

Mode-I Crack Control by SMA Fiber with a Special Configuration

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Abstract Crack propagation in solid members is an important reason for structure failure. In recent years, many research interests are focused on intelligent control of crack propagation. With the rise in temperature, contraction of prestrained SMA fiber embedded in matrix makes retardation of crack propagation possible. However, with the rise in temperature, separation of SMA fiber from matrix is inevitable. This kind of separation weakens effect of SMA fiber on crack tip. To overcome de-bonding of SMA fiber from matrix, a knot is made on the fiber in this paper. By shape memory effect with the rise in temperature, the knotted SMA fiber generates a couple of recovery forces acting on the matrix at the two knots. This couple of recovery forces may restrain opening of the mode-I crack. Based on Tanaka constitutive law on SMA fiber and complex stress function near an elliptic hole under a point load, a theoretical model on mode-I crack is proposed. An analytical expression of relation between stress intensity factor (SIF) of mode-I crack closure and temperature is got. Simulation results show that stress intensity factor of mode-I crack closure decreases obviously with the rise in temperature higher than the austenite start temperature of SMA fiber, and that there is an optimal position for SMA fiber to restrain crack opening, which is behind the crack tip. Therefore the theoretical model supports that prestrained SMA fiber with knots in martensite can be used to control mode-I crack opening effectively because de-bonding between fiber and matrix is eliminated. Specimen of epoxy resin embedded with knotted SMA fiber can be made in experiment and is useful to an

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analytical study. However, in practical point of view, SMA fiber should be embedded in engineering structure material such as steel, aluminum, etc. The embedding process in these matrix materials should be studied systematically in the future.

Keywords Crack control • SMA fiber • Actuator • Complex stress function

1 Introduction

Crack propagation in solid members is an important reason for structure failure. Many researchers are interested in methods of crack control. With the rise in temperature, contraction of prestrained shape memory alloy (SMA) fiber embedding in matrix makes retardation of crack propagation possible. Rogers et al. [1] proposed that embedded prestrained SMA fiber near crack tip may reduce the stress by contraction effect of SMA under increase of temperature. Photoelastic pictures and finite element analysis show that stress intensity factor (SIF) of a specimen embedded with prestrained SMA fiber decreased as temperature increases [2–4]. To overcome separation of SMA fiber from matrix, surface of SMA fiber is treated chemically [2]. Shimamoto et al. [5–8] and Umezaki et al. [9] indicated that stress at the tip of mode-I crack in epoxy matrix with SMA fiber can be reduced by heating SMA fiber. Quality change in matrix near SMA fiber is observed if the temperature is too high. This kind of quality change damages the contraction effect of SMA fiber on crack closure in matrix. Araki et al. [10, 11] performed a micromechanical model on crack closure considering crack bridging fibers and compared their numerical results with experiment. Experiments in these researches give a possible way in practical use to control crack propagation in a specimen.

However, with the rise in temperature, separation of SMA fiber from matrix is inevitable. This kind of separation weakens effect of SMA fiber on crack tip. In other words, effect of those applications is limited by the interfacial debonding. Umezaki [12] proposed a type of spiral SMA wire to prevent separation of SMA wire from matrix. In order to avoid the interface de-cohesion or sliding, Gabry et al. [13] used different surface treatments on SMA wires. Pullout tests and scanning electronic microscopy observations [14] indicated that increasing the number of turns of the twisted NiTi wires strengthens bonding strength between fiber and matrix. Zhou et al. [15–17] studied SMA fiber debonding in matrix. Wang et al. [18] proposed a theoretical model for SMA pullout from an elastic matrix. Hu et al. [19] studied interface failure between SMA fiber and matrix by shear lag model. Considering debonding at the interface between SMA wires and a polymer matrix, Wang [20] estimated the minimum volume fraction of SMA wires required to produce stress for the active crack closure. By finite element simulation, Burton et al. [21] indicated that reverse transformation of SMA wires during heating causes a closure force across the crack.

