

# Contents

<b>Adaptive Wavelet Methods</b>	1
Silvia Bertoluzza	
1 Introduction	1
2 Multiresolution Approximation and Wavelets	2
2.1 Riesz Bases	2
2.2 Multiresolution Analysis	3
2.3 Examples	9
2.4 Beyond $L^2(\mathbb{R})$	17
3 The Fundamental Property of Wavelets	21
3.1 The Case $\Omega = \mathbb{R}$ : The Frequency Domain Point of View vs. the Space Domain Point of View	22
4 Adaptive Wavelet Methods for PDE's: The First Generation	34
4.1 The Adaptive Wavelet Collocation Method	37
5 The New Generation of Adaptive Wavelet Methods	40
5.1 A Posteriori Error Estimates	41
5.2 Nonlinear Wavelet Methods for the Solution of PDE's	46
5.3 The CDD2 Algorithm	48
5.4 Operations on Infinite Matrices and Vectors	51
References	54
<b>Heterogeneous Mathematical Models in Fluid Dynamics and Associated Solution Algorithms</b>	57
Marco Discacciati, Paola Gervasio, and Alfio Quarteroni	
1 Introduction and Motivation	57
2 Variational Formulation Approach	67
2.1 The Advection–Diffusion Problem	67
2.2 Variational Analysis for the Advection–Diffusion Equation	68
2.3 Domain Decomposition Algorithms for the Solution of the Reduced Advection–Diffusion Problem	72
2.4 Numerical Results for the Advection–Diffusion Problem	77

2.5	Navier–Stokes/Potential Coupled Problem .....	80
2.6	Asymptotic Analysis of the Coupled Navier–Stokes/ Darcy Problem .....	82
2.7	Solution Techniques for the Navier–Stokes/Darcy Coupling .....	85
2.8	Numerical Results for the Navier–Stokes/Darcy Problem .....	90
3	Virtual Control Approach .....	94
3.1	Virtual Control Approach Without Overlap for AD Problems .....	95
3.2	Domain Decomposition with Overlap .....	105
3.3	Virtual Control Approach with Overlap for the Advection–Diffusion Equation .....	108
3.4	Virtual Control with Overlap for the Stokes–Darcy Coupling .....	114
3.5	Coupling for Incompressible Flows .....	119
	References .....	120
	<b>Primer of Adaptive Finite Element Methods</b> .....	125
	Ricardo H. Nochetto and Andreas Veiser	
1	Piecewise Polynomial Approximation .....	125
1.1	Classical vs Adaptive Pointwise Approximation .....	126
1.2	The Sobolev Number: Scaling and Embedding .....	127
1.3	Conforming Meshes: The Bisection Method .....	129
1.4	Finite Element Spaces .....	133
1.5	Polynomial Interpolation in Sobolev Spaces .....	134
1.6	Adaptive Approximation .....	139
1.7	Nonconforming Meshes .....	143
1.8	Notes .....	145
1.9	Problems .....	146
2	Error Bounds for Finite Element Solutions .....	148
2.1	Model Boundary Value Problem .....	148
2.2	Galerkin Solutions .....	149
2.3	Finite Element Solutions and A Priori Bound .....	150
2.4	A Posteriori Upper Bound .....	151
2.5	Notes .....	157
2.6	Problems .....	158
3	Lower A Posteriori Bounds .....	159
3.1	Local Lower Bounds .....	160
3.2	Global Lower Bound .....	166
3.3	Notes .....	167
3.4	Problems .....	168
4	Convergence of AFEM .....	170
4.1	A Model Adaptive Algorithm .....	171
4.2	Convergence .....	172
4.3	Notes .....	178
4.4	Problems .....	179

5	Contraction Property of AFEM .....	180
5.1	Modules of AFEM for the Model Problem .....	180
5.2	Basic Properties of AFEM .....	182
5.3	Contraction Property of AFEM .....	185
5.4	Example: Discontinuous Coefficients .....	189
5.5	Extensions and Restrictions .....	191
5.6	Notes .....	193
5.7	Problems .....	193
6	Complexity of Refinement .....	194
6.1	Chains and Labeling for $d = 2$ .....	195
6.2	Recursive Bisection .....	197
6.3	Conforming Meshes: Proof of Theorem 1 .....	199
6.4	Nonconforming Meshes: Proof of Lemma 3 .....	204
6.5	Notes .....	205
6.6	Problems .....	206
7	Convergence Rates .....	206
7.1	The Total Error .....	207
7.2	Approximation Classes .....	208
7.3	Quasi-Optimal Cardinality: Vanishing Oscillation .....	212
7.4	Quasi-Optimal Cardinality: General Data .....	215
7.5	Extensions and Restrictions .....	218
7.6	Notes .....	221
7.7	Problems .....	221
	References .....	223

## **Mathematically Founded Design of Adaptive Finite Element**

<b>Software</b> .....	227
Kunibert G. Siebert	
1 Introduction .....	227
1.1 The Variational Problem .....	229
1.2 The Basic Adaptive Algorithm .....	230
2 Triangulations and Finite Element Spaces .....	232
2.1 Triangulations .....	232
2.2 Finite Element Spaces .....	234
2.3 Basis Functions and Evaluation of Finite Element Functions .....	240
2.4 ALBERTA Realization of Finite Element Spaces .....	244
3 Refinement By Bisection .....	246
3.1 Basic Thoughts About Local Refinement .....	246
3.2 Bisection Rule: Bisection of One Single Simplex .....	248
3.3 Triangulations and Refinements .....	252
3.4 Refinement Algorithms .....	255
3.5 Complexity of Refinement By Bisection .....	260
3.6 ALBERTA Refinement .....	262
3.7 Mesh Traversal Routines .....	263

4	Assemblage of the Linear System .....	268
4.1	The Variational Problem and the Linear System .....	269
4.2	Assemblage: The Outer Loop .....	272
4.3	Assemblage: Element Integrals .....	276
4.4	Remarks on Iterative Solvers .....	283
5	The Adaptive Algorithm and Concluding Remarks .....	285
5.1	The Adaptive Algorithm .....	286
5.2	Concluding Remarks .....	294
6	Supplement: A Nonlinear and a Saddlepoint Problem .....	297
6.1	The Prescribed Mean Curvature Problem in Graph Formulation .....	297
6.2	The Generalized Stokes Problem .....	301
	References .....	308
	<b>List of Participants .....</b>	<b>311</b>

Multiscale and Adaptivity: Modeling, Numerics and  
Applications

C.I.M.E. Summer School, Cetraro, Italy 2009

Bertoluzza, S.; Nochetto, R.H.; Quarteroni, A.; Siebert,  
K.G.; Veese, A. - Naldi, G.; Russo, G. (Eds.)

2012, XII, 314 p. 72 illus., 24 illus. in color., Softcover

ISBN: 978-3-642-24078-2