two bidding strategies: (1) hourly benefit and (2) minimum stable output	California (pre-crisis i.e. before 2000)
Attaviriyanupap et al.	2005	Mixed-integer program/ evolutionary heuristics	Power and reserve markets are incorporated	
Swider	2007	Stochastic non-linear optimization	Power markets assumed to be price-taking; strategic behavior in reserve markets	Germany
Zhang et al.	2000	Mixed-integer programming/ Lagrangian relaxation	ISO problem is analytically solved	New England
Gross and Finlay	2000	Mixed-integer programming/ Lagrangian relaxation	Analytic solution under perfect competition	England and Wales
Guan et al.	2001	Mixed-integer program/ Lagrangian relaxation	Approximate solutions obtained via ordinal optimization theory	California
Bakirtzis et al.	2007	Mathematical program with equilibrium constraints (MPEC)/mixed integer programming	MPEC model is converted into a mixed integer program	Greece
Gountis et al.	2004	Bi-level program/genetic algorithms	Incorporates risk aversion and Monte Carlo simulation is used to compute expected profit	
Weber and Overbye	1999	Bi-level program	Transmission constraints are incorporated	

producing units with subsequent integration of these sets of decisions e.g. Wen and David [7] and Conejo et al. [19]. It is also seen that most producer models that include risk aversion are in models that have price-taking assumptions i.e. models without strategy.

6 Conclusion

Some representative models for producers (producers) for power in day-ahead markets were given in this chapter. The models have spanned across price-taking and non-price-taking assumptions. The primary focus has been on models that emphasize self-scheduling by a producer i.e. models that integrate offer decisions with unit commitment decisions. These models take of the form of non-linear

Table 2 Models without strategy

Non-strategic models	Year	Model-type/solution technique	Features	Intended market
Conejo et al.	2002	Mixed-integer program/branch and bound	Derives bidding strategy that achieves optimal self-schedule; requires estimation of day ahead hourly prices (probability distribution)	Spain/general pool type markets
Ladurantaye et al.	2007	Stochastic program/ successive linear programming	Integrates bidding with hydro-electric production	
Gonzalez et al.	2007	Mixed-integer linear program/under relaxation with branch and bound	Risk aversion is incorporated through the conditional value at risk (CVaR) measure	
Conejo et al.	2004	Mixed-integer quadratic program/branch and bound	Risk averse version of [19] where variance of market clearing price is a measure of risk	Spain/general pool type market
Yamin et al.	2004	Mixed-integer quadratic program/ Lagrangian relaxation	Risk aversion incorporated in self-scheduling where variance of market clearing price is a measure of risk	
Dicorato et al.	2009	Convex optimization model	Risk aversion is incorporated by using the CVaR measure. Hydro-electric and thermal units are considered.	
Fleten and Kristoffersen	2007	Stochastic integer program/ branch and bound	Integrates bidding with hydro-electric production	Nord Pool
Faria and Fleten	2009	Stochastic integer program/branch and bound	Similar to Fleten et al. 2007 but with incorporation of reserve markets	Nord Pool
Nowak et al.	2005	Stochastic integer program/ Lagrangian-based branch and bound	Incorporating bidding into hydro-thermal unit commitment. Main source of uncertainty is bids by competitors	Germany

integer programs, MPECs, and stochastic programming with recourse models. An important facet of many of the models is the incorporation of demand load and competitor behavior estimates. In addition, the incorporation of risk aversion into producer models is emerging and will continue to be an important development as preferences and utilities of producing agents are in general not the same. A universal assumption of the models was that the clearing mechanism by an ISO was performed as a single round auction, thus a produce model would need to be solved only once given all the relevant input parameters. An interesting future development will be day-ahead markets that have multiple round auction formats [28]. The benefits of such an auction would be in the information provided by results of a single round of the auction which could then be used to improve bidding for the producers in subsequent auctions. In such a case, a producer would have to repeatedly solve offer models based on updated results for a round of the auction.

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