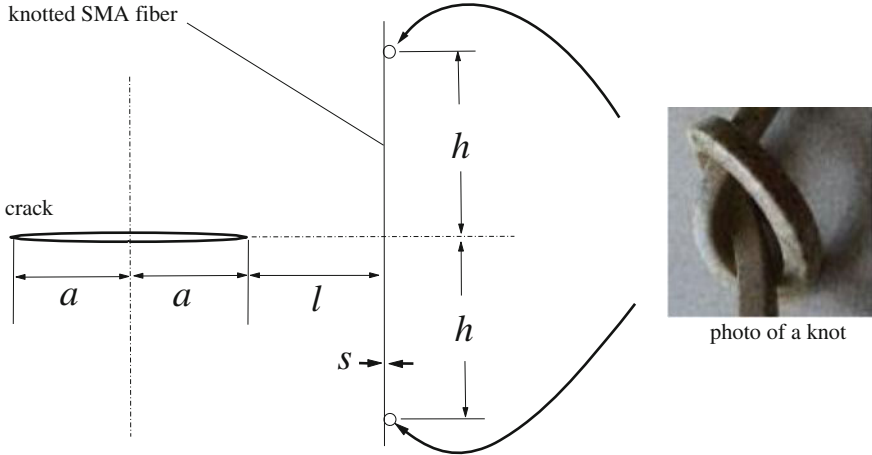


Fig. 1 A knotted prestrained SMA fiber perpendicular to crack surface

To improve effect of SMA fiber on matrix, small knots on SMA fiber are made by authors. Figure 1 shows configuration of a prestrained SMA fiber with knots at each end. With the rise in temperature, the SMA fiber contracts and the two knots on the fiber locking tightly on matrix by recovery stress. This recovery force may reduce the stress intensity factor at the crack tip and make the crack closure. In this process, the fiber cannot be debonded from matrix totally. Therefore, as an actuator in practical use, knotted SMA fiber can be embedded in matrix to control crack propagation. Relation of stress intensity factor with temperature is an important problem in crack control. In this paper, an approximate model on stress intensity factor of mode-I crack closure in matrix is proposed on a specimen with embedded SMA fiber with knots.

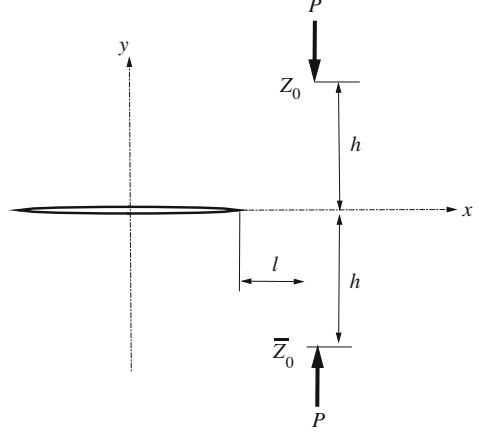
2 Relation of Recovery Force on Matrix with Temperature

See Fig. 1. Considering a specimen with a mode-I crack in matrix embedded with a prestrained SMA fiber with knots in martensite. SMA fiber is perpendicular to crack surface. Plane strain problem is considered.

$2a$ and $2h$ denote length of crack and length between the two knots, respectively. l denotes distance from the right crack tip to the fiber. With the rise in temperature, the knotted SMA fiber contracts due to phase transformation from martensite to austenite. The SMA fiber generates a couple of recovery forces $-P$ at upper knot Z_0 and P at lower knot $\bar{Z}_0$ acting on matrix where

$$Z_0 = a + l + ih, \quad (1)$$

Fig. 2 A couple of recovery forces, P acting on matrix



$\bar{Z}_0$ denotes conjugate of Z_0 ; and i denotes imaginary unit. See Fig. 2. Because bonding on interface between fiber and matrix is much weaker than locking between knot and matrix, interfacial shear stress is neglected in this paper. Therefore, the knotted SMA fiber is in simple tension by axial load, P . We have

$$P = s\sigma_f, \quad (2)$$

where σ_f and s denote recovery stress and area of the SMA fiber, respectively. In phase transformation, strain of SMA fiber is about several percent which is one order higher than strain of epoxy in elastic range. So we neglect strain of matrix when we consider strain of SMA fiber. Because each knot is locked in the matrix as the fiber contracts, increase of normal strain of SMA fiber embedding in the matrix is approximately zero, i.e.

$$\delta\varepsilon_f = 0, \quad (3)$$

where ε_f denotes normal strain of SMA fiber in phase transformation. Based on the Tanaka one-dimensional constitutive law on SMA [22], we have

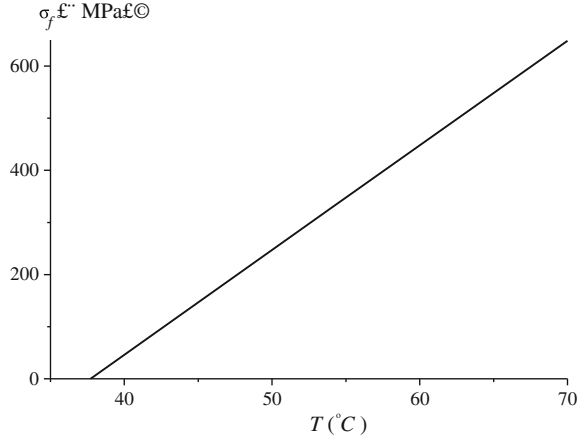
$$\delta\sigma_f = E_f \delta\varepsilon_f + \Theta \delta T + \Omega \delta\xi, \quad (4)$$

where E_f denotes Young's modulus; Θ denotes thermoelastic modulus; and Ω denotes phase transformation modulus. If ε_L denotes maximum residual strain, we have $\Omega = -E_f \varepsilon_L$ in SMA fiber. $\delta\sigma_f$, δT and $\delta\xi$ denote increase of normal stress, rise in temperature and increase of martensitic volume fraction of SMA fiber. Martensitic volume fraction, ξ is a function of stress, σ_f and temperature, T , i.e.,

$$\xi = \xi(\sigma_f, T). \quad (5)$$

Substituting Eqs. 3 to 5 into 4, we have increase of recovery stress

Fig. 3 Recovery stress, σ_f versus temperature, T



$$\delta\sigma_f = \frac{\Theta + \Omega \frac{\partial \xi}{\partial T}}{1 - \Omega \frac{\partial \xi}{\partial \sigma_f}} \delta T. \quad (6)$$

In phase transformation from martensite to austenite with rise in temperature, function (5) can be approximated by an exponential function [22] with respect to temperature, T , i.e.,

$$\xi = \exp[a_A(A_s - T) + b_A\sigma_f], \quad (7)$$

where $a_A = \frac{-2 \ln 10}{A_s - A_f}$, $b_A = \frac{a_A}{C_A}$, A_s and A_f are austenite start temperature and austenite finish temperature of SMA fiber, respectively; C_A denotes influence coefficient of stress on transition temperature.

Substituting Eq. 7 into 6, we have

$$\delta\sigma_f = \frac{\Theta - a_A\Omega \exp[a_A(A_s - T) + b_A\sigma_f]}{1 - b_A\Omega \exp[a_A(A_s - T) + b_A\sigma_f]} \delta T. \quad (8)$$

Before heating the SMA fiber, stress in the SMA fiber is zero as temperature, $T \leq A_s$. The initial value of σ_f in Eq. 8 is $\sigma_f|_{T=A_s} = 0$. Generally, we can solve initial value problem (8) numerically by the Euler iteration. However, in Appendix A, we have an analytical solution of initial value problem (8) on martensitic volume fraction, ξ

$$\begin{cases} T = \frac{\ln \xi - b_A\Omega(\xi - 1)}{b_A\Theta - a_A} + A_s \\ \sigma_f = \frac{\ln \xi - a_A(A_s - T)}{b_A} \end{cases} \quad (9)$$

where ξ decreases at 1. For a given ξ , we can get T and σ_f from Eq. 9. Substituting the value of σ_f into Eq. 2, we have relation of point load, P with temperature, T . Figure 3 shows curve of recovery stress, σ_f versus temperature, T .

Table 1 Material Properties of NiTi Fiber [16, 21]

E_f	ε_L	A_s	A_f	Θ	C_A
37.4 GPa	0.067	37.7°C	47.2°C	9.1 MPa/°C	20.3 MPa/°C

3 Stress Intensity Factor of Mode-I Crack Closure Under a Couple of Point Loads

Complex stress function of an elliptic hole in plane strain under a point load is given by [23]. The elliptic hole degenerates into a crack in Fig. 1 as perpendicular axis of elliptic hole tends to zero. Thus we can get complex stress function of the crack under a point load. Based on this complex stress function, we can get stress intensity factor of mode-I crack closure under a couple of point loads P in Fig. 2. The formulation can be seen in Appendix B. Considering Eq. 2, we have

$$\frac{\sqrt{a}}{s} K_I = \frac{\sigma_f}{\sqrt{\pi}} \left\{ 2\text{Im}\left(\frac{1}{\zeta_0 - 1}\right) + (1 + \nu')\text{Im}\left(\zeta_0 + \frac{1}{\zeta_0}\right)\text{Re}\left[\frac{\zeta_0^2}{(\zeta_0 + 1)(\zeta_0 - 1)^3}\right] \right\}, \quad (10)$$

where K_I is the stress intensity factor of mode-I crack closure, ζ_0 is given by $Z_0 = \frac{a}{2} \left(\zeta_0 + \frac{1}{\zeta_0} \right)$; $\nu' = \frac{\nu}{1-\nu}$ where ν denotes Poisson's ratio of matrix; $\text{Re}()$ and $\text{Im}()$ denote real component and imaginary component of $()$, respectively. Recovery stress, σ_f can be calculated numerically from Eq. 8 under a temperature, T or analytically from Eq. 9. Substituting recovery stress, σ_f into Eq. 10, we have the stress intensity factor, K_I of mode-I crack closure. Then we have relation of the stress intensity factor, K_I of mode-I crack closure with temperature, T .

4 Numerical Results

Material properties of SMA fiber are listed in Table 1, where E_f denotes Young's modulus; ε_L denotes maximum residual strain; A_s denotes austenite start temperature; A_f denotes austenite finish temperature; Θ denotes thermoelastic modulus; C_A denotes influence coefficient of stress on transition temperature. Poisson's ratio of matrix is $\nu = 0.3$.

Figure 3 shows curve of recovery stress, σ_f versus temperature, T . It is shown that recovery stress, σ_f reaches about 600 MPa as temperature rises to 70°C. Figure 4 shows martensitic volume fraction, ξ as a function with respect to temperature, T in phase transformation in Figs. 5, 6, 7, 8 and 9. It is shown that martensitic volume fraction, ξ decreases as the temperature rises from 40 to 70°C.

Fig. 4 Martensitic volume fraction, ξ versus temperature, T

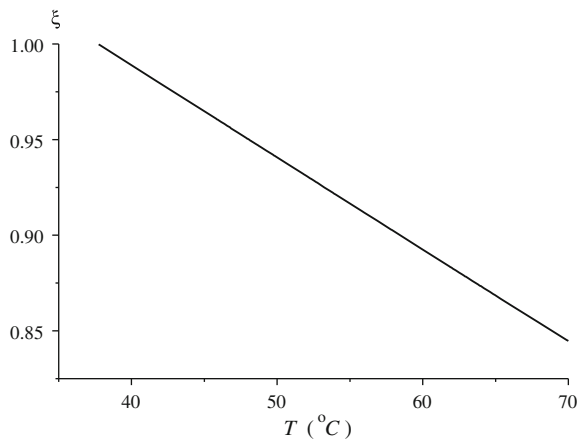


Fig. 5 The closure stress intensity factor, K_I versus temperature, T for different fiber position as $h/a = 1.0$

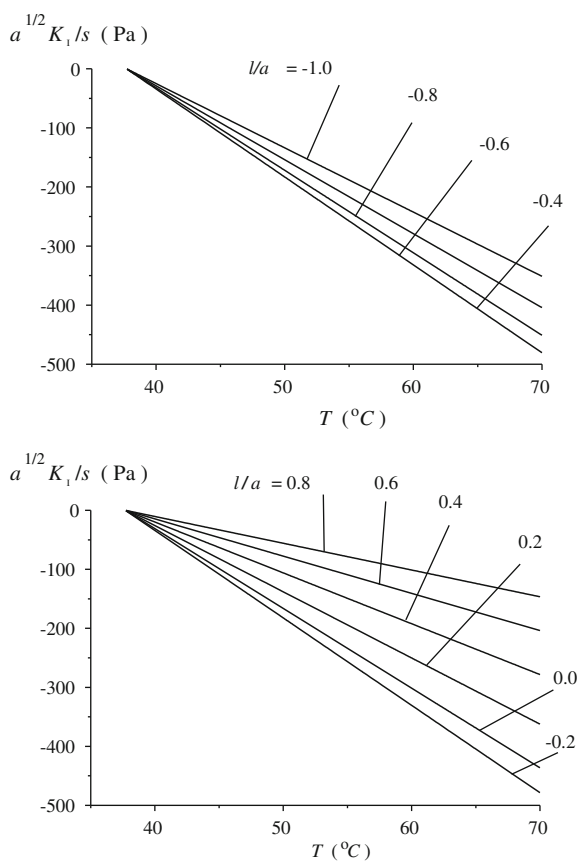


Fig. 6 The closure stress intensity factor, K_I versus fiber position, l/a at different temperature as $h/a = 1.0$

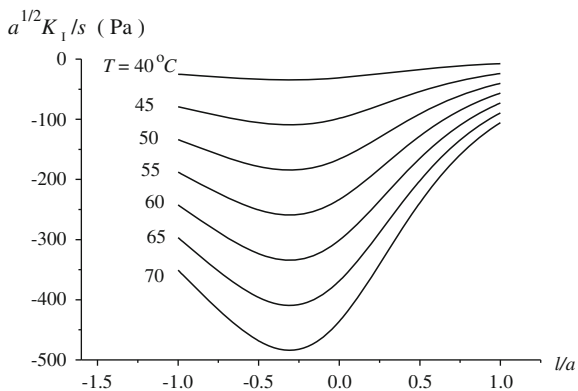


Fig. 7 The closure stress intensity factor, K_I versus fiber position, l/a at different temperature as $h/a = 0.1$

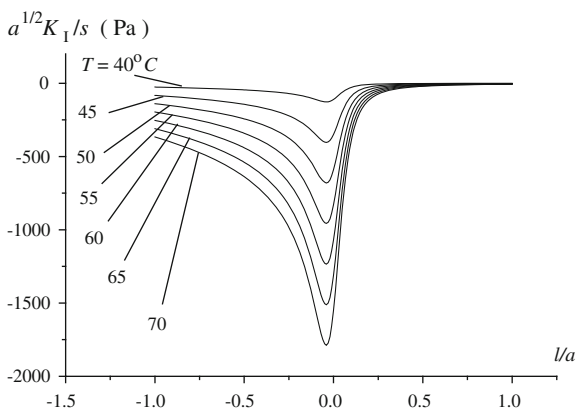
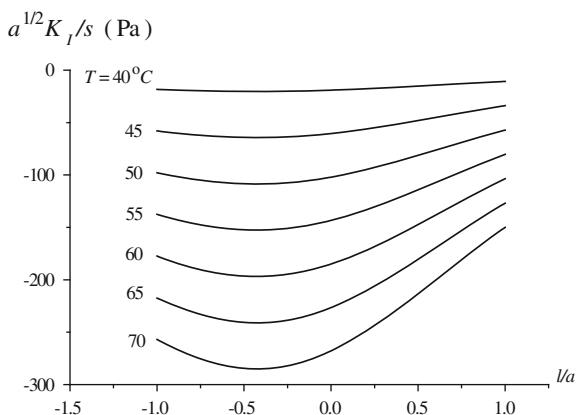
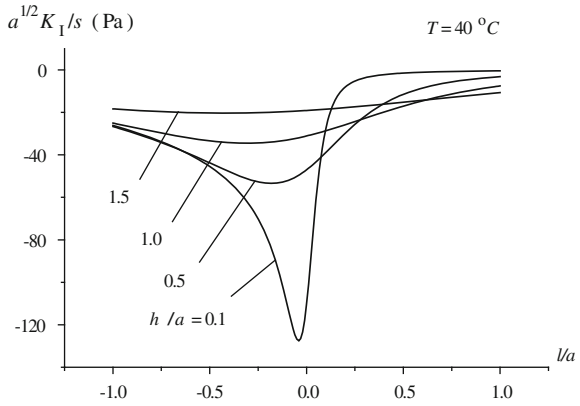


Fig. 8 The closure stress intensity factor, K_I versus fiber position l/a at different temperature as $h/a = 2.0$



As crack length is the same as fiber length, i.e. $\frac{h}{a} = 1.0$, Fig. 5 shows the stress intensity factor of mode-I crack closure as function with respect to temperature for different fiber position. It is shown that stress intensity factor decreases almost

Fig. 9 The closure stress intensity factor, K_I versus fiber position l/a at different fiber length



linearly with rise in temperature. It is shown that stress intensity factor may take the minimum value at the same temperature between $\frac{l}{a} = -0.2$ and -0.4 . Figure 6 shows closure stress intensity factor as a function with respect to fiber position at different temperature. At the same temperature, the nearer the fiber locates the crack tip, the little the stress intensity factor is. Effect of crack closure of the fiber behind crack tip is more obvious than that of the fiber in front of crack tip. It is also shown that there is an optimal position for fiber where the stress intensity factor takes the minimum value. For $\frac{h}{a} = 1.0$, this optimal position for effect of crack closure is approximately at $\frac{l}{a} = -0.3$ behind the crack tip. Figures 7 and 8 show the stress intensity factor of mode-I crack closure as a function with respect to fiber position at different temperature for $\frac{h}{a} = 0.1$ and 2.0, respectively. Approximately $\frac{l}{a} = -0.04$ and -0.45 are the optimal position for effect of crack closure for $\frac{h}{a} = 0.1$ and 2.0, respectively.

Figure 9 shows the closure stress intensity factor as a function with respect to fiber position $\frac{l}{a}$ for different fiber length $\frac{h}{a}$ at temperature 40°C. Near the crack tip, it is shown that, the more short distance of the two knots is, the more obvious the effect of crack closure is. In other words, the nearer the knot locates the crack tip, the more obvious effect of crack closure is.

5 Conclusions

Based on the Tanaka one-dimensional constitutive law on SMA, an approximate model on stress intensity factor of mode-I crack closure by prestrained knotted SMA fiber in martensite is proposed. Relation of the stress intensity factor of mode-I crack closure with temperature is given in Eqs. 8 and 9. Numerical results show that the stress intensity factor of mode-I crack closure increases as the prestrained knotted SMA fiber in martensite is heated. Optimal position of the knotted SMA fiber for crack closure is behind the crack tip. As an actuator in

matrix, prestrained knotted SMA fiber in martensite can be used to control mode-I crack opening.

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Appendix A

From derivative of Eq. 7 with respect to temperature, T , we have

$$\frac{\delta\sigma_f}{\delta T} = \frac{1}{b_A} \left(\frac{\delta\zeta}{\zeta\delta T} + a_A \right). \quad (\text{A1})$$

Substituting Eqs. 7 and (A1) into 8, we have

$$\left(\frac{1}{\zeta} - b_A\Omega \right) \delta\zeta = (b_A\Theta - a_A) \delta T. \quad (\text{A2})$$

Indefinite integral of Eq. A2 is

$$T = \frac{\ln \zeta - b_A\Omega\zeta}{b_A\Theta - a_A} + c. \quad (\text{A3})$$

where c is a constant. Substituting $\sigma_f|_{T=A_s} = 0$ into Eqs. 7 and A3, we have

$$c = \frac{b_A\Omega}{b_A\Theta - a_A} + A_s. \quad (\text{A4})$$

Substituting Eq. A4 into A3, we have the first equation in Eq. 9. The second equation in Eq. 9 can be got easily from Eq. 7.

Appendix B

Complex stress functions $\Phi(\zeta)$ and $\chi(\zeta)$ of an elliptic hole in plane strain under a point load P at Z_0 in Fig. 2. are given in Eqs. 4 and 5 in article of [23], respectively. As perpendicular axis of the elliptic hole tends to zero, we have derivatives of $\Phi(\zeta)$ and $\chi(\zeta)$ with respect to ζ

$$\frac{d\Phi(\zeta)}{d\zeta} = \frac{iPa^2}{8\pi} \left\{ (3 - \nu') \left(\frac{1}{\zeta - \frac{1}{\zeta_0}} - \frac{1}{\zeta} \right) + (1 + \nu') \left[\frac{1}{\zeta - \zeta_0} + \frac{\left(\bar{\zeta}_0 + \frac{1}{\zeta_0} \right) - \left(\zeta_0 + \frac{1}{\zeta_0} \right)}{\left(\bar{\zeta}_0^2 - 1 \right) \left(\zeta - \frac{1}{\zeta_0} \right)^2} \right] \right\} \quad (\text{B1})$$

and

$$\frac{d\chi(\zeta)}{d\zeta} = \frac{iPa^2}{8\pi} \left\{ \frac{3-v'}{\zeta-\zeta_0} + (1+v') \left[\frac{1}{\zeta-\frac{1}{\bar{\zeta}_0}} - \frac{1}{\zeta} + \frac{\left(\bar{\zeta}_0 + \frac{1}{\zeta_0}\right) - \left(\zeta_0 + \frac{1}{\bar{\zeta}_0}\right)}{(\zeta-\zeta_0)^2 \left(1-\frac{1}{\zeta_0^2}\right)} \right] \right\}, \quad (\text{B2})$$

where i is imaginary unit; ζ and ζ_0 are given by $Z = \frac{a}{2} \left(\zeta + \frac{1}{\bar{\zeta}} \right)$ and $Z_0 = \frac{a}{2} \left(\zeta_0 + \frac{1}{\bar{\zeta}_0} \right)$, respectively; $2a$ is the crack length; $v' = \frac{v}{1-v}$ where v denotes Poisson ratio of matrix; and $\bar{\zeta}_0$ denotes conjugate of ζ_0 . Perpendicular normal stress σ_y at Z in matrix under point load P at Z_0 in Fig. 2 is

$$\sigma_y = \frac{2\zeta^2}{a(\zeta^2 - 1)} \text{Re} \left(\frac{d\Phi}{d\zeta} + \frac{d\chi}{d\zeta} \right), \quad (\text{B3})$$

where $\text{Re}()$ denotes real component of $()$.

The closure stress intensity factor under point load P at Z_0 in Fig. 2 is

$$K_{I+} = \lim_{x \rightarrow 0} \sqrt{2\pi x} \sigma_y|_{y=0}, \quad (\text{B4})$$

Substituting Eqs. B1, B2 and B3 into B4, we have expression of closure stress intensity factor K_{I+} . Similarly, we have expression of closure stress intensity factor K_{I-} under point load $-P$ at $\bar{Z}_0$ in Fig. 2. Expression of the closure stress intensity factor K_I under point load P at Z_0 and $-P$ at $\bar{Z}_0$ together in Fig. 2 can be obtained by

$$K_I = K_{I+} + K_{I-} \quad (\text{B5})$$

Substituting Eq. 2 into Eq. B5, we have expression of K_I in Eq. 10.

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